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**ON LEIBNIZ'S  
PHILOSOPHICAL LEGACY**  
in the 350th Anniversary of His Birth

THE CHAIR OF LOGIC, INFORMATICS AND PHILOSOPHY OF SCIENCE  
UNIVERSITY OF BIAŁYSTOK  
Białystok 1997

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edited by:

The Chair of Logic, Informatics

and Philosophy of Science

University of Białystok

ul. Liniarskiego 4

15-420 Białystok

e-mail: logika@hum.uw.bialystok.pl

tel. (085) 457510

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WYDAWNICTWO UNIwersYTETU W BIAŁYMSTOKU

15-097 Białystok, ul. M. Skłodowskiej-Curie 14, tel. (085) 457059, 457058

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Introduction to this Volume  
by Witold Marciszewski  
**WHY SHOULD WE READ LEIBNIZ**  
in the 350th Anniversary of His Birth?

There is no reason that every philosopher read every philosophical classic. Leibniz, though, should be read by quite a many, at least those involved in the intellectual foundations of information society. It was he who saw the universe as an immense system of information machines. Such a philosophical approach and climate is found in the *Chair of Logic, Informatics and Philosophy of Science* – the team which most contributed to this book.

The dates as that celebrated in 1996 – the 350th Anniversary of Leibniz's birth – provide a special opportunity to reflect on his topicality. This volume (though appearing a year later) is to hint at some Leibniz' ideas which retained their vitality to our days. The present choice is very modest, indeed, for limitations of the *Studies in Logic, Grammar and Rhetoric* (recently transformed in a book series after some years of functioning as a periodical).

This series is a forum to display the research carried out in the said Chair; it proves a fitting device to forward its studies. Guest contributions are very welcome, but those of the local group are likely to prevail.

The first three contributions deal directly with Leibnizian issues, and the remaining are variously related, even if indirectly, to the main subject.

\* \* \*

Halina Świączkowska in the essay *Language as the Mirror of the Mind* offers a substantial motive to read Leibniz in our days. In AI and cognitive science, the vivid debate between symbolism and connectionism corresponds to a significant problem with Leibniz.

Leibniz proves a symbolist, namely the one who postulated a universal

system of symbols (*Characteristica Universalis*) to precisely mirror and enhance a system of thoughts. Thereby, Leibniz would have endorsed the recent notion of LOT (*Language Of Thought*), to which the behavioristic approach was so hostile. However, with Leibniz it is not clear whether every natural language should adequately mirror a system of thought or just a specially elaborated perfect language would match that system. Ms. Świączkowska's contribution consists in discovering and discussing two kinds of Leibniz's explicit statements, opposing each other in that matter.

\*

There is yet a more fundamental split in Leibniz's views on language, as discussed by Witold Marciszewski in his *Leibniz's Idea of Automated Reasoning Compared with Modern AI*. If we take Hilbert's programme for comparison, then Leibniz appears both as an eager forerunner of that programme and its eager opponent.

He represented a Hilbert-like approach when postulated algorithms to solve any problem whatever – if duly formalized, that is, stated in a manner as precise as should have been enabled by his *Characteristica*. On the other hand, in *Monadology*, he claimed infinite complexity of organic machines constituting the universe, and that should have produced problems to be hardly solvable in finite sequences of steps.

\*

Adam Drozdek's (Duquesne University, USA) paper *Leibniz: Struggles with Infinity* is an encouraging example of international cooperation due to *Studies in Logic* [etc]. It refers to a question raised in a previous volume (1993/94), that of reconciling Leibnizian finitism (as mentioned above) with infinity of nature.

Drozdek stresses the enormous role of infinity in Leibniz's outlook and research, and thoroughly examines approaches to continuum as found in his numerous texts. The very term 'continuum' is taken in its Leibnizian sense, proving vague enough when compared with its counterpart in modern set theory. The discussion offered by Drozdek encourages to attack the problem again, in the light of modern distinctions between infinities, while at its present stage it brings thought-provoking insights into some intricacies of Leibniz's thought.

\*

In the same year, there is a historical reason to commemorate Descartes as well, to wit the 400th anniversary of his birth. There are also reasons

to discuss his approach to science and to the universe in the context of commemorating Leibniz. In some points, understanding Leibniz does profit from understanding Descartes.

A reason of special import is inquired by Jerzy Kopania in the essay *Descartes' Great Thesis on Nature*. The author – a historian of philosophy with a rich logical background – avails himself of basic logical concepts to reconstruct Descartes' view on the structure of scientific theories, i.e., those concerning the material universe.

That view results in what is by the author called *the Great Thesis on Nature*. It is to the effect that *rational inquiry into nature cannot lead beyond it*, and is substantiated by the thesis of material homogeneity of nature, which means that material effects should be explained by material causes alone; and this, in turn, follows from the idea of extension as exhausting the essence of matter (hence no other factors, eg. non-material forms, as claimed by the Schoolmen, are needed to explain physical phenomena).

To fully understand this great thesis, the thesis complementary to it has to be considered – as recalled at the end of Kopania's essay – to wit: rational inquiry into *the mind* leads beyond it, to the transcendent mind as the cause of that of ours.

All that provides us with an excellent contrastive background to grasp the great thesis of Leibniz. Let it put in a nutshell, using the modern notion of code, or software. The thesis runs as follows: the inquiry into nature reveals that beyond extension there is a software, and that requires a mind as its author; and since matter is infinite as dividing into ever deeper and tinier structures, the corresponding software requires the infinite mind.

This is the very essence of Leibnizianism, namely – let us repeat – the combining of both software and structural infinity as attributes of matter, both denied by Descartes (his infinity of matter was purely geometrical, one could cut a body in an arbitrary way). Needless to say how close are these ideas to modern science in which some behaviour of matter has to be explained, eg, with recourse to genetic code, and the structural infinity of ever tinier particle structures is seriously considered by scientists.

Such insights into Leibniz are available just through those insights into Descartes which we owe to Kopania's penetrating examination. This is why his essay so nicely fits into this volume.

\*

Now, let us think about Leibniz's dream of reasoning machines. He believed in practicability of such a project, encouraged by the success of his calculating machine as well as theoretical considerations concerning

the closest similarity between computing and reasoning. This dream is being materialized owing to computer programs called provers. Ms. Anna Zalewska's paper *A Criterion of Decidability of some Algorithmic Formulas* is based on a work to result in a prover which she constructed for Salwicki's algorithmic logic (included in a broader category, called logic of programs).

The construction of a prover required the adjusting of algorithmic logic to constraints of automated proving. For this modified system, the author states a criterion of decidability which makes it possible to automatically prove validity of algorithmic logic formulas.

\*

Algorithmic logic appears again in two communications completing each other: *The Norms from the Point of View of a Certain Logic of Programs* by Anna Zalewska, and *Norms and Programs* by Andrzej Malec. The former is more concerned with logical foundations, the latter with legal applications. The authors take advantage of the fact that both a legal norm and a computer program transform an existing situation into a postulated situation. Such formal analogies make it likely that each member of the pair would profit from a joint development.

This idea is only sketched in the said texts, but even in a sketchy form it proves its belonging to the Leibnizian legacy. As for algorithmic logic, its relation to the great Leibniz's project of an automatic prover is mentioned above. As to legal issues, their approaching from a logical point was Leibniz's concern as well (as shown in his *De casibus perplexibus in lege*). Though he was far from algorithmic approach to the law, the first step in this direction was made in his precisizing the language of that discipline.

\*

If we look for most significant points of Leibniz philosophical creed, then – besides the great thesis on the ubiquity and infinity of software – we encounter his radically deterministic approach. At the same time, we observe how much attention he paid to the notions of space and time.

The last two items in the volume are concerned with logical connexions between determinism and conceptions of time. These are: *Some Remarks about Intuitionistic Tense Logic* by Dariusz Surowik, and *Omniscience, Omnipotence and Related Notions* by Kazimierz Trzęsicki. Their role for understanding Leibniz is a bit similar to that played by the study of Descartes as considered above, namely that of a contrastive background.

Both authors develop a version of *indeterminism* which goes back to Jan Łukasiewicz. However, in spite of the fact that Łukasiewicz himself provided

a logic to precisely state indeterministic insights, namely his multi-valued logic, our authors prefer other logical devices: those of tense logic (created after Łukasiewicz). Surowik combines them with intuitionistic logic (thus resorting to multi-valuedness but not in Łukasiewicz's style). Trzęsicki develops some insights concerning the notion of freewill; these are based on thorough historical erudition, and have a formal logic of tenses in their background.

\* \* \*

Let me sum up with a more general comment on the approaches found in this volume. They may resemble what is called *Whig interpretation of history*. This notion was used by Volker Peckhaus (University of Erlangen), a thorough historian of logic, in a review of him concerning W. Marciszewski's and R. Murawski's *Mechanization of Reasoning in a Historical Perspective* (Amsterdam 1995, Rodopi). V. Peckhaus does not share this approach, and he has good reasons for that. This is why a comment on this volume's intentions will be in order.

When applied to the history of ideas, Volker's expression is like a metaphor, since literally it refers to political history. First it was used by H. Butterfield in his *The Whig Interpretation of History* (London 1931). However, it is a fitting metaphor if, for example, one refers to Jan Łukasiewicz's programme for history of logic, followed by quite a number of logic historians.

In his book, Butterfield examined critically the tendency of historians to see the past as the story of the conflict between progressives and reactionaries, in which the progressives, or Whigs, win and bring about the modern world. For Łukasiewicz, for instance, Stoic logic was more 'progressive' than Aristotelian logic since in our times the former has proved more general and more fundamental.

In this volume, to take a most recent example, J. Kopania presents Descartes as a 'progressive' in methodology of natural science, contrasting his attitude with that of the Schoolmen, while the present author in his comment to Kopania's contribution suggests that it is Leibniz who proves more 'progressive' (in some respect, at least).

To hint at a rational core of the 'Whiggish interpretation', let me first observe that it proves more reasonable in the history of science than in the political history. For the former is a cumulative process in which previous achievements contribute to later ones. Nevertheless, a caution is needed.

We should cautiously distinguish between a *historical reconstruction of the past* and what may be called a *diachronic comparative research*. In the

volume commented, it is the study by Świączkowska which is closest to the former (though not without a modern perspective) while Marciszewski's approach exemplifies the latter.

In the latter one does not claim that, for instance, Leibniz's projects belonged to a causal chain to result in modern computer science. Instead, one compares two systems of ideas, distant in time, to recognize their logical relations. Once having done so, one can ask whether logical relations have influenced the actual progress or have not. The mere fact of logical connexions does not yield any historical answer, it just may assist a better understanding of the concepts to be used in a genuine historical research.

The Editors of this volume hope that it does contribute to such a logical and philosophical enterprise.

Halina Świączkowska

## LANGUAGE AS THE MIRROR OF THE MIND

The Leibnizian attitude toward language is most fully expressed in the conviction that language is the finest mirror of the mind (UG, §1, NE, III, vii, §6) and therefore the analysis of the substance of language should thus lead to a recognition of the mechanisms of the process of thinking itself. Language capacity, in a like manner as the capacity to think, is a function of the same intensity and the same driving force which differentiates Man from other creatures. If this interpretation is on target, it leads us to the conclusion that any natural language system may be the subject of analysis. This results from the fact that every language "reflects" the natural order of ideas which potentially belongs to the intellectual equipment of all substances capable of reflecting about themselves as "Me". (GP, II, 53; L, 237).

Indeed, this conclusion finds confirmation in many statements made by Leibniz regarding the relation between language and thinking and the knowledge accumulated in the language. The following is a fragment of *Analysis Linguarum* of 11 September, 1678 in which Leibniz writes:

"Although there are many human languages, all of them sufficiently developed to be suitable for the transmission of any science whatsoever, it is enough, I think, to consider one language: any nation can in fact make discoveries and direct the sciences in its own backyard." (C, 252; Dascal, 1987, 162).

Consequently, independent of the level of civilizational development of a given language society, it is capable of making discoveries and registering in the language the intellectual process whose aim is to broaden knowledge. Such thesis indeed lies at the basis of classical social anthropology.<sup>1</sup> Despite the fact that language studied in its historical perspective registers, as Leibniz acknowledges, the history of our discoveries, which is reflected in the example of those "who as Copernicans continue to say that the sun rises and sets" (GP, IV, 459; L, 320); this does not impair Leibniz's conviction

<sup>1</sup> See Malinowski, 1931.



that “languages are the best mirror of the human mind, and that a precise analysis of the significations of words would tell us more than anything else about the operations of the understanding”. (NE, III, vii, §6)

The motif of the mirror image appears in a number of Leibniz’s texts. Let us consider how the relationship of reflection is to be understood and what is the role assigned to the “mirror” itself. It may be assumed that signs as well as language are essential to the understanding of the functioning of the mind only when they are the basis of some direct method or when they are the only accessible means of reaching the contents of the mind itself. And also that the mind reflects itself in a mirror, even when this is the only means of learning anything about its nature. This does not lead us to the assumption that this mirror plays a role in the functioning of the mind itself. It seems that Leibniz has gone far from such superficial formulation of the relationship between signs and thinking.<sup>2</sup> Language in Leibniz is integrally connected with the mind because according to him:

“all human reasoning is performed by means of certain signs or characters. Indeed, it is neither possible nor desirable that the things themselves or even the ideas of them be always distinctly observed by the mind. So, for reasons of economy, signs are used for them”. (GP, VII, 204; Dascal, 1987, 181)

Matters connected with how language involves itself in thinking and what functions are assigned by Leibniz in this process will be considered in a separate study. Below we concentrate on the concept of expression which is fundamental for Leibniz.

## Expression

The relation of reflection becomes clear in the context of another, wider concept – “representation” or “expression” – constituting the core of Leibnizian metaphysics. Let us note that: “each created monad represents the whole universe” (GP, VI, 620; L, 649) and “each monad is a living mirror, or a mirror endowed with an internal action, and (...) it represents the universe according to its point of view and is regulated as completely as is the universe itself”. (GP, VI, 598; L, 637) because, since monads are, essentially, closed to any external influence in the process of representing the world, they turn to the content of their interior, finding there the ideas of things.

“Expression is common to all the forms and is a genus of which natural perception, animal feeling, and intellectual knowledge are species. In natural perception and feeling it suffices that what is divisible and material and is found dispersed among several beings should be expressed or represented in a single indivisible being or in a substance which is endowed with a true unity. The possibility of such a representation of several things in one cannot be doubted, since our soul provides us with an example of it. But in the reasonable soul this representation is accompanied by consciousness, and it is than that it is called thought.” (GP, II, 112; L, 339)

In Leibniz the notion of expression is a technical concept; therefore, it needs some clarification. In his essay *What is an Idea*, Leibniz explains that, what is common to different forms of expression is that from the learning of the relations which it expresses we may achieve knowledge of properties corresponding to the thing expressed. The idea is what is in the mind. But as Leibniz writes:

“There are many things in our mind, however, which we know are not ideas, though they would not occur without ideas – for example, thoughts, perceptions, and affections. In my opinion, namely, *an idea consists, not in some act, but in the faculty of thinking*, and we are said to have an idea of a thing even if we do not think of it, if only, on a given occasion, we can think of it.” (GP, VII, 263-64; L, 207)

According to Nicolas Jolley, Leibniz reduces the discussion of ideas to the discussion of the ways of thinking due to formulation of ideas as disposition to think in a given way.<sup>3</sup> The idea assumes therefore a certain closer ability or ease of thinking about a thing, the idea of which we have inside, although it not a thought itself. This ability must lead however not only to thinking about the thing, but it has to also express this object. In *Discourse on Metaphysics*, Leibniz develops this concept by writing:

“As a matter of fact, our soul always does have within it the disposition to represent to itself any nature or form whatever, when an occasion arises for thinking of it. I believe that this disposition of our soul, insofar as it expresses some nature, form, or essence, is properly the idea of the thing, which is in us and is always in us whether we think of it or not.” (L, 320)

The means of expressing things is based on the analogy of relations between a thing and what constitutes its expression. And therefore “speech expresses thoughts and truths, characters express numbers, and an algebraic equation expresses a circle or some other figure”. (GP, VII, 263-64; L, 207) Resemblance is of no need here, for even if the idea of a wheel would not resemble a wheel, it is still possible to elicit from it the truth about the wheel itself.

<sup>2</sup> See Dascal, 1987, Foreword, X.

<sup>3</sup> See Jolley, 1990, 162.

The mind therefore thinks about a thing at that moment when its idea is formulated by it in a certain way. A question arises: does such interpretation assume thinking only at the level of consciousness? The idea of a thing summoned by the mind leads however, as Leibniz says, to thinking about the thing itself under one condition: when simultaneously the idea somehow expresses that thing. If an idea is a disposition to think in a particular mode, then these modes of thinking about a thing are connected with a set of possible forms for the expression of a given thing. Indeed the concept of the idea of the thing seems to be in Leibniz integrally connected with a certain skill of expression. If we however are not able to “grasp” the idea of a thing in any form, does this mean that we do not possess this idea? Leibniz would strongly deny this. He would presumably answer that this idea, although present in the mind, is still in a state of “slumber”, is potential, and as such contains all possible forms of expressing things. This results from the fact that

“we have all these forms in our own minds, and even from eternity, for et every moment the mind expresses all its future thought and already thinks confusedly of everything of which it will ever think distinctly.” (L, 320)

It is just a matter of activating an appropriate cognitive process which will release the disposition to reach out for one of the forms of expression. In relation with the above, one can not assume that we are talking about ideas as dispositions to think about things, which is correlated with expressing them: it is always conscious thought. These dispositions may lead to bringing about their actualization and then they may be interpreted as active dispositions. This does not contradict the opinion that the mind of an infant is able to grasp for example the idea of Turing’s universal machine or express the quantum theory of gravitation. In the case of infants as well as the majority of people, these ideas are in the sphere of potentiality, and although it is not known if they will ever achieve disposition to express these ideas on the conscious level, still their minds are programmed to achieve this or any other disposition leading to the expression of idea of things in question. We find the confirmation of such interpretation in the theory of substance. For Leibniz assumes that a given disposition is the result of cooperation of the so-called “minute perceptions”, imperceptible impressions which undergo processing of the mind. (NE, II, i, §15) The disposition, finding a basis in the actions of the mind, leads to expression – the representation of things. Admittedly, the ability for representation is vested in all substances, since “every individual substance in its own manner expresses the universe”. (GP, I, 383-84). But, as we remember, in the thinking soul representation merges with consciousness and only then is it named thinking. (GP, II, 112;

L. 339) Such a formulation may lead however to the conclusion that the inner system of representation in any monad is the same system which is vested also in thinking beings – the difference lying only in the fact that the thinking being is able to recognize the system thanks to the ability to think. This conclusion however is false, on the grounds that only minds are vested in the natural system of representation, labeled by Leibniz the natural order of ideas. An indispensable condition of thinking is the existence of something which one may think about. This “something” is for Leibniz the idea itself. Animals, having no ideas, are unable to think. They do however have some system of representation other than ideas, because their souls, just like other monads, express the world.<sup>4</sup>

### Ideas as dispositions

Leibniz’s theory of ideas is interpreted as a disposition theory. Such a manner of interpretation is indicated by numerous statements made by Leibniz, for example the one that ideas and truths are innate in us – as inclinations, dispositions, tendencies or natural potentialities. (NE, Preface, 52). The human mind by its nature contains the basis of certain concepts and theories only to be awakened in appropriate conditions by outside objects. (NE, Preface, 49) Inborn ideas are inborn in the way that the mind draws them from itself; this does not mean that every mind begins as if from the whole stock of ideas and innate truths, or even, that every mind ever reaches the clear knowledge of all those truths which can be drawn from it. According to Leibniz however, there is no such barbarian who, in a matter important to him, would not reflect on the behavior of a liar contradicting himself. (NE, I, i, §4) But such an almost instinctive use of the principle of contradiction does not mean that everyone knows this principle out of necessity, it means only that it is potentially innate, as are other mathematical truths. The proof of learning these truths is to become aware of them and the adequate expression of them. This potential character of innate ideas assumes a parallel invariability of the equipping of the human mind with the invariability of the whole human race. It is also suggested that aboriginal man had at his disposal the same set of truths and

<sup>4</sup> Leroy E. Loemker in the introduction to his translation of selected writings of Leibniz (L, 42) expresses the opinion that on the level of unconscious perceptions, Man basically does not differ from other creatures. Thus he accepts that obscure knowledge (*cognito obscura*) combined with access to unclear notions or ideas is vested also in animals. This however is in contradiction with the Leibnizian assumption concerning the lack of ideas in substances other than the mind.

ideas which also constitute the intellectual stock of people living today. But this conclusion contradicts Leibniz's principle of individualization. The same equipping of minds results in their lack of differentiation. Leibniz safeguards himself against this consequence by treating ideas as dispositional. This differentiation corresponds to the range of dispositions, and it is possible to treat every single disposition as building from a certain initial state which we may call predisposition that finds its grounding in the unconscious perceptions of the mind and may be reduced to them.<sup>5</sup> In accordance with this, though the mind of aboriginal man already had the predisposition to discover integral and differential calculus, only in Leibniz do we find that this predisposition has evolved into a full disposition.

Dispositional features of ideas are illustrated by Leibniz by the example of the marble block in which the form of Hercules already exists as if potentially contained within, although the sculptor has to make a big effort to extract it from the block. This allows us to assume that the mind possesses a certain disposition to find in itself such truths and principles for which it is initially programmed, just as the grain of the marble conditions the movement of the sculptor's chisel. We see here a fairly close analogy with the thesis of Descartes which says that certain ideas are inborn in the same way as certain family diseases, not because the "infants of these families suffer from these diseases in their mother's womb, but because they are born with a certain disposition or propensity for contracting them".<sup>6</sup>

### Implicit knowledge

Let us note that in the description of the thesis of disposition, the assumption of Leibniz fulfills a very important role concerning the implicit character of inborn features. The fact that the concepts are inborn *implicite* in the mind means that "[this] should signify only that the mind has a faculty for knowing them; (...) it has in addition a faculty for finding them in itself, and the disposition, if it is thinking properly, to accept them". (NE, I, i, §21). Subsequently, the ability of learning notions alone is something different than disposition. As Jolley notices, Leibniz assigns to the concept of implicit knowledge a different sense than to dispositional knowledge. Jolley interprets dispositional knowledge as

"express knowledge",<sup>7</sup> whereas implicit knowledge is knowledge containing, for example, axioms. "The statement that the body is greater than the trunk differs from Euclid's axiom only in that the axiom restricts itself to precisely what needs to be." (NE, IV, vii, §10) "Thus, we use these these maxims without having them explicitly in mind. It is rather like the way in which one has potentially in mind the suppressed premisses in enthymemes, which are omitted in our thinking of the argument as well as in our outward expression of it." (NE, I, i, §4)

Implicit knowledge is therefore, as Leibniz claims, united with the certain ability to learn it or gain it. However, does this ability always lead to its revelation, or does the mind have access to it? We are referring here to Leibniz's differentiation between truths of reason and truths of fact. Truths of reason are necessary statements in the sense that they are apparent statements or they are reduced to them. Among the truths of reason, Leibniz differentiates primary truths which he names "identities", "because they seem to do nothing but repeat the same thing without telling us anything". (NE, IV, ii, §1) Among them he distinguishes two categories: affirmative or negative. Assuming the implicit character of these principles, he describes the mechanism of affirmative statements such as: "an equilateral rectangle is a rectangle" or "a square is not a circle". Leibniz claims that these statements have features of innate truths because when considering them we implement the principle of identity (and the principle of contradiction respectively), "for in thinking it, one applies the principle of identity and the principle of contradiction to materials which the understanding itself provides." (NE, I, i, §18). Thus they constitute the exemplification of these innate principles and should not be treated as various truths, instead "regarding the axiom as embodied in the example and as making the example truth" (NE, IV, vii, §10). A child's acceptance of the statement "the mother is not the father" (meaning that it has the certain disposition to find this truth), consequently reveals the general logical principal of which the above statement is an example. The fact that a child has at its disposal certain implicit knowledge, which consists of, as Leibniz assumes, the principles of classical logic, is not sufficient however to its actualization. Thus it is doubtful if a child, even after long training, could achieve the ability to reveal these principles, though it uses them efficiently on the intuitive level. In other words a child does not possess the disposition to implement them consciously. Despite being devoid of this disposition, it cannot possibly to be able to reveal them.

<sup>5</sup> See Jolley, 1990, 162.

<sup>6</sup> R. Descartes, *Notes Against a Certain Programme*, trans. E. S. Haldane and G. R. T. Ross, in: *The Philosophical Works of Descartes*, Cambridge University Press, 1911, I, p. 442.

<sup>7</sup> See Jolley, 1984, 172.

By this interpretation, having implicit knowledge is not sufficient to be transformed into disposition connected with its expression. But we should also remember that Leibniz discussing ability, in particular the ability to learn, did not have in mind exclusively “the bare faculty”. As he has written:

“inactive faculties – in short, the pure powers (...) are (...) mere fictions, unknown to nature (...). For where will one ever find in the world a faculty consisting in sheer power without performing any act?” (NE, II, i, §2)

The ability to learn activates itself thanks to the potential characteristic of every mind which Leibniz calls effort (*conatus*, *appetition*). Ability joined with this power leads to a certain result – this result being the revelation of an appropriate disposition.

In *Meditations on Knowledge, Truth, and Ideas* (GP, IV, 422-26, L, 291-94) with reference to the division of ideas in Descartes, Leibniz conducts a detailed classification of ideas from the point of view of the role they play in cognition. The content of this treatise is also repeated in *New Essays...* in book II, Chapter 29-31. The starting point of this classification is the division of ideas – and the knowledge which corresponds to them – into obscure (*cognitio obscura*) and clear (*cognitio clara*). An idea is obscure when it is not sufficient to recognize the thing represented by it. We deal with this then when we consider “some term which the Scholastics had defined poorly, such as Aristotle’s *entelechy* (...) or other such terms of which we have no sure definition”. (L, 291) We may therefore assume that the mind remains in certain contact with such an idea as the representation of thing; however, this state of knowledge does not lead to the identification of thing. Obscure ideas can be interpreted on the ground of Leibniz’s metaphysics as perceptions which have not yet transformed themselves into *aperception*, although they may lead to it. Let us point out that Leibniz, accepting the division into obscure and clear ideas as a starting point, assumes at the same time that the mind always finds itself in a certain state of knowledge at which the lowest level marks obscure knowledge and not the lack of it. The mind therefore has to always possess from the beginning of its existence access to the ideas of things which is the consequence of the implicit character of these ideas. In the face of this we may assume that there exist many truths we have access to on the perception level, however the disposition to grasp them is lacking. For “the faculty of knowing innate notions” alone is only a prerequisite to having the “faculty for finding them in itself and the disposition to accept them”.

## Disposition and *aperception*

Leibniz clearly connects the ability of *aperception* with disposition when he writes that

“to be aware of what is within us, we must be attentive and methodical (...). For thoughts are actions whereas items of knowledge (or truths), in so far as they are within us even when we do not think of them, are tendencies or dispositions; and we know many things which we scarcely think about”. (NE, I, i, §25-26)

Undoubtedly, the only means of reaching the content of the mind is to direct it to its natural abilities. Leibniz illustrates this process adducing the Platonic dialogue *Menon* in which Socrates, guiding a boy by asking questions, helps him to discover in himself innate laws of geometry. (NE, I, i, §5) *Aperception* alone demands however a certain impetus or a whole series of stimuli which direct the mind to its content – thanks to, for example the properly formulated questions by Socrates. It does not seem probable however that the state of full consciousness of the laws of geometry could be achieved by an infant, and Leibniz did not want to accept “that every innate truth is known always and by everyone”. (NE, I, ii, §11) although cases occur of children reaching these and other laws.<sup>8</sup> He was protected from such formulation by the assumption concerning the potential and possible features of ideas.

Leibniz in a way confirms the above understanding of the process leading to *aperception* by writing that “innate maxims make their appearance only through the attention one gives to them” (NE, I, i, §27) Admittedly every man is equipped with the ability of reaching the innate set of ideas, but the minds of children and savages on which Leibniz bases his examples are less spoiled and corrupted by custom and less molded by the learning which shapes this attention. (NE, I, i, §27) At this point we need to again return to the notion of implicit knowledge. We may assume that this knowledge is vested in every mind. Supposing however it does not become the subject of reflection of the mind – the mind will not activate the disposition to reveal it; this means that it will remain in the area of those activities of the mind which are responsible for thinking on the level of unconsciousness. We are speaking here of activating the disposition, bearing in mind the fact that dispositions to act are based in Leibniz on a whole series of minute perceptions which cause “the mind has a disposition (as much active as passive) to draw (truths) from its own depths”. But for this to happen certain stimuli are needed “to give the mind the opportunity

<sup>8</sup> Example supplied by Leibniz in NE, I, i §55.

and the attention for this, and to direct it toward certain necessary truths rather than others". (NE, I, i, §5) Maybe therefore, implicit knowledge is interrelated with a series of predispositions or passive dispositions which properly stimulated allow the elevation to the state of consciousness. This is what Leibniz has in mind when he writes, "that the less one knows the closer one comes to sharing with blocks of marble and bits of wood the advantage of being infallible and faultless. But unfortunately that is not the respect in which one comes close to them; and so far one is capable of knowledge, it is a sin to neglect to acquire it, and the less instruction one has had the easier it is to fail in this". (NE, I, i, §27)

Thus Leibniz assumes that although in the initial phase, let us suppose at the time of birth, we are potentially equipped with all possible knowledge and we are ready to obtain it, the learning mechanism activates itself at the moment we receive proper impulses. At the beginning these are impulses originating from the outside – from the senses – because

"children should attend more to the notions of the senses, because attention is governed by need. However, we shall see later that nature has not 'taken pains to no purpose' in imprinting us, innately, with items of knowledge; for without these there would be no way of achieving actual knowledge of necessary truths in the demonstrative sciences, or of learning the reasons for facts; and we should have nothing over the bests." (NE, I, i, §25)

In this context Leibniz's comments connected with the intellectual capabilities of children become interesting. According to him children in their first years of development do not differ much from animals in the externalized usage of the mind. Therefore, teachers who shape their behavior may implement the same methods as trainers of animals. These tools do not suffice however when we deal with those minds which are capable of taking advantage of them. Taking advantage means achieving the capability of remembering, discovering and judging, and these features are characteristic only of intelligent beings. But since every mind in fact is closed to external influence any impulse originating from the senses according to the principle of pre-established harmony corresponds with certain activities of the mind. If we assume that "every action of the mind is thought", a child's mind, as opposed to an animal's, is capable of transferring certain unconscious actions to the conscious level, while at the same time sorting them and finding explanations of them. (AA, VI, i, 275; L, 88) For Leibniz notices that:

"The human mind is analogous to a sieve: the process of thinking consist in shaking it until all the subtlest items pass through. Meanwhile as they pass through, Reason acts as an inspector snatching out whatever seems useful." (C, 170)

What fundamentally differentiates man from animals is the ability to prove, to find rational connections between particular events. I. Hacking writes that "to be human is to be able to prove a little. A man 'who will not reason about anything' is, as Aristotle says, 'no better than a vegetable'". (Hacking, 1975, 69) However, the approaching of the content of the mind itself is irrevocably connected, according to Leibniz with the process of the shaping of the mind. He argues this in the following way:

"If someone looks attentively at more pictures of plants and animals than another person, and at more diagrams of machines (...), and if he reads more imaginative novels (...), then he can be said to have more knowledge than the other, even if there is not a word of truth in all that he has seen and heard." (NE, IV, i, §2)

Knowledge then is a certain process which reveals itself on its particular levels and Leibniz claims with full conviction that for example the knowledge of the rules of classical logic can be achieved only after one acquires the knowledge of mathematics, history and other subjects for how can one order one's thoughts who has never thought of much? (L, 468) But in another place he also notes that logic is built into thoughts (AA, IV, i, 286; L, 89), and consequently the science of logic is in effect a process of discovering what is already inside of us.

Let us inquire here: on what basis are we able to evaluate how much a child or any other intelligent being knows? A simple answer obviously is: potentially it knows everything. The above question however concerns essentially the dispositional knowledge which is able to activate the given mind. Approaching a possible answer will be the statement that the mind knows that which it is able to grasp in the act of aperception. But the state of gaining knowledge in truth is known only to oneself and the only way to examine this state by another mind is through its externalization; for example, through language. And it seems that essentially this is what Leibniz means when he says that language is the best mirror of the human mind. We may therefore assume that the knowledge which we have at our disposal in an aware or conscious form (active dispositional knowledge) is that part of potential knowledge which is possible to express in some language. Dispositional character of knowledge is closely correlated with the ability of revealing it in the act of speech.

Leibniz clearly indicates that the "activation" of dispositions takes place through the process of education. A Man learning about the world through the senses learns to find a basis for the data referring to the content of his mind. One could obviously assume that everyone of us is capable of discovering many truths independently of any contact with the world

through the senses – this is what Leibiz claims with reference to arithmetic and geometry when he writes that

“one could construct these sciences in one’s study and even with one’s eyes closed, without learning from sight or even from touch any of the needed truths; although it is true that if one had never seen or touched anything, one would not bring to mind the relevant ideas. For it is an admirable arrangement on the part of nature that we cannot have abstract thoughts which have no need of something sensible, even if it be merely symbols such as the shapes of letters, or sounds; though there is no necessary connection between such arbitrary symbols and such thoughts”. (NE, I, i, §5)

Focusing on language, Leibniz in his argumentation refers to the metaphysical principle of pre-established harmony, according to which traces of the senses are indispensable for this harmony between the soul and the body to take place. It seems that this argument is of too high a caliber. One could ask how else than through language the process of learning takes place. We must agree here with the view expressed by the contemporary Platonist Hans George Gadamer that consciousness never faces the world without language.

“Wir sind vielmehr in allem Wissen von uns selbst und allem Wissen von der Welt immer schon von der Sprache umgriffen, die unsere eigene ist. Wir wachsen auf, wir lernen die Welt kennen, wir lernen die Menschen kennen und am Ende uns selbst, indem wir sprechen lernen. Sprechen lernen heißt nicht: zur Bezeichnung der uns vertrauten und bekannten Welt in den Gebrauch eines schon vorhandenen Werkzeuges eingeführt werden, sondern es heißt, die Vertrautheit und Erkenntnis der Welt selbst, und wie sie uns begegnet, erwerben.” (Gadamer 1967, 96)

This view is in accord with Leibniz’s assumption present in his theory of the mind and his theory of language that linguistic interpretation of the world proceeds any thought and any learning.<sup>9</sup> Activation of the mind and activation of its disposition are in Leibniz inseparably connected with the existence of language. Language stands at the source of the process of learning which begins at the moment of our birth. It has a very important activating function. Throughout all of our education we are exposed to the impact of the word. Unconsciously immersed in the language we look for things and meanings corresponding to its particular words. At first

“he takes care to put the words in in the order that others customarily do, and contents himself with the thought that he could grasp their sense if the need arose. Thus a person is sometimes – oftener indeed than he thinks – a mere

passer-on of thoughts, a carrier of someone else’s message, as though it were a letter”. (NE, III, ii, §2)

Having however the possibility of “grasping” the thought itself with the assistance of other means of expression than the thought itself, we are capable of reflection on the content of the thought. For as Leibniz writes,

“although characters are arbitrary, their use and connection have something which is not arbitrary, namely a definite analogy between characters and things, and the relations which different characters expressing the same thing have to each other. This analogy or relation is the basis of truth”. (L, 184)

### The natural and the psychological order of knowledge

Leibniz makes a clear distinction between the natural and the psychological order of knowledge. He assumes the existence of a natural order of ideas which is “common to angels and men and to intelligences in general”. (NE, III, i, §5) This order being fixed and unchangeable is the potential equipment of every mind. This assumption in consequence leads to the acceptance of the inner mind system of representation – a certain language of thoughts or ideas common to all minds. If, as Leibniz claims, the mind always thinks, from this it results that it thinks in a certain inner language. This inner mind system of representation corresponds with the whole aperception of the thinking substance – the soul and the mind. This does mean that all perceptions are conscious. The mind perceives the world according to its structure which finds its projection in the system of inner mind representation. It is not conscious of the whole structure and is able to interpret only those fragments which are perceived in the act of aperception.

It is difficult to judge if the above interpretation enables one to recreate Leibniz’s real views connected in particular with the representational character of unconscious perceptions. However, there are arguments which make this interpretation admissible. The first, is the assumption that lowest on the cognitive scale is the unclear knowledge which is connected with unclear ideas; from this it appears that even an unclear idea is some sort of disposition to think and leads to expression which however is not possible to reach from the level of consciousness. The second, is the Leibnizian idea of a universal algorithm which is labeled as the thread of thought – *filum cogitationis*.<sup>10</sup> Finally the third, being supplemental to the

<sup>10</sup> See Marciszewski 1994, see also C, 351, and C, 153-57.

<sup>9</sup> In this context Leibniz’s comments contained in *Préface à la science générale* (C, 156) become essential – particularly the one which states that language precedes thought.

previous one, is the belief concerning the existence of a language which was created by Adam, a language which is indeed unknown but whose essence consisted of a natural relationship between names and things.<sup>11</sup> In contemporary research connected with artificial intelligence appears an assumption that “unconscious processes could well be algorithmic, but at a very complicated level that is monstrously difficult to disentangle in detail”. (Penrose, 1989, 411) However the algorithm itself must operate on something and this something is presumably indicated by Leibniz as the natural order of ideas.

The assumption of the existence of the natural order of ideas which is the inner mind’s system of representation, a certain language of the mind, is in accord with the rational tradition, and although Leibniz develops his theory of the activity of the mind enriching it with the theory of unconscious perceptions, its general assumptions refer however to the representational theory of the mind laid out by Descartes.<sup>12</sup> We find the source of this theory in the philosophy of Plato. Plato describes thinking as a silent conversation of the mind with itself. The process of thinking is understood as a discourse, which must assume language.<sup>13</sup> Wittgenstein, analyzing the views of St. Augustine notices that:

“Augustine describes the learning of human language as if the child came into a strange country and did not understand the language of the country; that is, as if it already had a language, only not this one. Or again: as if the child could already *think*, only not yet speak. And ‘think’ would here mean something like ‘talk to itself’”. (Wittgenstein 1958, 15e-16e)

Wittgenstein treats this opinion as absurd although it is fully accepted by present-day representational theory. J. Fodor, a representative of this trend of research, writes that

“one cannot learn a language unless one has a language. In particular, one cannot learn a first language unless one already has a system capable of representing the predicates in that language *and their extensions*. And, on pain of circularity, that system cannot be the language that is being learned.

<sup>11</sup> Adamic or paradisaical language was the subject of great speculation in the 17th century. Extensive discussion of this issue may be found in: Aarsleff, 1982, Losonsky, 1992, and Losonsky, 1993.

<sup>12</sup> Chomsky, 1990, 629-30.

<sup>13</sup> See: Plato, *Theaetetus*, 189e-190a. Z. Vendler in his article *Wordless Thoughts* (Vendler, 1977, 29-30), by interpreting this passage of the dialogue, maintains that Plato treats thinking as a conversation conducted by the mind in some ethnic language, for example Greek. Such interpretation seems to be controversial compared with the fact that, for Plato, language is secondary in relation to thinking itself and constitutes only the initial phase of the learning process. See Plato, *Letter VII*, see also F. Sontag 1954, 823-830.

But first languages are learned. Hence, at least some cognitive operations are carried out in languages other than natural languages.” (Fodor, 1976, 64)

The assumptions of the existence of the inner language of thought is the starting point for many theories of representation developing at present, among which the theory of Noam Chomsky has received the widest acclaim.

Let us turn now to the sequence of discoveries singled out by Leibniz which is a certain psychological order and which Leibniz calls the history of our discoveries, different with different people. (NE, IV, vii, §9) Reaching aperception or reflection assumes a proper act of dispositions leading to thinking of this or that fragment of reality. Disposition, as we have written above, requires a certain impulse; Leibniz presumably would say: a certain anxiety to such reflective activity. These impulses which usually come from the external senses enable one, when they are properly strengthened, to reach the knowledge of a given thing which may be revealed on many levels of learning. Whichever level of cognitive advancement a given mind has reached, it can be evaluated at the moment this knowledge is externalized. It is unquestionable that language plays here a first rank role. Revealing the content of the mind may be possible through art or an act of construction, a gesture or behavior, but it most fully reveals itself through natural language. One can obviously recall as an argument here the fact that language is the result of aspiration of the thinking substance. It is enough to state here that, by characterizing various levels of learning, Leibniz refers in fact to the language itself. Necessary premises are supplied here by the classification of the learning states set forth in *Meditations of Knowledge, Truth, and Ideas* (GP, VI, 422-26; L, 291-95) which is as follows:

Knowledge is either obscure or clear. (*Est ergo cognito vel obscura vel clara*). Clear knowledge (*clara*) is either distinct (*distincta*) or confused (*confusa*). We deal with distinct knowledge when we have at our disposal “means” which enable us to recognize the thing represented (by them). Clear and distinct (*clara et distincta*) knowledge consists of the ability to enumerate features (*notae*) sufficient to recognize things. Clear and confused (*clara et confusa*) knowledge occurs then when we are not able to enumerate one by one those features (*notae*) which results in us being able to distinguish a given thing from others. Leibniz employs here the example of colors, tastes and smells, which are in fact distinguishable thanks to the evidence of the senses but not through features which are possible to be worked out (*non vero notis enuntiabilibus*).

Clear and distinct knowledge further divides into adequate and inadequate. Adequate knowledge consists of the ability to enumerate all

features (*notae*) constituting the analyzed thing among which each feature individually is known also distinctly and clearly. If any of these features does not meet the criterion of distinctness then the knowledge is inadequate.

Let us draw our attention to the fact that the criterion of the ability to create definitions becomes essential in distinguishing particular learning states. Most difficult to interpret is the lowest state of knowledge (*cognitio obscura*) because examples indicated by Leibniz do not explain much here. It seems that obscure knowledge indicates a lack of whatever ability to define. This is a knowledge which we have at our disposal only on the level of unconscious perceptions and as such it is not verbalized. We can interpret this type of knowledge as implicit knowledge, which is the nucleus of other learning states of the mind. We can describe the remaining levels of learning on the basis of the ability to create various kinds of definitions. For example, clear and confused knowledge (*cognitio clara et confusa*) reveals itself at least as the skill to create an ostensive definition of the object whose idea we possess inside. Partial definitions indicate clear and distinct (*cognitio clara et distincta*); nominal definitions indicate clear, distinct and inadequate knowledge (*cognitio clara, distincta et inadequata*); and real definitions indicate clear, distinct and adequate knowledge. We should however stipulate that this assignment illustrates only in a very large approximation the possibility of revealing the particular learning states of a thinking mind. These symptoms of what is invisible and takes place inside of the thinking mind itself are external, possible to observe and measure. Leibniz does not claim that we know only so much as we are able to put into words; this is contradicted by the assumption of the existence of obscure knowledge which is non-verbalizable. However it can be assumed that all remaining forms of knowledge are possible to be reduced to the ability of expressing them in a language; that is, some external language.

Here a question arises: in what relationship to the inner language of thoughts does the natural language in all its variants remain? There is no doubt whatsoever that according to Leibniz “speech expresses thoughts and truths” (GP VII, 263; L, 207). Language is the result which must have its source and for Leibniz this is the permanent and invariable order of ideas. According to him, natural languages have historical character and therefore are carriers of the order of discoveries. This order has a historical as well as psychological dimension. Language is vested only in a human being who is secondary to the mental structures, but consists of one of the most important means of expression of particular learning states of individual users, societies and the whole of Mankind. Language expresses these states thanks to the fact that there exists a certain analogy between all which is enclosed in the

language and all that is contained in thoughts. Language refers directly to all which constitutes its source, to the inner world of ideas. Reference to the inner system of representation enables a progressive discovery of new areas which are gradually elevated to the level of consciousness thanks to the capabilities of their verbalization.

### The problem of translatability

In this context one is astounded by Leibniz’s strong conviction expressed in *Unvorgreifliche Gedancken* that every language, even most limited, is able to express everything. He admittedly notices a certain difficulty, but it is for him rather a matter of form and statistics and not a matter of content because although everything may be expressed through paraphrases or description, the length of the utterance results in the fact that both the speaker and the listener lose the whole pleasure and the proper distribution of accents because it takes up the mind for too long. (UG, §59)

Every language can express everything: this means that it is possible to express all levels of learning accessible to the mind in any language with the assistance of means of expression attainable in that language. But such an approach to language leads to an assumption that it is so to say timeless and is equivalent to the language of the mind. External language as perfect as the natural one reflecting the order of ideas given by God does not exist. Leibniz writes about “any” language having in mind “ethnic” languages. Assuming their timeless character would contradict the thesis of the historical features of natural languages.

The problem of “expressing” is for Leibniz resolved in this context into appropriate rules of language translation while he ignores fully the question of cultural distinction or the level of civilizational development of particular societies. These issues were of course a topic of consideration for 17th century science that was due to the great openness of Europe to the world at that time. Leibniz himself was known for his great fascination with Chinese culture. Therefore, one is surprised at the simplified approach to this so very complicated matter especially when we compare it with Leibniz’s views on the problem of the origin of human speech. Since the original image of the world is imprinted mainly in onomatopoeic words, let us remember that according to Leibniz, primitive peoples had more instinct than reason. (D, IV, ii, 187). How can it then be possible to establish and coordinate the rules of language translation as well as, in a wider sense, the rules of cultural translation? We could ask: is such coordination of the diametrically different



states of learning of Man at various levels of civilizational development at all possible?

Leibniz was not ready to give an answer to a question formulated in such a way. He also did not notice a certain contradiction occurring at the theoretical level. The theory of innate ideas assumes the invariability of the equipment of the mind, which means that from the very beginning of his existence, Man had at his disposal a full set of ideas projecting the universe and the changes taking place there. That which determined and still determines the individual character of every mind was the difference in the scale of activated learning dispositions accepted by Leibniz. This difference should also reveal itself in language. Of course this is possible to observe: it includes all verbal behavior of the user of the language, it also enables research of the consecutive stages of his intellectual development. This difference also manifests itself in full language competence through stylistic difference, richness of vocabulary, grammatical competence, etc. In the historical perspective, changes in the domain of the same language take place mainly in the lexical system but also do not exclude grammar. Finally, we have differences between national languages. If, then, an indispensable condition of Leibniz's principle of individualization is the difference in the scale of learning dispositions of the mind which reveal themselves at the language level – and thanks to it become measurable – how then can one can at the same time assume that what is expressed in the individual language of every user is reciprocally translatable? Since there are no two identical minds, then there are no two identical languages. We could further inquire about the translatability of different language systems, language in its original form and the German language of the 17th and 20th centuries. The only answer is the Leibnizian idea of prior established harmony thanks to which every change taking place in any given substance, and every movement of the mind results in the change of the remainder. But this is not a satisfying answer.

Leibniz, by accepting the fact that languages are the best mirror of the human mind, based this assumption on developed language systems known to him which he researched throughout many years. The conclusions concerned this material which was preserved in written accounts, manuscripts, documents and other sources. He was not able however to interpret facts, as evidenced by his comments concerning languages of the newly discovered tribes: "they differ so much among themselves and are so different from us that we can say that these are a completely different breed". (AA, I, vii, 399) One may presume that Leibniz had fundamental doubts here and it is not clear if he would

admit without hesitation that in these languages also, everything could be expressed. Conclusions educed by contemporary social anthropology rather rule out the possibility of such an approach. For it is accepted that "language in its primitive forms ought to be regarded and studied against the background of human activities and as a mode of human behaviour in practical matters. We have to realize that language originally among primitive, non-civilized peoples was never used as a mere mirror of reflected thought. The manner (...) in which the author of a book, or a papyrus or a hewn iscription has to use it, is a very far-fetched and derivative function of a language. In this, language becomes a condensed piece of reflexion, a record of fact or thought. In its primitive uses language functions as a link in concerted human activity (...). It is a mode of action and not an instrument of reflexion". (Malinowski, 1956, 312)

The above arguments may lead to the conclusion that there exist essential problems impeding or making the translation of a primitive language to another developed language system impossible. One essential obstacle is, among other things, the reconstruction of the semantic system which is entangled in the pragmatic context.<sup>14</sup> The problem of translatability is one of the main objections formulated against the contemporary theory of representation. It is pointed out that every description of a learning state of another mind is inadequate due to the lack of objective rules of translation as well as difficulties in establishing the tools of measurement of the accuracy of such translation. Questions of cultural translatability, of the learning states of people with damaged brains, and of children in the pre-verbal phase are also raised.<sup>15</sup>

However, we may assume that all human language systems remain in a certain relationship with the same ideal system of representation projecting the order of the universe and that this system attempts to decipher Man. For Leibniz assumes that

*"all individual created substances, indeed, are different expressions of the same universe and the same universal cause, God. But these expressions vary in perfection as do different representations or perspectives of the same city seen from different points". (L, 269)*

Human languages are the best observable result of the learning activity of the thinking substance. Because they exist there also must exist justification for this existence. This constitutes the previously mentioned inner system of expression. The imperfectness of our language fully renders the complexity

<sup>14</sup> See Malinowski, 1956.

<sup>15</sup> See Churchland, Smith Churchland, 1990, 301-302.

of the learning processes taking place in our mind. But language refers only to those areas of thinking which are conscious thinking. Leibniz was aware of this fact, and its consequence was his theory of the mind whose essence is the assumption of the gradation of consciousness.<sup>16</sup> Assuming that the mind always thinks, Leibniz clearly indicated that all which we are unable to examine is unconscious thinking, for it is not verbalized. Thus the process of thinking goes beyond language, but language reflects all that which appears to us in the act of consciousness, and in fact is the only tool enabling us to define the scope of the learning dispositions of a thinking mind.

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Witold Marciszewski

### LEIBNIZ'S IDEA OF AUTOMATED REASONING COMPARED WITH MODERN AI<sup>1</sup>

Gottfried Wilhelm Leibniz (1646-1716) is duly merited as the first who anticipated Artificial Intelligence. If so, let us ask: which AI? That called *strong* which — following Turing [1950], Newell [1980], etc. — answers in the affirmative the famous Turing's question '*can machines think?*'? Or rather that which opposes that claim, and reduces AI to a device to assist people in solving problems of some restricted kinds? (Cf. Gams [1995].)

There is no one simple answer concerning Leibniz's position. Instead, there are hints that he stuck to the both opposite views, and showed no signs of being aware of their incompatibility. It is the purpose of this essay to present that dilemma and look for its sources. To make the matter simpler, the most general question '*can machines think?*' will be limited to a more specific crucial issue, to wit: '*can machines reason?*'.

This limitation to reasonings fairly reduces the number of themes to be handled. But there is more in it. It results from a fundamental point as stated by Fodor [1976, p. 202 f]. What Fodor says about cognitive psychology holds for AI as well; here is his statement (italics mine – WM). "Cognitive explanation requires not only causally interrelated mental states, but also mental states whose *causal* relations respect the *semantic* relations that hold between formulae in the internal representational system. The present point is that there may well be mental states whose etiology is precluded from cognitive explanation because they are related to their causes in ways that satisfy the first condition but do not satisfy the second."

Fodor lists some phenomena that satisfy the first condition alone. Instead of quoting them, let me add another example — that of dreams. These typically exemplify information processing subjected to some causal

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laws but hardly related to semantic rules. Such a contrastive background helps to appreciate the case of *automated reasoning* as carried out by computers. It yields a paradigmatic case of conformity between causal physical laws and semantic entailment. Such a perfect agreement is due to the fact that the same laws of *algebra* are mirrored both in electric circuits and in propositional calculus, thus forming causal relations in the former domain, and semantic relations in the latter. Even if we are not sure that human reasonings comply to such a 'preestablished harmony' (to use Leibniz's key word), the guess that they do is a brilliant working hypothesis — as can be seen in the pioneering study by McCulloch and Pitts [1943] (this is why in Marciszewski and Murawski [1995] so a great emphasis is put on algebraization of logic as the turning point in the history of mechanization of reasoning). In this perspective, the study of automated reasoning is of highest importance for Artificial Intelligence.

Now, the question put at the start should be rephrased as follows: Would the problem '*can machines reason*' be by Leibniz answered in the affirmative or in the negative? As hinted above, one should expect two responses opposing each other. A guess to explain this split in Leibniz's thought might take into account that Leibniz was an earnest engineer of knowledge as well as a metaphysician occupied with the mind-world relation. In his capacity as an engineer, he set up most ambitious goals, while as a metaphysician he must have acknowledged the incommensurability between the human mind and the immense complexity of the world.

Apart from such psychological guessing, a more theoretical explanation will be suggested, to wit Leibniz's mistaken treating of the reasoning (supposed by him as liable to mechanization) and the perception (seen by him as entirely non-mechanical) as mutually exclusive mental activities. Thus, he might have held the mechanistic view in the former, and the opposite in the latter issue.

In what follows, the prospective conclusion is anticipated with a typographical device to distinguish, where necessary, between 'two Leibnizs'; let one of them be referred to as Leibniz<sub>e</sub>, for engineer, and the other one as Leibniz<sub>m</sub>, for metaphysician.

1. LEIBNIZ<sub>e</sub> ATTITUDE TO AI. At the very start, it should be noted that Leibniz<sub>e</sub> — as a forerunner of strong AI — did by no means anticipate the limitative results of Gödel [1931] and Turing [1936-37]. Sharing the epistemological attitude of his century, he was even more optimistic than the early Hilbert School as for the possibility of solving any problem properly posed.

While Hilbert's contention was concerned with mathematics alone, Leibniz<sub>e</sub> believed that all scientific and philosophical problems can be definitely solved in a foreseeable time. In this respect, he was confident like Descartes. However, while Descartes attributed the power of reasoning to the mind alone, and discounted linguistic devices, Leibniz<sub>e</sub> extended that power to the mind-imitating machines equipped with a suitable symbolic system.

Thus that claim concerning AI shared by Leibniz<sub>e</sub> with Turing rests on a stronger epistemological assumption than that of Turing. Turing, owing to his own [1936-37] and Gödel's [1931] mathematical results, was aware of the existence of problems undecidable for any machine; that is to say, undecidable with the use of a purely *formalistic scheme* in proving, in which solely the physical form of symbols is referred to. However, he opposed those who used these results to argue for the minds's superiority to the reasoning machine. In his [1950, Sec. 4] essay, he wrote as follows.

The short answer to this argument [that there is a disability of machines to which the human intellect is not subject] is that although it is established that there are limitations to the powers of any particular machine, it has only been stated, without any sort of proof, that no such limitations apply to the human intellect.

When Turing speaks of the lack of any proof, he means a mathematical proof. However, there may be other reasons to believe in a greater ability of the human intellect, for instance, those adduced by Penrose [1989, p. 111 f].

It seems to me that it is a clear consequence of the Gödel argument that the concept of mathematical truth cannot be encapsulated in any formalistic scheme. Mathematical truth is something that goes beyond mere formalism. This is perhaps clear even without Gödel's theorem. For how are we to decide what axioms or rules of procedure to adopt in any case when trying to set up a formal system? Our guide in deciding on the rules to adopt must always be our intuitive understanding of what is 'self-evidently true', given the 'meanings' of the symbols of the system. How are we to decide which formal systems are sensible ones to adopt — in accordance, that is, with our intuitive feelings about 'self-evidence' and 'meaning' — and which are not? The notion of self-consistency is certainly not adequate for this. One can have many self-consistent systems which are not 'sensible' in this sense, where the axioms and rules of procedure have meanings that we would reject as false, or perhaps no meaning at all. 'Self-evidence' and 'meaning' are concepts which would still be needed, even without Gödel's theorem.

However, without Gödel theorem it might have been possible to imagine that the intuitive notions of 'self-evidence' and 'meaning' could have been employed just one and for all, merely to set up the formal system in the first place, and thereafter dispensed with as part of clear mathematical argument for determining truth. Then, in accordance with a formalist view, these 'vague' intuitive notions would have roles to play as part of the mathematician's

*preliminary* thinking, as a guide toward finding the appropriate formal argument; but they would play no part in the actual demonstration of mathematical truth. Gödel's theorem shows that this point of view is not really tenable in a fundamental philosophy of mathematics. The notion of mathematical truth goes beyond the whole concept of formalism.

This quotation is to testify how an intensely practicing mathematician like Penrose (a mathematical physicist) may see the human intellect's superiority to a reasoning machine. Such a testimony is no decisive argument but it should counterbalance Turing's belief, not being supported by a decisive proof either, that the human mind equals a sufficiently involved machine. One can rewrite Turing's words as cited above, just omitting 'no', and so his own positions will be equalled with that of his opponents: *it has only been stated, without any sort of [mathematical] proof, that such limitations apply to the human intellect.* It does not seem possible to decide which side is here obliged by the *onus probandi* rule. However, Turing's opponents, as represented by Penrose, may have arguments which are not mathematical but take advantage of mathematicians' experiences.

Now, what about Leibniz<sub>e</sub>? Let the following biographical event renders his attitude toward what we nowadays call strong AI (cf. Ross [1984, p. 12]). After he constructed his mechanical calculator in 1670, he was so proud of his invention (applauded, indeed, by most brilliant minds in Paris and London) that he thought of commemorating it with a medal bearing the motto *Superior to Man*. To understand this emphasis, one should recall that at Leibniz's time "even educated people rarely understood multiplication, let alone division (Pepys had to learn his multiplication tables when already a senior administrator)". In this respect, Leibniz's calculator, which surpassed Pascal's machine (1641) as it was capable of multiplying and dividing, was actually superior to quite a number of people.

However, as will be reported later (Sec. 2), Leibniz<sub>e</sub> believed that his arithmetical machine is just the beginning of a development that should result in logical machines to match humans in the ability of reasoning. And that, in principle, there are in science and philosophy no unsolvable problems, either for men or for logical machines, but in practice (he presumably thought) the machines should act better (as carefully equipped for their cognitive tasks).

Thus, though on different premisses, Leibniz<sub>e</sub> shared with Turing and other strong AI proponents the belief in the prospective likeness of the reasoning power with humans and machines.

Once having been so identified, Leibniz<sub>e</sub>'s point that the human reasoner would not surpass mechanical reasoning devices should be

attentively examined, and then compared with Leibniz<sub>m</sub>'s view. The latter is to the effect that *each organic body is a kind of divine machine, or natural automaton, which infinitely surpasses all artificial automata* because of its infinite complexity (see *Monadology*, 64). Hence the human mind should surpass artificial machines in dealing with complexity of problems to be addressed.<sup>2</sup>

2. ON 'FILUM COGITATIONIS' — AN ALGORITHMIC METHOD OF REASONING. The algorithmic procedure of problem-solving consists in mechanically following a set of fixed instructions which describe transformations of characters (ie, symbols) treated as physical objects. Hence any introduction of algorithm has to be preceded by, so to speak, physicalization of language. This idea of algorithm is clearly stated by Leibniz<sub>e</sub> in the following passage.<sup>3</sup>

*Filum autem meditandi voco quandam sensibilem et velut mechanicam mentis directionem, quam stupidissimus quisque agnoscat. [...] Scriptura enim et meditatio pari passu ibunt, vel ut rectius dicam, scriptura erit meditandi filum.* — GP vii, 14; *Briefwechsel*, I, 102, to Oldenburg. See Couturat [1901], p. 91, fn. 2; p. 96, fn. 4.

"What I call a *thread of thought* is a certain sensory and machine-like guidance to the mind to be practicable even for most stupid ones. For, the following of a text and the thinking will proceed in step, that is, a written text will be a thread for thought."

That even most stupid beings can profit from this method, means that no intelligence is needed to algorithmically solve problems; just mechanical rules, concerning sensible and palpable properties, should be followed, like commands followed by a computer carrying out a program. Such palpability is due to the sensible qualities of characters which constitute the language used in reasonings. Owing to a suitable correspondence between notions and characters, as postulated by Leibniz, and owing to the mechanical characters-processing ('*caeca cogitatio*'), one safely arrives at the solution to be found.

Here are other statements of the same programme.

<sup>2</sup> This remarkable split does not seem to have attracted due attention in Leibniz literature. When discussing it in my talks addressed to some audiences of Leibniz scholars, I had enjoyed their kind interest, but — except for some Breger's and Schnelle's publications (cf. References) — the problem of Leibniz's relation to AI does not enjoy a treatment it deserves.

<sup>3</sup> This point becomes more conspicuous when seen against the contrastive background of the Cartesian Method; see Marciszewski [1994a, Chap. 3]; another Leibniz's metaphor to render algorithmic procedures is that of *caeca cogitatio* — the blind thinking (see *ibid.* p. 61, 178). A clear and insightful discussion of the concept of algorithm as related to the decision problem and modern computers is found in Gandy [1988] and Davies [1988].

*Ad inventionem ac demonstrationem veritatum opus est analysi cogitationum, quae quia respondet analysi characterum..., hinc analysin cogitationum possimus sensibilem reddere, et velut quodam filo mechanico dirigere; quia analysis characterum quoddam sensibile est. — Analysis linguarum, 11 Sept. 1678. (C, 351)*

“What is required for finding and proving truths, it is an analysis of thoughts. Since it corresponds to analysis of characters [...], the analysis of thoughts can be physicalised through characters, and proceed as if guided by a mechanical thread.”

*Erit enim in promptu velut Mechanicum meditandi filum, cujus ope idea quaelibet in alias, ex quibus componitur, facillime resolvi possit; imo caractere alicujus conceptus attente considerato, statim conceptus simpliciores, in quos resolvitur, menti occurrunt: ... resolutio conceptus resolutioni characteris ad amussim respondet. — GM iv, 461; Briefwechsel, I, 380, to Tschirnhaus, May 1678. See Couturat [1901], p. 91, fn. 4.*

“There will be on hand something like a mechanical device to assist thinking, such that with its help any idea could be most easily resolved in its constituents; to wit, with careful examining a character denoting a concept, at sight the simpler concepts in which that one resolves will appear to the mind: the resolution of concepts exactly corresponds to the resolution of respective symbols.”

In other texts there appear the notions of *calculus* and of *machine*. They should be also considered in order to compare the whole Leibniz’s programme with the nowadays concepts of algorithm and of formalized and mechanized reasoning; formalization, i.e. the rendering of a reasoning in symbols to be processed by an algorithmic calculus, forms a prerequisite for mechanization. Here are the statements in question.

*Nihil enim aliud est Calculus, quam operatio per characteres, quae non solum in quantitate, sed et in omni alia ratiocinatione locum habet. — GM iv, 462, Briefwechsel, I, 381, to Tschirnhaus, May 1678. See Couturat [1901], p. 96, fn. 2.*

“The calculus is nothing else but operating with characters, what occurs not only in computing quantities but also in any other reasoning.”

This Leibniz<sub>e</sub>’s insight, not without Thomas Hobbes’ influence, does anticipate the modern notion of logical calculus as dealing with characters but not those which refer to numbers or quantities. He is aware of the novelty of these ideas, as in another place he remarks:

*[...] calculus ratiocinator, seu artificium facile et infallibiter ratiocinandi. Res hactenus ignorata. — GP vii, B, II, 8. See Couturat [1901], p. 96, fn. 5.*  
the reasoner calculus, that is, a device for reasoning in an easy and infallible way — the thing unknown so far.

This kind of statements includes the famous “Calculemus” which occurs in the following context:

*Quando orientur controversiae, non magis disputatione opus erit inter duos philosophos, quam inter duos Computistas. Sufficiet enim calamus in manus*

*sumere sedereque ad abacos, et sibi mutuo dicere: Calculemus!* — GP vii, 200. See Couturat [1901], p. 88, fn. 3.

“When arise controversies, no more a dispute will be necessary among two philosophers than among two calculators. For it will be enough to take pencils and abacuses in hands, and say to each other: *let us compute!*”

When the notion of *machine* gets added to those of *sensible characters* and of *calculus*, Leibniz’s theory of mechanical problem-solving becomes surprisingly close to the modern idea of logical computing, even in such details as printouts of inferences produced by a computer operated by a suitable software (at present called a prover):

*Ut Veritas quasi picta, velut Machinae ope in charta expressa, deprehendatur. [...] Illud Criterion [...] quod velut mechanica ratione fixam et visibilem reddit veritatem. — GP vii, 10, Briefwechsel I, 145, to Oldenburg, 28 Dec. 1675. See Couturat [1901], p. 99, fn. 2; p. 100, fn. 3.*

“[A device should enable that] the truth like in a picture, as if by a machine printed on a chart, be perceived. [This would be] that criterion which will produce the truth as if established in a mechanical way and made clearly visible.”

In such insights and images, Leibniz<sub>e</sub> might have been inspired by his successful construction of arithmetical machine. Once reasoning is conceived as computing, a reasoning machine can be seen as a computing machine. However, to obtain a deeper insight into Leibniz’s views, we should start not from the concept of a machine but rather from the concept of a formal language. It is the latter in which we are to look for the core of Leibniz’s programme. For, once we obtain a language defined in a purely formal manner, i.e., with respect to physical forms of expressions (and their combinations) alone, we can arithmetize it through assigning numbers to expressions, and to syntactic constructions, and then take advantage of a mathematical machine.

This is why a formalization of inferences has to be prior to their mechanization. Hence, a modern counterpart of the Leibnizian *filum cogitationis* is the Hilbert programme of formalizing the language of mathematics, the programme stated to ensure decidability of mathematical problems. It should have resulted in an algorithm to check whether a reasoning in question is logically correct. To commit such a task to the care of machines is the next step whose success would be granted provided both a suitable formalization and an advanced data-processing technology.

2. LEIBNIZ<sub>e</sub> COMPARED WITH HILBERT. It was Hilbert who at the Second International Congress of Mathematics held in Paris in 1900, expressed an extreme optimism like that of Leibniz — a faith in the mathematician’s

ability to solve any problem he might set for himself. This Leibniz-Hilbert analogy is crucial for the point of this essay; Leibniz in his methodological programme is as close as possible to Hilbert's formalism and finitism while his philosophical views on mind and matter are as remote as possible from finitistic conceptions.

Let us examine the analogy in question. With Hilbert and Bernays [1934-39], the first step was to formalize the language of mathematics. That is to say, an artificial symbolic language and rules of building well-formed formulas ought to be fixed. Furthermore, axioms as well as formal rules of inference, i.e., the rules referring only to the physical form of formulas (not to their meaning), should be stated. Both the set of primitive expressions, being building blocks of formulas, and the set of rules, are finite, and every proof is to be performed in a finite number of steps.

Leibniz<sub>e</sub>'s stress on the sensibility and palpability of characters used in reasoning resembles the formalistic point of Hilbert, and his belief expressed in the famous *Calculamus* is to the effect that every demonstration can be performed in a finite number of steps. As for differences, these are as follows. With Leibniz<sub>e</sub>, (i) axioms are certain real definitions in the form of equality, (ii) the sole inference rule is that of definitional replacement; thus, (iii) the proof reduces to an analysis of defined concepts which terminates in some semantic primitives.<sup>4</sup> With Hilbert, both the form of axioms and the form of inference rules is much more differentiated, in accordance with the modern methodology of deduction. Moreover, (iv) Leibniz<sub>e</sub> is even more optimistic than Hilbert, for the method postulated is by him regarded as universal, i.e., applicable to the whole knowledge, not only to mathematics.

However, these variations do not affect the analogy crucial for the present discussion, namely that consisting in formalism and finitism, as far as a theory of proof is concerned. The postulate of finitism, for its practicability, requires a more precise statement than that found in Hilbert himself; however, for the present comparison it is enough to conceive it as demanding possibility to mechanically solve each problem in a finite number of steps, which amounts to the existence of a suitable algorithm.<sup>5</sup>

<sup>4</sup> The problem of the creative role of real definitions as axioms in a reasoning is discussed in my [1993] and [1995b] essays.

<sup>5</sup> A thorough examination of Hilbert's approach is found in Murawski [1994] while my paper [1994b] involves a discussion on how formalization of inferences is related to their mechanization.

3. LEIBNIZ<sub>m</sub>'S INFINITISTIC THEORY OF MATTER. One may object a lack of coherence between the finitistic *filum cogitationis* methodology, as reported above, and Leibniz<sub>m</sub>'s infinitistic approach in his metaphysical conceptions of matter and mind. The relation between these two points in the Leibnizian thought deserves a closer examination.

The first expression of the infinitistic point of view is found in the dissertation *De Arte Combinatoria*, 1666, where, advised by Erhard Weigel, he appended "Demonstratio existentiae Dei ad mathematicam certitudinem exacta". The last axiom in this demonstration reads:

*Cujuscumque corporis infinitae sunt partes, seu, ut vulgo loquuntur, Continuum est divisibile in infinitum.*

"Every body has infinitely many parts, or, as it is commonly said, the continuum is infinitely divisible."

This statement constitutes the very core of the demonstration: were it not so that every body whatever has an infinite number of parts, then it would not be necessary to resort to the infinite power, hence to that of God, which in Definition 3 is meant as an original capacity (*potentia principalis*) to move the infinite. Note that Leibniz<sub>m</sub>, unlike Aquinas and other masters of *Gottesbeweise*, does not claim the necessity of closing the chain with the First Mover, but the necessity of infinite power to be possessed by the Mover; and such a power is required because of the infinite difficulty of the task to move the infinite set of pieces of matter.

This crucial statement will be confusing, unless the activity of moving is conceived as an intellectual operation (again, a point alien to other authors trying to prove God's existence). In fact, each of us is able to physically move a piece of matter, say a chair, even if the matter which it consists of is divisible in infinitum; that we exercise such a power is independent of whether the question of infinite divisibility of matter proves answered in the negative, as with atomists, or in the affirmative, as with Leibniz<sub>m</sub> and, possibly, modern physics.

Let the following quotations support the point that Leibniz, in a way, anticipated some recent guesses as to the actual infinite divisibility of matter.

"So we know that particles thought to be 'elementary' twenty years ago are, in fact, made up of smaller particles. May these, as we go to still higher energies, in turn be found to be made from still smaller particles? This is certainly possible." (Hawking [1992, p. 66].) "For modern particle physicists, [...] every new accelerator, with its increase in energy and speed, extends science's field of view to tinier particles." (Gleick [1991, p. 115].)

According to Ulam [1976, Ch. 15], the most interesting question in physics is whether there exists actual infinity of ever tinier structures. He suggests to consider that we deal with a strange structure having infinitely many levels, each level possessing its specific nature. This is – he continues – not only a

philosophical riddle but also a fascinating vision in physics. When mentioning quarks, he comments that we may have reached the point in which we should consider an infinite sequence of structures.

Ulam's picture of the physical world is astonishingly close to that of Leibniz<sub>m</sub>. It becomes even closer if we take into account the limitations of physical research as implied by Gleick's account. One cannot increase the energies applied in laboratories ad infinitum. Hence, for this reason too, there must be a limit of structural complexity beyond which no human mind can penetrate. The lack of knowledge of those so deep levels of complexity puts up an impassable barrier to human technological power to control processes at those levels. Other barriers have to arise from the finiteness of human memory and of the time given the mind for its operations; these limitations affect both humans and computers.<sup>6</sup>

Hence, if 'moving' (in the mentioned *Demonstratio*) means controlling, like a software moves a computer, the more involved the object to be controlled, the greater intellectual power is required. Thus, the increasing human capacity to change the world, and in this sense to move things, is proportional to the advances in knowing ever more minute structures of matter, be it genetic code, be it atomic structure. Hence, in the case of an infinite structural complexity, an infinite intellectual power is required to handle it, i.e., to influence a course of events according to an intended plan.

Thus – let us emphasize this once more – any finite mind, when inquiring into more and more involved structures, has to meet a limit of its cognitive capabilities. The more forceful is a mind, the more distant is that limit, but somewhere it must exist; the infinite mind alone is free of any such limitations. This implies a failure of the belief that for every problem challenging the human mind there is a suitable algorithm, i.e. *filum cogitationis*, to solve the problem mechanically in a finite number of steps. This is a consequence of the three Leibniz's tenets, viz. that (i) controlling the material world requires an intellectual power to the extent relevant to the degree of complexity, (ii) the material world possesses infinite computational complexity, and (iii) the human mind's power is not infinite.

The question of coherence between the above points and the programme for logic (to make logic capable of mechanical solving any problem whatever) cannot be settled by assigning each view to a different stage of Leibniz's

<sup>6</sup> That incommensurability between the human mind and the unboundness of universe, as seen by Leibniz<sub>m</sub>, is extensively discussed by Drozdek [1997] (this book). His thought-provoking analysis should be continued by taking into account the set-theoretical distinction between dense ordering and continuous ordering; it was alien to Leibniz<sub>m</sub> himself, but when rendering his term *continuum* in modern concepts, we are to consider which concept corresponds to his intentions.

development. The juvenile insight regarding the infinite complexity of bodies persisted through all the changes of his views, and is found in the final phase, that of *Monadology*; also his insistence on formalization of reasoning is constant.

As for the concept of matter, in 1686 he wrote that "every body, however small, has parts which are actually infinite, and in every particle there is a world of innumerable creatures",<sup>7</sup> and that "there is no body so small that it is not actually subdivided".<sup>8</sup> A similar statement in a text of 1689 reads "there is no portion of matter so small that there does not exist in it a world of creatures, infinite in number".<sup>9</sup> In Leibniz's article "Système nouveau [...]" published in *Journal des Savants*, 1695, we read that "everything in matter is but a collection or accumulation of parts *ad infinitum*".<sup>10</sup> In *Monadology* of 1714 it is stated in item 65 what follows.

*Et l'Auteur de la Nature a pû practiquer cet artifice divin et infiniment merveilleux, parce que chaque portion de la matière n'est pas seulement divisible à l'infini comme les anciens ont reconnu, mais encor sous-divisée actuellement sans fin, chaque partie en parties, dont chacune a quelque mouvement propre: autrement il seroit impossible, que chaque portion de la matière pût exprimer tout l'univers.*

"The Author of nature was enabled to practise this divine and infinitely marvellous artifice, because each portion of matter is not only infinitely divisible, as the ancients recognised, but is also subdivided without limit, each part into further parts, of which each one has some motion of its own: otherwise it would be impossible for each portion of matter to express the whole universe." — (GP vi, 607-23; P. [1973], p. 190.)

4. ON SCALE-DEPENDENT LIMITATIONS OF RESEARCH. The texts quoted above testify Leibniz<sub>m</sub>'s persistency in his infinitistic theory of matter. One of them, the juvenile *Demonstratio existentiae Dei*, hints at the connexion between a theory of matter and a theory of mind. According to *Demonstratio*, the infinite mind alone is capable of managing the infinitely complex universe. It is in order to consider later texts to express the same idea.

A text on *Metaphysical consequences of the principle of reason* (ca. 1712) attributes the full knowledge of matter to the omniscient, hence infinite, mind alone.

<sup>7</sup> GP vii, 309-18. See P. [1973], p. 82.

<sup>8</sup> C 518-23. See P. [1973], p. 91.

<sup>9</sup> E. Bodeman, *Die Leibniz-Handschriften der Königlichen Bibliothek zu Hannover*, Hannover 1889, 115-17. See P. [1973], p. 108.

<sup>10</sup> GP iv, 477-87. See P. [1973], p. 16.



“Each corpuscle is acted on by all the bodies in the universe, and is variously affected by them, in such a way that the omniscient being knows, in each particle of the universe, everything which happens in the entire universe. This could not happen unless matter were everywhere divisible, and indeed actually divided *ad infinitum*.” (P. [1973], p. 176)

The latest testimony of the connexion between Leibniz’s philosophy of matter and his theory of mind is found in the correspondence with Clarke, 1715-16. Let it be discussed against the contrastive background of scientific atomism as represented by the physicist Herman von Helmholtz (1821–1894). He shared the view close to the philosophy of ancient atomism that the whole material world can be simply explained by interactions of material particles.<sup>11</sup>

We come to the conclusion that the task of science consists in reducing natural phenomena to invariable forces of attraction and repulsion whose intensity entirely depends on the distance. The decidability of this problem constitutes the condition of complete comprehensibility of Nature. [...] The task of science will be completed when all the physical phenomena get reduced to those simple forces, and when a proof is provided that it is the sole possible solution.

Thus, atomistic philosophy of matter implies finitistic epistemology. This relation is discussed by Leibniz – in order to oppose the finitistic view – in the following passage of the *Correspondence with Clarke*.

“On this [i.e., atomistic] theory a limit is set to our researches; reflexion is fixed and as it were pinned down; we suppose ourselves to have found the first elements — a *non plus ultra*. We should like nature to go no further; we should like it to be finite, like our mind; but this is to ignore the greatness and the majesty of the Author of things. The last corpuscle is actually subdivided *ad infinitum* and contains a world of new created things, which the universe would lack if this corpuscle were an atom, that is a body all of a piece and not subdivided.” (GP vii, 352 ff; P. [1973], p. 220)

In Helmholtz’s finitistic paradigm no additional assumptions are necessary to justify the belief in solvability of any physicists’ problem, provided a sufficient advancement of the research in question. (Even if the number of atoms is infinite, in each particular problem one deals with a finite number of them.) Is it possible to reach such conclusion within Leibniz<sub>m</sub>’s infinitistic paradigm?

The answer depends on some additional assumptions. Let us imagine that besides his infinitistic contention Leibniz<sub>m</sub> endorsed what should be called an “explanation by downward resolution”, *downward explanation*, for short, and should be defined as follows: *for each system, a downward explanation of its structure and functions takes into account structures*

*and functions of its constituents: first, of immediate constituents, then (if necessary) of those forming the immediate constituents, and so on.* (While upward explanation would refer to the structure and functions of that whole whose the system in question is a constituent). In such a reductionist way, we explain activities of organic cells by activities of their molecules, and those, in turn, by activities of atoms, and those by motions of elementary particles, and so on.

In the infinitistic framework of Leibniz<sub>m</sub>, the chain of levels of complexity is infinite. This implies that solely the infinite mind, in the sense of actual infinity, can deal with *all* of them. But what about dealing with *each* of them? This would require less from the mind, only potential infinity. Suppose, the human mind has potentially infinite ability of development. Then for each problem there may come a time in which the mind’s capability will match its complexity.

Then, though a new problem may require going still deeper “down”, again it would be a finite number of steps. For each new problem one could devise a special algorithm, in accordance with the *Calculamus* postulate. The more involved a problem, the more steps must be done to solve it, but always there exists its solution in a finite number of steps.

Are there any reasons to suppose that Leibniz might have cherished such an optimistic perspective? If so, then Leibniz<sub>m</sub> and Leibniz<sub>c</sub> might be reconciled through attributing the human mind a potentially infinite power (the actual infinity being reserved for the divine mind). No explicit statements of such a kind are found in his writings, but this conjecture does not seem to be inconsistent with the rest of his views; as he believed in the eternal existence of minds, he might have hoped that the eternal life involves a potentially infinite intellectual development.

However, when we return to more earthly affairs, and consider the development of science in a finite perspective, such eschatological speculations have to be put aside. The historical development of science reveals limitations of our research, up to the unsolvability of some problems which result from a change of scale; let them be called *scale-dependent limitations*.

The most famous of them is Heisenberg’s uncertainty principle, another one is the increase of observation abilities only with the increase of energies to be adopted (note, we cannot increase energies infinitely). Another scale limitation, most relevant to the present issue, is due to the fact of memory size limitations in a computer; analogous limitations must be supposed to exist in human brains. An intuitive reasoning requires much less brain memory than a formalized, or algorithmized, reasoning. Hence some

<sup>11</sup> Reported after Einstein and Infeld [1947, Ch. 1].

problems which are solvable in an intuitive way, without a verbalization, may prove unsolvable in a verbalized form (while verbalization is necessary to formalize a reasoning, and this, in turn, is necessary for its mechanization).

When holding a finistic opinion like that of Helmholtz, one may *a priori* hope to evade scale-dependent limitations. However, Leibniz<sub>m</sub>'s infinitistic conception of matter, when linked with assuming the human mind's finiteness (cf. point (iii), Sec. 3), should have resulted in a limitative theorem — to the effect that there occur unsolvable problems. They occur because of reaching, when going sufficiently long along an infinite path of resolution, such an increase on the scale of complexity which dramatically changes the relationship between the subject of research and the finite mind's capabilities.

However, Leibniz<sub>c</sub> did not draw such consequences from what was being held by Leibniz<sub>m</sub>. The explanation of this riddle in its entirety does not seem possible without a very thorough and extensive study on this subject. But the problem becomes more manageable when restricted to the issue of relations between reasoning and what Leibniz<sub>m</sub> called perception.

Let us try this way.

5. AN OVERLOOKED RELATION BETWEEN PERCEPTION AND REASONING. When there appears inconsistency with a great thinker, one may try to explain this by resorting to Bergson's idea that the philosopher's insights exceed any verbal means of their adequate expression, hence the apparent cleavage (this is the strategy adopted in my [1996b, p. 245 ff] paper).

However, when following this advice, a historian or a follower should conjecture what deficiencies in the thinker's means of expression — likely, may be, to be remedied with a newer conceptual equipment — are responsible for the fact in question. The conjecture to be here offered for discussion runs as follows.

There is in *Monadology* a direct attack on the claim that the functioning of the mind can be explained in terms of mechanism (which looks as if addressed to Leibniz<sub>e</sub>). It is found in item 17.

*On est obligé d'ailleurs de confesser que la perception et ce qui en dépend est inexplicable par des raisons mecaniques, c'est à dire, par les figures et par les mouvements. Et feignant qu'il ait une Machine, dont la structure fasse penser, sentir, avoir perception; on pourra la concevoir aggrandie en conservant les mêmes proportions, en sorte qu'on y puisse entrer, comme dans un moulin. Et cela posé, on ne trouvera en la visitant au-dedans, que des pieces qui se poussent les unes les autres, et jamais de quoi expliquer une perception. Ainsi c'est dans la substance simple, et non dans le composé ou dans la machine qu'il la faut chercher.*

“We are moreover obliged to confess that *perception* and that which depends on it *cannot be explained mechanically*, that is to say by figures and motions. Suppose that there were a machine so constructed as to produce thought, feeling, and perception, we could imagine it increased in size while retaining the same proportions, so that one could enter as one might a mill. On going inside we should only see the parts impinging upon one another; we should not see anything which would explain a perception. The explanation of perception must therefore be sought in a simple substance, and not in a compound or in a machine.” (P. [1973], p. 181.)

Note, there is not necessarily an inconsistency in supposing both that the system in question is a machine (as put down in the second sentence) and that its functioning cannot be explained mechanically. For, as we know from the item 64 of *Monadology*, there are two kinds of machines, and the system endowed with perception is a divine machine, not being a machine in the sense of a human artifact.

Unfortunately, Leibniz<sub>m</sub> yields no argument why we should not observe anything which would mechanically explain a perception; he simply states that. Neither mentions reasonings as activities of such ‘non-mechanical machine’, though he lists thought, feeling and perception. Should, for instance, reasoning fall under what he calls thought?

Suppose, there is possible the following agreement between Leibniz<sub>m</sub> and Leibniz<sub>c</sub>: the former accepts the latter's claim that reasonings can be explained mechanically, as in an artificial logical machine, while the latter acknowledges the irreducibility of perception to mechanical moves. This solution assumes that no perception is involved in reasoning, since its involvement would deprive reasoning of mechanical character.

Now suppose that, after a time, Leibniz<sub>m</sub> finds a new evidence, namely to the effect that there are mathematical proofs necessarily involving perception which could not be adequately verbalized. Had Leibniz<sub>m</sub> lived not earlier than Georg Cantor, he could have produced a nice example of perception-involving argument; to wit, the diagonal slash procedure to prove that the set of real numbers is not countable (see Cantor [1890/91], Kertész [1983], Penrose [1989]).

This argument requires both an evidence supplied with one's eyes as well as bold imagination to extend the perceived picture towards infinity. Is not either of them an indubitable instance of perception? No verbal statement seems necessary to render the course of that reasoning which results in the firm conviction that there are more real numbers than natural numbers.

Thus we reach a double conclusion — a historical conjecture and a theoretical suggestion. The former is to the effect that Leibniz would have reasonably limited his AI project concerning a logical machine, had he

become aware of the role of perception in reasonings, eg. those of the diagonal kind.

The theoretical suggestion – independent of a possible course of Leibniz's thought, if confronted with modern mathematical proofs – is to the effect that one should check Leibniz's project as being close to the modern strong AI project. This is to be done so that one picks up those mathematical demonstrations which evidently involve perceptions, as once upon a time tried by Immanuel Kant, and then launches an attack on the problem of their automatizing.<sup>12</sup> Does the attack succeed, this should shed light both on the modern issue of automatization of reasoning and Leibniz's engineering dream.

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The references to this author's contributions are listed apart for their additional function: they should round off the text with minute notes to hint at some preparatory steps leading to the point of this essay.

- Marciszewski [1993]. *Arguments Founded on Creative Definitions* in: E. C. W. Krabbe et al. (eds.) *Empirical Logic and Public Debate. Essays in Honour of Else M. Barth*, Rodopi, Amsterdam 1993.  
A study of the role of non-verbal abstraction processes in concept-formation, being an important kind of what Leibniz calls perceptions, and contributing to reasonings in a way comparable to that of axiomatic definitions (cp. [1994a]).
- Marciszewski [1993/94]. *Why Leibniz Should not Have Believed in *flum cogitationis**, "Studies in Logic, Grammar and Rhetoric" 12/13, Białystok 1994, 5-16. The same text appeared in: *Leibniz und Europa. VI. Internationaler Leibniz-Kongress. Vorträge, I. Teil*, Leibniz Gesellschaft, Hannover 1994, 457-464.  
It forms a considerable part of Sections 2-4 of the present essay.
- Marciszewski [1994a]. *Logic from a Rhetorical Point of View*, Walter de Gruyter, Berlin - New York 1994.  
The book contributes to the problems of the present essay through extensive discussion of formalization procedures as contrasted with non-verbal reasonings, while axiomatization procedures are used as a device to study non-verbal processes of concept-formation.
- Marciszewski [1994b]. *A Jaśkowski-Style System of Computer-Assisted Reasoning* in: J. Woleński (ed.) *Philosophical Logic in Poland*, Kluwer, Dordrecht etc. 1994 (Synthese Library vol. 228).  
The paper comments on the proof-checker Mizar MSE, being a preliminary part of the system Mizar. Special attention is paid to how that computerized system of reasoning renders obviousness of inferences (belonging to the Leibnizian domain of perception).
- Marciszewski, Witold and Murawski, Roman [1995a]. *Mechanization of Reasoning in a Historical Perspective*, Rodopi, Amsterdam - Atlanta GA 1995.  
The book tells the history of logic as leading to mechanization of reasoning — through (i) the medieval nominalism merited for the ideas of logical form and of extensionality, (ii) the ideas of formalized language and logical calculus due to Leibniz, Lambert et al., (iii) the success of those ideas with algebraization of logic by Boole et al., (iv) the modern methods of formalization and mechanization of reasoning.
- Marciszewski [1995b]. *Real Definitions and Creativity* in: Vito Sinisi and Jan Woleński (eds.), *The Heritage of Kazimierz Ajdukiewicz*, Rodopi, Amsterdam - Atlanta GA, 1995.

The problem, handled also in [1993], is here related to Ajdukiewicz's theory of real and nominal definitions.

— Marciszewski [1996a]. *Leibniz's Two Legacies. Their Implications for Knowledge Engineering*, "Knowledge Organization" (Frankfurt), 23 (1996) No.2, 77-82.

This text (invited by KO Editors to commemorate Descartes' and Leibniz's anniversaries in 1996) presents Leibniz's views on knowledge against the contrastive background of those of Descartes, and then relates them to some Turing's [1950] and von Neumann's [1951], [1958] ideas.

— Marciszewski [1996b]. *Hätte Leibniz von Neumanns logischen Physikalismus geteilt?*, "Beiträge zur Geschichte der Sprachwissenschaft" (Münster), 6 (1996), 245-259.

The text identical with the author's lecture read at a session of *Leibniz Gesellschaft* in Hannover (1995, June). In a part, it is a German version of [1996a].

Adam Drozdek

## LEIBNIZ: STRUGGLES WITH INFINITY

In his insightful paper on Leibniz, Witold Marciszewski raises an interesting problem of reconciling finitism with infinity of nature<sup>1</sup>. If nature is assumed to be infinite, then how our finite mind can explain anything, how the mind's finite reasoning faculties can match the unboundedness of universe? It is a problem which Leibniz wrestled with all of his life and solved it by assuming infinity as the foundation of both scientific and philosophico-theological considerations.

### 1.

The concept of infinity is already used prominently in Leibniz' juvenile *Ars combinatoria* (1666). Definition 1 says that "God is incorporeal substance of infinite power" and axiom four states that "every body whatsoever has an infinite number of parts" (*Ars*, L 73-74). Hence, the concept of infinity underlies the assumptions of his system and is used in all subsequent proofs. Importantly, the two senses of infinity are used: infinitely large and infinitely small. This understanding of infinity was far from new – both senses can be found at least since Anaxagoras.

Infinity was a concept assumed to be understood in *Ars combinatoria* and as such it was used in proving, among other things, God's existence. Also, if only in passing, Leibniz used the concept of decreasing infinity when pronouncing in Aristotelian spirit that a continuum is infinitely divisible (L 74). As rightly observed by Kabitz, the statement on infinite divisibility of a continuum should be understood literally, otherwise the following proof concerning God would be unintelligible<sup>2</sup>. But what are components of the

<sup>1</sup> Witold Marciszewski, *Why Leibniz should not have believed in 'flum cogitationis'*, "Studies in Logic, Grammar and Rhetoric" 12/13 (1993/94), 5-16.

<sup>2</sup> Willy Kabitz, *Die Philosophie des jungen Leibniz*, Heidelberg: Carl Winter 1909 [reprint Hildesheim: Georg Olms 1974], p. 54.

continuum? Are they themselves divisible? If yes, would this contradict Leibniz' early atomism? Are these components dimensionless? These types of questions made Leibniz soon realize that a continuum is truly a labyrinth.

Infinity was also a link between the natural and supernatural. In *The confession of nature against atheists*, he admits that explanations in science should not constantly resort to supernatural causes; however, this should not mean that a reference to such causes is avoidable, and Leibniz shows that a natural body "is not self-sufficient and cannot subsist without incorporeal principle" (L 110). In his proof, which is "without obscurity and detours," he refers to the chain of causes of motion, and the full reason of motion cannot be given if one body is considered a cause of motion of another body, since such a reply "will be followed by a question through all infinity" (L 111). Hence, an infinite regress is a mark of impossibility in giving a full, natural explanation. Also, the cohesion of the body can be explained, in the spirit of Democritus, by saying that atoms composing this body have hooks which hold the body together, but the hooks must be tenacious enough to enable this. "Whence this tenacity? Must we assume hooks on hooks to infinity?" (L 112). Hence, infinity is used here to demolish naturalist explanations. This infinity is assumed as obvious. However, infinity is worth studying in its own right, and natural sciences are not a proper tool for this study. Mathematics, on the other hand, is.

But even before his Paris period, Leibniz tried to come to grip with infinity, and probably the most serious attempt before 1672 was undertaken in *Theoria motus abstracti* (1671). The problem was that, as we observe in nature, each body and motion have a beginning and end in time and space. Also, as expressed in the first two principles, continuum has actual parts which are actually infinite (L 139). But if each interval can be infinitely divisible, then a beginning or an end of motion or a body would be impossible. Leibniz uses here, very unconvincingly, continued bisection of an interval as a proof, since such a bisection is supposed to leave us with nothing. Therefore, as expressed in the fourth principle: "there exist indivisibles or unextended entities." Hobbes solved this problem by having the smallest magnitudes of time, space, and motion: *conatus* (tendency) was a motion taking place in the smallest imaginable space (*punctum*) and smallest time (*instans*). Leibniz refers here to Cavalieri, who said that there is a spirit more powerful than ours that can number elements of a continuum and thus isolate its constituent parts. We may suppose that this spirit would reach the level of indivisible atoms. However, these parts are indivisibles and yet they are not minimal, since these indivisibles have parts – in direct opposition to the notion of points in Euclidean geometry.

So Leibniz also says in the third principle, "there is no minimum either in space or in body." The troubling point that Leibniz addressed here was that if there were minimal parts (of magnitude zero), then "there are as many minima in the whole as in the part, which implies a contradiction" – a contradiction with the unspoken assumption that a part is always smaller than the whole<sup>3</sup>.

In his discussion of indivisibles, Leibniz relied on their use by Cavalieri, or rather on the aura of reliability of mathematics in which Cavalieri applied this concept. Cavalieri, however, gives no explanation of the indivisible in his *Geometria indivisibilibus* (1635), and stating that certain non-Euclidian facts about the point are "obviously demonstrated" by Cavalieri (principle 5) only betrays that Leibniz knew Cavalieri's method second hand<sup>4</sup>.

In any event, Leibniz is convinced that by using this approach he is able to escape the labyrinth of continuum composition (e.g., to de Carcavy, June 22, 1671, AA ii i 126; to van Velthuysen, early May 1671, AA ii i 97), and his solutions have bearing not only on explaining natural phenomena, as exemplified in *Theoria motus concreti*, but also, and foremost, on psychological and theological issues. For example, *conatus* can last only for a moment – except in the mind, otherwise memory would be impossible. Using this concept, he defines body as a momentaneous mind (*mens momentanea*) (principle 17, also to Oldenburg March 11, 1671, AA ii i 90), which quite elegantly goes beyond Cartesian mind-body dualism<sup>5</sup>. The nature of *conatus*, or, more generally, the nature of the indivisible, is also to be used as a launching pad for proving the immortality of the soul, the existence of God, and the defense of such mysteries of faith as the Eucharist (to Oldenburg, 1670, GM i 46, to Arnauld, GP i 71, cf. iv 225). For example, it can be said that "the mind (*gemüth*) exists in one point, so that it is indivisible and indestructible" (to Johann Friedrich, May 21, 1671, AA ii i 108).

<sup>3</sup> Thus the reference in *Ars combinatoria* to Cardan's statement that "one infinite is greater than another" can be considered spurious, all the more because this reference is removed from the revised version of this dissertation.

<sup>4</sup> Probably from Hobbes' *De corpore* 1.15.2; and *De principiis* 1 or from "Angelus" preface to Digby's *Demonstration immortalitatis animae* (1664), cf. Joseph E. Hofmann, *Leibniz in Paris 1672-1676. His growth to mathematical maturity*, Cambridge University Press 1974, p. 8; Kurt Ufermann, *Untersuchungen über das Gesetz der Kontinuität bei Leibniz*, Berlin: Funk 1927, p. 27.

<sup>5</sup> As pointed out by Rudolf Hahn, this definition of the body "seems to indicate that *conatus* should be understood as some spiritual force," whereby a way for Leibniz' later spiritualism is opened", Rudolf Hahn, *Die Entwicklung der Leibnizischen Metaphysik und der Einfluss der Mathematik auf dieselbe, bis zum Jahre 1686*, Halle 1899, pp. 16, 20.

Philosophical analyses alone did not seem to resolve the problem of infinity. It was possible to escape the labyrinth of continuity because some assumptions were made about infinitely small quantities, hence the problem of infinity was solved only because infinity was accepted at the beginning. Even if it was not an actual infinity, at least an assumption was made that, actually or potentially, infinitely small quantities can be obtained, and, therefore, infinitistic thinking precedes the solution of the problem of infinity. Is there a better way out of the labyrinth? It is true, “the whole labyrinth about the composition of the continuum has to be unraveled as soon as possible... We must see whether it can be demonstrated that there is something infinitely small yet not indivisible,” jotted Leibniz down in his *Paris notes*, “from the existence of such a being there follow wonderful things about infinity” (Feb. 11, 1676, L 159). Interestingly, this note was made after some results of differential calculus had already been obtained. However, it is mathematics which should shed some light on this problem. After all, as Leibniz wrote, “it is not possible to get a thread through the labyrinth concerning the composition of the continuum or concerning the greatest (maximum) and the least (minimum) and the unnameable and the infinite unless geometry gets it; in fact, no one arrives at a sound metaphysics except the man who comes over to it by that way” (GM vii 326). But did mathematical analysis solve the problem of infinity?

In *Nova methodus pro maximis et minimis* (1684), his first published account of differential calculus, Leibniz introduces basic rules of differentiation based on the definition of a differential –  $dy$  is a differential which is to some arbitrary  $dx$ , as ordinate  $y$  is to subtangent  $x$ ; however, this definition relies on the definition of a tangent which is “a line that connects two points of the curve at an infinitely small distance or the continued side of a polygon with an infinite number of angles”, the polygon taking the place of the curve. “This infinitely small distance can always be expressed by a known differential like  $dy$ ” (Struik 272, 276; GM v 220, 223). This is clearly a *circulus vitiosus* in definition which will plague Leibniz’ attempts to build a solid foundation for calculus – and the attempts of his successors until Cauchy. Also, it is interesting to observe that, at least in *Nova methodus*, differentials are finite intervals, and yet Leibniz refers in the definition of tangent to an infinite polygon and to points placed infinitely close to each other, which in turn is to substantiate differentials. Infinity thus precedes the finite; infinity is an assumption on which to build the foundation of the calculus. Although mathematically pregnant, Leibniz’ mathematical analyses did not solve the problem of infinity; at best they showed its depth and richness, at worst they indicated that without

assuming it even mathematical problems concerning finite elements are unmanageable.

Leibniz did not build a solid mathematical theory of the infinite. He explored the foundations of mathematics as a philosopher and theologian and laid foundations for calculus which were developed in a mathematically much sounder fashion by Bernoulli brothers and de l’Hospital. But although Leibniz did not solve the problem of infinity, since he treated infinity as a datum, and did not find an escape from the labyrinth of continuum, mathematics, and especially his own contributions, made him keenly aware of the place of continuum in the whole of philosophical system.

## 2.

Leibniz never had any doubt about an orderliness of the universe and about God being its source. He says, for example, that “a beautiful order” arises in nature “because it is the timepiece of God” (to Thomasius, April 20 1669, L 101). Incidentally, he mentions some traditional expressions of the order in nature, one being that “nature strives for continuity.” He rejects them, since they smack of deism or pantheism, by attributing wisdom to nature, not God. In 1699, he passes lightly over this principle concerning continuity, but he returns to it later, when he does not ascribe it to nature but sees in it a foundation of the order of nature. This principle is the principle of continuity.

In earliest form it appears as an unnamed principle in a 1679 letter in the context of critiquing Descartes’ dynamics: “when causes differ from one another as little as you would wish, and so approach each other that one stops at another, so their effects approach each other also indefinitely so that the difference would become smaller than any assignable value, and one stops at another” (to Craanen, AA ii i 470). However, this principle was given for the first time in print in 1687 under the name of a principle of general order: “when the differences between two instances in a given series of that which is presupposed can be diminished until it becomes smaller than any given quantity whatever, the corresponding difference in what is sought or in their results must of necessity also be diminished or become less than any given quantity whatever” (GP iii 51; L 351)<sup>6</sup>. Hence, the effects are ordered since the causes are ordered. Predictable behavior of causes also

<sup>6</sup> If this wording is expressed in a formal fashion, as in: if for any  $x$  and  $y$ ,  $|x-y| \rightarrow 0$  then  $|f(x)-f(y)| \rightarrow 0$ , then we see that the principle of continuity resembles very closely the definition of uniform continuity of function  $f$ .

makes effects predictable. Orderliness is both in causes and effects, whereby chaos is excluded and any semblance of chaos in nature and society is only what it is – a semblance, an appearance, a result of insufficient knowledge, of missing data, or of imperfect perspicuity of the observer.

Continuity is built into the world as a guiding principle which enables the world to develop, to progress. “The present is full of the past and is pregnant with the future” (to Arnauld, July 1686), and in Alexander the Great we can find “marks of all that had happened to him and evidences of all that would happen to him” (*Discourse* §8). The future is fully predictable, although only God can do this. The history of the world in its entirety and of each substance individually hides no surprises for God, for the infinite God, for the God of order, who – because of his perfection – could not create the world other than perfect, that is, orderly, and the primary means of ensuring this orderliness is the infinity of the world and the principle of continuity that rules in it. God would not have been almighty if the world he created had not been orderly; it would not have been orderly if the principle of continuity had not been the principle of order whose manifestation is a part of each substance. The principle of continuity could not have worked if the world had not been infinite; therefore, the world has to be infinite. Infinity is more perfect than the finite; true finitude is a distortion of the infinite. The infinite is and has to be primary in metaphysical and epistemological order. The finite is a mark of imperfection and everything is as imperfect as it is finite.

The validity of the principle of continuity is also acknowledged in biology. Since no jumps should occur in nature, one can expect transitions between species to be blurred to the extent that the dividing line between plants and animals does not exist (to Bourguet, Aug. 5, 1715, L 664; *NE* 4.16.12). Also, there is no dividing line between life and death. Death is infinitely small life<sup>7</sup> so as rest is infinitely small movement or equality is infinitely small inequality. “Generation and corruption are nothing but transformations from small to great or the reverse,” and because “all matter must be filled with ... living substances,” yet “there is no particle of matter which does not contain a world of innumerable creatures”; a ram burned for offering is transformed into another form, like a caterpillar into a butterfly, and not annihilated (to Arnauld, October 9, 1687, L 345-7; also, L 455, 557). In *Pacidius Philalethi*, Leibniz uses another analogy: nature is like a tunic or a shell with an infinite number of folds, and these folds are also

fold. Folds never become flat (AA vi iii 535). This principle can be found today in the theory of fractals. Fractals repeat themselves indefinitely and any level of magnification reveals the same pattern as the one the process of generating the fractals has begun.

Psychology is not free from the rule of the continuity principle either; in particular, our perceptions are under this rule. Each perception is a ground of the next perception; the sequence of perceptions has no gaps. Although the number of perceptions is infinite, we cannot – because we are finite beings – be conscious of all of them at the same time. But thereby they do not disappear. They still exist as “little perceptions,” as the domain of the subconscious<sup>8</sup>. Conscious acts are only a proverbial tip of the iceberg, the iceberg itself being the subconscious acts. Consciousness exists thanks to subconsciousness; subconscious acts constitute a glue for conscious acts without which the latter acts would be a disordered and hence incomprehensible heap of mental events. So, the infinity of the world and the infinity of our mental structure in conjunction with the principle of continuity leads to the discovery of the subconscious, since the conscious cannot by itself bear the burden of the infinite<sup>9</sup>. Had the subconscious not existed, the conscious would have collapsed. The orderliness of cognition is due mainly to the underlying current of the subconscious events. Only in God is there no need for the subconscious, but it takes an infinite supreme being for this to be possible.

The principle of continuum can be also used as a methodological guideline: If there exist two different events, then we are bound to find an event that mediates them, either synchronically (e.g., a new biological genus between two existing genera) or diachronically (e.g., finding a historical event between two other events). This is possible since, in Leibniz’ system, there are two kinds of continuum, one being an ideal continuum and one being its faint reflection in the real world<sup>10</sup>. Ideal continuum has no parts, or rather its parts are indeterminate; it is infinitely divisible, but not divided. The actual continua are the aggregates of substances. The world’s continuum would be the same as a continuum defined in a

<sup>8</sup> In the words of Ufermann, this understanding of the subconscious was a trump-card Leibniz used against Locke’s psychology, *op. cit.*, p. 66.

<sup>9</sup> Leibniz’ psychology “stems from the infinity and the continuum”, Ufermann, *op. cit.*, p. 72.

<sup>10</sup> Hermann Weyl writes about perceptual and mathematical continua (1918), Friedrich Kaulbach writes about abstract and concrete continua (Das Prinzip der Kontinuität bei Leibniz, in P. Schneider, O. Saame (eds.), *Das Problem der Kontinuität*, Mainz: Krach 1970, p. 12), but cf. Herbert Breger, Leibniz, Weyl und das Kontinuum, in *Leibniz. Werk und Wirkung. IV Internationaler Leibniz-Kongreß. Vorträge*, Hannover: Gottfried-Wilhelm-Leibniz-Gesellschaft 1983, pp. 80-81.

<sup>7</sup> This view is by means new; a comentator says that according to Psalm 143:3 “death is thought of as a gradual ebbing of life which continues even in the grave,” J. H. Eaton, *Psalms: Introduction and commentary*, Bloomsbury: SCM Press 1967, p. 307.



standard set theory according to which the continuum is a dense and ordered collection of points. The ideal continuum, although much less frequently used in today's mathematics, can still be very rigidly defined, as exemplified by Brouwer's and Hermann Weyl's intuitionistic mathematics or Paul Lorenzen's constructivist approach.

The ideal continuum is a top-down entity, which encompasses an infinite number of indefinite parts. The continuum itself precedes the parts, the subcontinua. The real world continuity is bottom-up, it is being produced from an infinity of elements to result in a dense set. A similar distinction is true also about infinity. A true infinity, a perfect infinity, which can be found also in God, is characterized by its wholeness, by "being anterior to all composition," by "not being formed by the addition of parts" (*NE* 2.17.1). Infinity of points or of instants, on the other hand, is an infinity of a lesser kind. It is like an infinity of numbers which is an accumulation or aggregate of numbers, "not a whole any more than the infinite number itself, whereof one cannot say whether it is even or uneven" (*Theodicy* §195). In this sense, an actual infinity encountered in the world is less perfect than the ideal infinity found in God and in his ideas, the infinity which contains parts only potentially.

Both real and ideal worlds are continuous, but continuity is understood differently. Therefore, the difference is made not between discrete and continuous, but between two kinds of continuity. This should be clear from the statement that "all repetition is either discrete as where parts are discriminated ... or continuous when the parts are indeterminate and can be assumed in infinite ways" (1702, GP iv 394). Hence, the ideal continuum is undiscriminating, the actual is discriminating. We can describe matter as discrete, meaning thereby not a discrete set but a set composed of parts, and each part is, to be sure, different than another part, and in this sense discrete or discriminate. So the gap between the ideal and actual worlds is not unbridgeable, since continuum can be found in both of them<sup>11</sup>.

"Uniformly ordered continuity, although it is nothing but supposition and abstraction, forms the basis of the eternal truths and of the necessary sciences ... Matter appears to us [to be] a continuum, but it only appears so, just as does actual motion" (to princess Sophia, Nov. 30, 1701, GP vii 564).

<sup>11</sup> Therefore, there is no need to limit the number of objects in the world to the countable. Even if the set of objects in the world were not dense, but discrete, it would not imply that "there are at most countable number of objects in the universe", as claimed by Monika Osterheld-Koepke, *Der Ursprung der Mathematik aus der Monadologie*, Frankfurt: Haag u. Herchen 1984, p. 80, since the number of discrete objects can well surpass any cardinality (the sequence of all cardinals being a good example).

Matter has an appearance of *ideal* continuity since it is already actually continuous. And so does motion. It is like watching a movie that invokes in us an illusion of continuous motion although the movie is just a sequence of discrete frames shown at the rate of 24 frames per second. However, the actual motion does not have 24 frames per second but a dense set of frames. Motion so understood arouses in us an impression of continuity. But this is only continuity of a lesser kind. Our reality is perceived as truly continuous since continuity is within us, in our minds, as an abstraction, as an idea. "Space, just like time, is a certain order ... which embraces not only actuals, but possibles also" (to des Bosses 1709, GP ii 379, also 336). "Space is something continuous but ideal" (*ib.*), or rather it is continuous because it is ideal. What is actual, can only be, so to speak, a breakable continuity, since true continuity would defy the reality of the phenomenal world. "In the real world, if matter were not divided up, there would be no distinct things" and this actual division and discrimination presupposes simple substances (to de Volder, GP ii 276). The order in the real world can be preserved if the mind sees the world through the mirror of continuity, if the law of continuity is assumed to be effective. The mind requires perfect continuity, and yet the nature of the world refuses it. In this, atomism was never truly abandoned by Leibniz, since to create real unities, a "real and animated point, or an atom of substance" is needed (*Système nouveau* (1695), L 460 note 3).

These atoms of substance are the only true realities since even matter has a borrowed reality. Matter is a phenomenon and is something between the mental things and the real things. There is an infinite number of these things, and hence, matter can be infinitely divisible. At the bottom of this division, which we never reach, there are real things. In the order of nature, these things constitute matter; these atoms of substance are primary, and only thanks to them matter emerges as a phenomenon. In the realm of mental things, continuum, a whole, is primary, and parts are secondary. These parts are *in potentia*; they are indefinite.

These atoms of substance also indicate that the two kinds of continuum are not separate entities, real continuum being an approximation of the ideal continuum. We should remember that the monad is without windows, and as such its knowledge comes from within, it is inscribed, if only in the form of little perceptions, in the monad itself. Leibniz ascribes actual infinity even to the smallest elements as mirrors of the universe. "The present is full of the past and is pregnant with the future" also in the form of the knowledge the monad possesses, so that the knowledge of the present is full of the past knowledge and is pregnant with the future

knowledge<sup>12</sup>. That is, ideal continuum is within us; it is inscribed in us. The atoms of substance which constitute a continuum of substances are based on the ideal continuum; the ideal continuum is within them. So a real continuum is an infinite and dense aggregate of entities, each one of them including ideal continuum. Continuum within continuum, infinity within infinity. It is somewhat analogical to an infinite set of real numbers from the interval (0,1), each number defined as an infinite sequence of digits following zero and the decimal point. Or, in Brouwer's mathematics, it is similar to an infinite set of points, each point being defined as an infinite and converging sequence of rectangles. In fact, a similar approach is also found in Leibniz, for whom the irrational numbers are an infinite series of rational numbers (*Nova algebrae promotio*, GM vii 156, v 308), whereas for Descartes they are segments of a line (hypotenuses).

It can be claimed now that such understanding of irrationals is a predecessor of the monad concept. The concept of the irrational incorporates the concept of infinity, and so does the concept of a windowless monad which possesses the concept of infinity and continuity. This influence of mathematics on philosophical concepts is magnified by an impact of the infinitesimal concept on that of the monad. An infinitesimal also included an infinity, as exemplified already in *Theoria motus abstracti*. Infinitesimal can be smaller than any number, hence smaller than an infinity of numbers. Consequently, although implicitly, infinity is included not only in the name of infinitesimal but also in its concept. In this sense we may agree that infinitesimal "became a foundation of the world in the concept of monad"<sup>13</sup>.

Although the real world is not truly continuous, it is nevertheless infinite, and so is the ideal continuum. The infinity constitutes the link between the real and ideal worlds, and it allows us to apply the continuity principle in our world as well. Infinity can be ordered or disordered, it can be continuous or discrete; therefore, infinity by itself is an insufficient key to the universe. An order has to be added to it, and the order is supplied by the continuity principle, by the principle of general order. This order, the continuous order, on the other hand, would be impossible if it were not for an underlying infinity since finite universe cannot be ordered in the sense required by this principle. After all, the principle of continuity "has its origin in the infinite" (1687, L 351), and consequently in God, because only he is truly infinite. God's infinity is, therefore, poured into the world's infinity.

<sup>12</sup> Cf. Kaulbach, *op. cit.*, p. 14: The fact that each being "reflects in itself the totality of the world with all its content" was to be an exit from the labyrinth of continuum.

<sup>13</sup> Miodrag Cekić, *Infinitesimalni račun i monadologija*, Niš 1980, p. 49; cf. review by Radmila Šajković in "Studia Leibnitiana" 13 (1981), pp. 151-154.

Our human task is exploration of the world. This exploration is the means of glorifying God through recognizing his greatness in the greatness of the universe. This exploration requires sharp tools, and God's means of creations indicate what tools they should be. One tool necessary for exploring the world is infinity; thus, the better we know the latter, the fuller our knowledge can be. Therefore, an exploration of infinity in its own right has its merit. Furthermore, since God "acts as a perfect geometrician" (L 351), since it is true that "God uses geometry and that mathematics makes up a part of the intelligible world," mathematics "is therefore more fit to be an entrance into it" (1702, L 585) and this knowledge of the world, both real and ideal, can be acquired best through geometry, or, more generally, mathematics.

Such a stance is, by the way, a break with the tradition of Cartesianism. For Descartes, infinity was a sacred property ascribable only to God. Mathematics was separate from theology; it investigated its own world of numbers and figures; therefore, infinity, as a sacred property, did not belong to mathematics. Thus, because Descartes did not consider infinity to have a legitimate place in mathematics, he did his best to avoid using it in his proofs. Occasionally, Descartes showed that he can be quite proficient with the use of infinity in proofs (cf. his proofs of de Beaune problem), but such proofs were to him inadmissible. For Leibniz, mathematics was an extension of theology, a field allowing for deeper investigation and gaining better understanding of concepts that theology also may ponder upon, in particular, the concept of infinity.

However, Leibniz agrees with Descartes in the priority of the idea of infinity in epistemological order. According to Descartes, because "there is more reality in an infinite substance than in finite substance," "there is in me somehow in the first place understanding of the infinite before the finite, i.e., [understanding] of God before myself" (third *Meditation*, AT vii 45). This principle was not expressed more explicitly by anybody before Descartes; consequently, it can be called Descartes' principle<sup>14</sup>.

A similar thought is also found in Leibniz: Infinity can be understood as an infinity that unfolds itself, as in numerical series. But it can also be understood as a complete whole, a positive infinity. "The positive infinity is nothing but the absolute," hence we have an idea of positive infinity, and "this [idea] precedes the [idea] of the finite" (*Sur l'Essay de l'entendement de Monsieur Locke*, GP v 17). "The idea of infinity does not come from stretching the finite ideas" (*NE* ii 23). We have an idea of this infinity,

<sup>14</sup> Adam Drozdek, Descartes: *Mathematics and sacredness of infinity*, "Laval théologique et philosophique" 52 (1966), 167-178.

since we know about the absolute, because we simply participate in it, and thereby we possess some measure of fullness (*Entretien de Philarete et d'Ariste*, GP vi 592).

In the light of Descartes' principle and of the infinity built into each monad, we have to disagree with the statement that according to Leibniz, "our finite mind can take it [infinity] only as a *sign* abbreviating some operation which is impossible for us" to conduct<sup>15</sup>. If infinity played only this symbolic role in the human mind, then such an infinity would not be infinite any longer, all the more that an operation for which it stands is undoable. If it is reduced to a mere sign, then no cognition is possible since finite can be understood only in the light of the infinite. However, because the "intelligible world" of ideas "is in God and in some way also in us" (L 585)<sup>16</sup>, there is much more to the concept of infinity than a symbolic side. Infinity is in us not only as an idea, but also in the form of sequence of perceptions. Furthermore, if we refer to the latter, then this gives us a meaning in which infinity can be understood as a sign, namely as a blind or symbolic intuition (e.g., *Meditationes de cogitatione, veritate et ideis* 1684, L 291-292). We are not aware of everything at the same time, but this does not preclude the subconscious area of thinking from existence. Infinity is very real in us although, because the mind is finite, it cannot make the full use of infinity. But it is indispensable, otherwise, by Descartes' principle, no cognition would be possible. We can do a great deal with it, and calculus is but one proof of that.

Descartes' principle also has a methodological offshoot in Leibniz. To him, an entity can be perfectly distinguished from another entity if it is completely described, but such a description is nothing short of an infinite list of characteristics. "Individuality involves the infinite, and only he who understands the latter can have first-hand knowledge of the principle of individuation of this or that thing; this arises from the influence of all things in the universe on one another" (NE 3.3.6). Full description of an entity requires giving all relationships of this entity with everything else in the world, this world which is infinite. In this way, infinity again has to precede the finite.

<sup>15</sup> Gilles-Gaston Granger, *Philosophie et mathématique leibnizienne*, "Revue métaphysique et morale" 86 (1981), p. 29.

<sup>16</sup> Cf. also "insofar as our intellect is a reflection of his, we may say that God has an intellect similar to ours and that God understands things as we do; but there is this difference, namely, that he understands them simultaneously in an infinite number of ways, but we only in one" (1676, AA vi iii 400, cf. p. 523).

### 3.

The real world is but one realized possibility; the real is a manifestation of the possible. The possible precedes the real, and in this sense the possible can be considered more real than the real itself. All possibilities are in God's mind, and although they become instantiated to be real, real to us, they are real as ideas in God's mind, they are not illusions or figments of an imagination. The law of continuity can be fully applied to the ideal, and although it is not fully applicable to the actual, it is not thereby suspended; only its manifestation is limited, and in our eyes it may appear to have limited validity. This law is as valid in the ideal as in the real, but because the real is just a carved-out portion of the immensity of the possible, manifestation of this law is not full; it is also carved out<sup>17</sup>. After all, it is "quite true ... that the existence of intelligible things ... is incomparably more certain than the existence of sensible things and it would thus not be impossible ... that there should exist at bottom only intelligible substances, of which sensible things would be only the appearances. Instead, our lack of attention causes us to take sensible things for the only true ones" (1702, L 549).

"There is an intelligible world in the divine mind," "the region of ideas." Minds "are produced as images of divinity. The mathematical sciences, which deal with eternal truths rooted in the divine mind, prepare us for knowledge of substances" (1707, L 592). Thus, although it is true that "spatial concepts have become prior in knowledge to one's concept of bodily extension," this, in Leibniz' view, cannot be reconciled with the view that "bodily extension is metaphysically more fundamental than space"<sup>18</sup>. The order of the mental is prior to that of the material; consequently, the concept of space and time, i.e., of continuum, precedes the concept of extension and duration. "[T]he seeds of the things we learn are within us – the ideas and the eternal truths which arise from them." The innate ideas are much to be preferred over the concept of *tabula rasa* (1707, L593), and Plato's reminiscence thesis is "a well-founded doctrine" (*Discourse* 26). Abstraction of space and time, therefore, does not consist of creating these concepts from observations of extension and duration. The latter may sharpen the former, may make us realize their existence, but not create them. They may

<sup>17</sup> "The ideal is inherent both to the possible and the actual – so far as the latter can be considered the possible", Alexandru Giaculescu, *Der Begriff des Unendlichen bei Leibniz und Cantor*, in *Leibniz* 1983, p. 880.

<sup>18</sup> Glenn A. Hartz and J. A. Cover, *Space and time in the Leibnizian metaphysics*, "Nous" 22 (1988), p. 511.

accompany them since “there is never an abstract thought which is not accompanied by some images or material traces” (1702, L 556; also 551), but in no wise are they “metaphysically more fundamental.”

Although “continuity is something ideal ... the real never ceases to be governed perfectly by the ideal and the abstract. ... This is because everything is governed by reason” (1702, L 544). The abstract determines the structure of the universe, of the possible, and of the real. The essence of being can be found in the abstract, hence the abstract is the basis of all its possible instantiations, including the real world<sup>19</sup>. Without reason the world would not exist; the abstract precedes its manifestations in chronological and metaphysical order, and therefore it can be considered more real than reality itself. The abstract encompasses infinity and is embodied in an infinite number of possibilities.

Leibniz is careful in distinguishing these two different modes of reality. God’s understanding, the source of essences, contains ideas of possibilities, whereas his will, which chooses the best possible world, is the domain of existences (*Theodicy* §7). The modes of existence of possibilities and their actualizations are different, but both possibilities and existences are nevertheless real.

What is thus the difference between these two realities? “[T]here can be nothing real in nature except simple substances and the aggregates resulting from them” (1706, L 539). Having discriminate parts is the distinctive feature of this world of ours. On the other hand, “continuity is something ideal”; there is nothing perfectly uniform in nature (1702, L 544). Our reality is the realm of the discrete, the continuous is something ideal and is related to the possible, since it is indefinite and indeterminate, whereas there is nothing indefinite in actual things (to de Volder, Jan. 19, 1706, L 539). But although true continuity cannot be found in this world, the principle of continuity is applicable in it, since infinity permeates both the world of ideal and the world of existent and makes both these worlds real. So, for example, aggregates can have any number of objects, hence the number of objects in the world is infinite, and there is an infinity of planets like ours in the universe (*Theodicy* 2.19). Infinity is a platform on which these two worlds meet; it enables possibilities to be realizable as existences, it enables the existences to be considered as emerging from possibilities. Therefore, “the knowledge of the continuous (*scientia continuorum*) ... contains eternal

<sup>19</sup> Cf. Osterheld-Koepke, *op. cit.*, 77: “The abstract being is the structure of being, the structure of both possible and real being; it includes all possibilities of thought. So it is not independent, but it is an essence of being. The abstract being can recur in infinitely many appearances.”

truths which are never violated by actual phenomena, since the difference is always less than any given assignable amount” (1706, L 539); this situation would have been impossible had the world been finite.

Infinity is also a platform enabling an order in these two worlds. First, the realms of the possible and the actual are ordered through continuum; and the principle of continuity expressly states the nature of this orderliness of continuum. A harmony is only possible through continuum, and hence, through infinity. Continuum *is* order; it *is* harmony<sup>20</sup>. So, for instance, “space and time taken together constitute the order of possibilities of the one entire universe.” Because the actual is secondary to the possible, it cannot break out of the laws to be found in the realm of the possible, and “the actual phenomena of nature are arranged, and must be, in such a way that nothing ever happens which violates the law of continuity” (L 583). The world could not exist if it were not ordered, i.e., if it were not under the rule of the law of continuity. However, the world can be under the rule of this law only because the actual world is infinite, and so is the world of the possible. Infinity is an underlying assumption of this orderliness: there can be an unordered infinity, but not a non-infinite continuum. Infinity is, so to speak, a foundation upon which the order worthy of the infinite author can be built. The world exists because it comes from an author of order; it exists because it is ordered, and because it is infinite. Infinity of continuum enables this order, and since the actual world is not continuous, the order is ascertained through its infinity. An infinite author, God, is its source, and its infinity is the surest link between the world and God. Actual infinity affects nature everywhere “to better mark the perfection of its author. I also believe that there is no part of matter which would not be, I don’t say divisible, but actually divided, and consequently the smallest particle should be considered as a world full of an infinity of diverse creatures” (to Fouchet, 1693, GP i 416).

#### 4.

What is the result of Leibniz’ investigations as far as the concept of infinity is concerned? The concept led him from philosophy to mathematics and then back to philosophy. However, these philosophical and scientific peregrinations did not subdue the problem of infinity. Leibniz for a

<sup>20</sup> “The activity of God’s reason established an order as continuity”, Rainer Piepmaier, *Aspekte der Erinnerung bei Leibniz*, in *Leibniz* 1983, p. 601.

very good reason was very proud of his new calculus. This is reflected, among other things, in terminology he uses. He says namely in *A new method for maxima and minima* that his method “also covers transcendental curves.” As remarked by Struik, “this may be the first time that the term “transcendental” in the sense of “nonanalytical” occurs in print<sup>21</sup>. Leibniz’ distinction between transcendental and algebraic curves corresponds to Descartes’ division of curves into mechanical and geometrical, but for Leibniz, unlike for Descartes, mechanical lines can be analyzed in mathematics. By naming mechanical curves transcendental, Leibniz alludes to scholastic tradition in which the concept of infinity is transcendental, since it surpassed man’s powers. From the fact that transcendental curves and functions are those for which no finite number of algebraic operations suffices, and from the fact that Leibniz’ calculus can grapple with these lines, a seemingly inescapable conclusion may be drawn, to the effect that Leibniz tamed infinity. This, however, is not the case, since the concept of infinity underlies Leibniz’ definitions as already indicated in this paper. Infinity cannot be elucidated by mathematical means unless some knowledge of the infinite is assumed. And this fact may have been one of the most important realizations to which Leibniz was led by his mathematical investigations. Infinity is unconquerable with finite means. Some understanding of infinity must precede any attempts to build an axiomatic system, and this understanding is simply a given, a part of human cognitive apparatus.

This turned out to be true in philosophy and even more so in theology. His struggles with the labyrinth of continuity could be crowned with some measure of success only if infinity underlay all his attempts. In this he is an heir of Descartes by assuming more or less explicitly that infinity is clearer than the finite, and even grappling with the finite must assume some insight of the infinite. Hence, Descartes’ principle is always used by Leibniz.

Two most difficult problems for Leibniz were the composition of a continuum and the problem of freedom. Leibniz recognized the source of this problem, which was infinity. This is an interesting tie to Kant who looked with wonderment at two things: the starry sky above him and moral law within himself. Leibniz was puzzled by the infinitely small and by morality. Kant wondered about the infinitely large and also about morality. Both of them were concerned with infinity and the moral dimension of man. Philosophically, they have Pascal as an intermediary, for whom the human

being, a trembling reed, is suspended between two infinities. Infinity is a part of human condition wherever human beings lay their eyes. Infinity cannot be conquered; if it can, then only partially and only because we are saturated with infinity, only because Descartes’ principle is at work, only because it is the foundation of all.

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<sup>21</sup> D. J. Struik (ed.), *A source book in mathematics, 1200-1800*, Cambridge: Harvard University Press 1969, p. 271, note 8. Cf. also Wilhelm Wundt, *Leibniz. Zu seinem 200jährigen Todestag*, Leipzig 1917, p. 30.

Jerzy Kopania

## DESCARTES' GREAT THESIS ON NATURE

### I

Carrying out the reasoning process of total methodical doubt Descartes came to the conclusion that the only primary and undoubtful thesis was: "I think, therefore I am". However, what can I think of if everything is doubted with respect to its existence? It is clear that I can think of my own thinking only. Thinking is thus a totally immanent process, which determines its self-sufficiency, because directing it toward external reality would be incompatible with immanence. In other words: the inner content of the mind is the only object of cognition.

But if thinking is a totally immanent process, is it possible to demonstrate that there exists anything except the mind of knower? That is how a certain problem arose in Descartes' system, a problem which had never been posed before – at least not so clearly and consciously. Descartes formulated the question of the existence of the universe and it was a philosophical question, that is to say: a theoretical, not a practical one.

Contrary to the scholastic followers of Aristotle, which one and all based on the uncritical confidence in the objective obviousness of the world, Descartes was of the opinion that philosophy should prove the existence of the independent world rather than assume it dogmatically. In order to produce such a proof it was necessary to assume that the reality of the mind was not augmented by anything derived from the reality of the world, which meant assuming that the human mind had a complete set of innate ideas at its disposal, ideas being both the only object of cognition and the necessary condition of its possibility and validity. Lack of this assumption portends a vicious circle in demonstration: by claiming that the content of the mind is augmented by some elements of extra-mental reality one assumes the existence of this reality, which is exactly what was to be proved.

Additional confirmation of such a standpoint was the mechanistic vision of the universe held by Descartes. If all events taking place in nature are of purely mechanistic character, than anything that is not mechanistic cannot affect the mind from outside. And since the mind is aware of not only this kind of influence – we experience the objects, and not the stimuli, as Kant will say later – everything else must be innate in it. In other words: the elements of independent world only create an opportunity for the mind to become aware of what is innate in it. In this way the mechanistic standpoint started to play the role of a verifying factor of the idealistic viewpoint in philosophy.

## II

It might seem that some basic contradiction emerged in Descartes' reasoning: as philosopher he propagates the thesis that the only direct object of cognition is the inner content of the mind, while as man of science he does empirical research in natural reality in a variety of its aspects. If all cognitive operations are carried out on ideas only, what can the role of empirical research be?

Descartes thought that on the ground of his philosophy he proved the existence of the universe. Yet, the approach to it was exclusively indirect through the inner cognitive structures of the mind. That, however, brings about the appearance of some essential problem – how to distinguish that which is only possible (viz. only inside the mind) from that which is realized in the world outside the mind. The adequacy between the cognitive structure of the mind and the structure of the universe is not simply established but still has to be achieved. The point is that only one of the possible explanations is actually connected in a necessary way with the phenomenon being explained and it has to be determined somehow which one it is. An ordinary experiment is not sufficient here and one has to resort to crucial experiments. That is why emphasizing in *Discourse on Method* the fact that he does not know any physical phenomena which could not be explained by means of his principles, at the same time Descartes expresses his reservation: "But I must also confess that the power of nature is so ample and so vast, and these principles are so simple and general that I observed hardly any particular effect as to which I could not at once recognize that it might be deduced from the principles in many different ways; and my greatest difficulty is usually to discover in which of these ways the effect does depend upon them. As to that, I do not know any other plan but again to try to find

experiments of such a nature that their result is not the same if it has to be explained by one of the methods, as it would be if explained by the other."<sup>1</sup> Thus the point is in planning experiments in such a way that they might produce various results, depending on the methods of explanation employed. The difficulty of which Descartes speaks consists in time consumption and big costs of carrying out such experiments. Hence, it is necessary to limit them to several selected issues and to cooperate with other researchers.

The investigative method of Descartes was in its assumption supposed to be a synthesis of deduction and experiment because it was to discover the necessary and unchangeable relation between the *datum* of experiment and the *quaesitum* of theory, which seen from the methodological viewpoint resolved itself into discovering the essential connection of foreseeing and explaining. On the basis of experimental data subjected to initial analysis and classification, hypothetical suppositions concerning their cause are introduced. The complexity and magnitude of nature cause the suppositions to be varied and numerous, and therefore empirical conclusions deduced from them will also be varied and numerous. Hence, to eliminate false suppositions one needs to use the crucial experiment (Bacon's *instantia crucis*) and – speaking in Mill's language – the methods of agreement, difference, and concomittant changes. If the elements of a theory precisely correspond with the elements of a phenomenon, then the true case was discovered. In such a way the theory was on the one hand supposed to explain facts, while on the other – facts were to confirm the theory.

## III

The question of how to understand the proposition that theory explains facts, being at the same time confirmed by them, Descartes touched upon in the *Discourse on Method*. "If some of the matters of which I spoke in the beginning of the *Dioptrics* and *Meteors* should at first sight give offence because I call them hypotheses and do not appear to care about their proof, let them have the patience to read these in entirety, and I hope that they will find themselves satisfied. For it appears to me that the reasonings are so mutually interwoven, that as the later ones are demonstrated by the earlier, which are their causes, the earlier are reciprocally demonstrated by the later

<sup>1</sup> AT VI, 64-65; HR I, 121. References beginning with AT are by volume and page to *Oeuvres de Descartes*, ed. by Adam and Tannery, Paris 1974-1983, and with HR are to *The Philosophical Works of Descartes*, transl. by E. S. Haldane and G. R. T. Rose, Cambridge University Press, 1967.

which are their effects. And it must not be imagined that in this I commit the fallacy which logicians name arguing in a circle, for, since experience renders the greater part of these effects very certain, the causes from which I deduce them do not so much serve to prove their existence as to explain them; on the other hand, the causes are explained by the effects.”<sup>2</sup>

The above text could cause misunderstanding. Jean-Baptiste Morin had his doubts in this respect and he expressed them (along with a number of other scientific problems) in a letter to Descartes written on February 22, 1638. In his reply Descartes tries to give explanation of the terms used in the above quotation from the *Discourse*. He naturally admits that proving effects by means of causes and then proving causes by the same effects actually means a vicious circle in reasoning. However, in the case under consideration, the problem consists in something else. “I do not agree that it is circular to explain effects by a cause and then prove the cause by the effects; because there is a big difference between proving and explaining. I should add that the word ,demonstrate. can be used to signify either, if it is used according to common usage and not in the technical philosophical sense. I should add also that there is nothing circular in proving a cause by several effects which are independently known, and then proving certain other effects from their cause.”<sup>3</sup>

Descartes defends himself pointing out that he used the word “demonstrate” (*démontrer*) in its everyday and not technical sense; whenever he means demonstration in mathematical or philosophical sense, he uses the form “prove” (*prouver*) and it is this understanding of demonstration that he contrasts with explanation. Stating in the *Discourse* that effects demonstrates causes and causes demonstrate effects, Descartes includes two meanings (viz. prove and explain) in one word “demonstrate”. In spite of that he does not think he might be charged with ambiguous pronouncements because of the immediately expressed reservation that the causes from which effects are deduced serve not so much as a means of proving those effects, but rather as a means of explaining them. Hence it is the causes that are proved by effects. Descartes says in addition: “And I put ,serve not so much to prove them. rather than ,do not serve at all., so that people could tell that each of these effects could also be proved from other effects. I do not see what other term I could have used to explain myself better.”<sup>4</sup>

<sup>2</sup> AT VI, 76; HR I, 128-129.

<sup>3</sup> AT II, 198; K, 57-58. References beginning with K are by page to: Descartes, *Philosophical Letters*, transl. and ed. by A. Kenny, Oxford, Clarendon Press, 1970.

<sup>4</sup> AT II, 198; K, 58.

The explanations given by Descartes are formulated in a somewhat complicated stylistics, though one can grasp his understanding of the relations between cause and effect. Between statement C ascertaining a cause and statement E ascertaining an effect there must be the relation of logical inference. Then statement C is the reason of statement E and statement E is the consequence of statement C, viz. statement E follows from statement C. That is how one should understand Descartes' ascertainment that he deduces effects from causes. The relation of inference can be characterized only syntactically, though semantic characteristics can also be given. On the grounds of pure syntax one determines inference, the so-called syntactic derivability, in the following way: statement S1 is a logical consequence of statement S2 if and only if the implication whose antecedent is S1 and consequent is S2 is a substitution of a logical tautology, viz. of such a sentential schema which turns into a true statement at each substitution. Semantic consequence, in turn, is characterized by means of the notion of a model: statement S1 is a semantic consequence of statement S2 if and only if in each model in which S1 is true S2 is also true. Syntactic consequence is thus characterized with respect to the given deductive system while semantic consequence with respect to the models of that system. When Descartes says that he deduces an effect from a cause, such a pronouncement should be interpreted as referring to semantic consequence. After all, he does not build a formal system on the ground of which inference would take place. He has at his disposal a set of laws composed of the so-called eternal truths.<sup>5</sup> It is a heterogeneous set made of logical and mathematical truths as well as ontological theses. If, however, we treat this set as a specific system, the world of nature will be its model. Then the ascertainment that statement E follows from statement C should be understood in the sense that statement C cannot be true and statement E false in any conceivable world being a model for those laws. This is of course secured by logical relations between the elements of the set of truths and it is in this sense that semantic consequence would be equivalent to syntactic consequence if we treated that set as a formal system.

Then one may speak of deducing, just as Descartes does. Here we have a situation in which only true statements follow from true statements and thus our inference is infallible, that is deductive. It is in this context that

<sup>5</sup> These laws are eternal but created by God. Descartes identified the creation of laws with the creation of essences, viz. the eternal truth is the same as the essence of what the truth is about, e.g. the essence of the circle lies in its being a space enclosed by a curved line every point of which is equidistant from the center, and this thesis is the eternal truth about the circle.



investigative procedure should be understood as consisting in selecting for a given effect such a cause from which it will logically follow. Of course, such a procedure is not any demonstration, and Descartes emphasizes it. The cause selected in such a way does not prove the effect. Proving refers to statements not recognized yet as true statements; they will be done so only after producing the proof. In other words: on the basis of statements recognized as true, the truthfulness of statements being their logical consequence is proved. In the case under consideration it is the statement ascertaining the effect that is recognized as true and not the statement ascertaining the cause. If statement C were in some way proved, thus at the same time recognized as true, and since E is a logical consequence of C, then statement E could be proved by means of statement C. That is the situation to which the latter of the quotations from Descartes refers. He emphasizes the fact that such a procedure would be recommended if any doubts concerning the truthfulness of the statement E arose. Demonstration is necessary when the truthfulness of the statement being demonstrated is not obvious.

A question arises immediately as to the purpose of selecting a cause for a known effect, viz. selecting a statement ascertaining the cause for the statement ascertaining the effect which is known to be true statement. The purpose is to give explanation of the effect; for even if we recognize the statement ascertaining the effect as true, we do not know the reason underlying the truthfulness. The explanations of some state of affairs consists in answering the question why this state of affairs came into being, which is the question concerning its cause. The question can only be answered in such a way that we give the statement whose logical consequence is the statement ascertaining that state of affairs. The direction of explanation is thus in accord with that of logical inference, i.e. deduction, and the explanation concerns the statement already recognized as true.

However, the very occurrence of the relation of logical inference does not suffice, viz. it is not always so that if statement S2 is a consequent of statement S1 we may recognize S1 as explaining the state of affairs ascertained in S2. The occurrence of the relation of logical inference between statements is a necessary condition, though not sufficient for the relation of explanation to occur between them. Moreover, the very explanatory statement must be of a special type, viz. it must possess a certain feature thanks to which it explains the statement that follows from it. In the case of one single fact the statement explaining it will be the statement ascertaining its cause. When we deal with a recurrent phenomenon, the statement explaining this phenomenon must be a general

statement expressing some law of nature for it is only then that an adequate conditional statement will describe the relation of causal determination. In other words: statement C explains statement E if and only if statement C is a law whose logical consequence is statement E. The conditional statement "if C, then E" describes the relation of causal determination occurring between the state of affairs ascertained by the law expressed in statement C and the state of affairs ascertained by statement E. Obviously, because of the complex character of natural phenomena and the entanglement of explanatory statements in the explanatory theory, in the actual process of explanation we mostly deal not with a single statement C but with a conjunction of statements from which statement E follows.

The above settlements make it possible to give a certain interpretation of those parts of the *Discourse* and the analysed letter in which Descartes claims that the effect proves cause. The statement ascertaining the cause is selected in such a way that the statement ascertaining the effect might follow from it and an adequate conditional statement might be the description of the relation of causal determination thanks to which the explanation of the analysed state of affairs is given. How should one understand in this context the statement claiming that the effect proves the cause?

Proving consists in the fact that on the basis of statements recognized as true one demonstrates the truthfulness of statements which are their logical consequence. Hence, statement E ascertaining the effect is obviously recognized as true, but statement C ascertaining the cause does not follow from it (the direction of inference is opposite). If, then, the effect is to demonstrate the cause, the very statement E cannot be a premise; most likely the point is in some conjunction of statements where statement E is one of its elements and statement C is its logical consequence. Since the conditional statement "if C, then E" expresses the relation of causal determination, we may assume that an additional conjectural premise also concerns this relation.

Descartes was very firm in the way he propounded his thesis according to which no thing exists without the cause of its existence. He put forward the following thesis as his axiom: "Nothing exists concerning which the question may not be raised – ,What is the cause of its existence?"<sup>6</sup> In the terminology derived from Aristotle the cause bringing about the realisation of being (the fact of beginning to be) used to be called the efficient cause. The acting of the efficient cause was expressed in the form of the so-called principle of causality. Most generally it can

<sup>6</sup> AT VII, 164; HR II, 55.

be formulated as follows: "everything that comes into being has its cause", which is tantamount to saying: "every effect has its cause". In the methodological procedure described by Descartes for statement E ascertaining the occurrence of a certain state of affairs one selects reason C explaining this state. Consequently, one formulates a certain description of causal determination dependence. Thus, one assumes here the principle of causality, though without expressing it overtly, and after accepting statement C as explanatory we actually obtain two statements: (1) "the state of affairs ascertained in statement E did occur", and (2) "if the state of affairs ascertained in statement E occurred, then its cause described in statement C also existed". By virtue of the rule of *ponendo ponens* a logical consequence of the above two statements is the statement: (3) "as described in statement C, the cause of the state of affairs ascertained in statement E did exist". Thus, the above inference is an act of proving because on the basis of statements (1) and (2) recognized as true one comes to recognize statement (3) as true, too. If Descartes claims that the effect proves the cause, then according to the presented interpretation one should understand it in such a way that the statement ascertaining the occurrence of a given state of affairs proves the statement ascertaining the existence of the cause of this state of affairs.

#### IV

Any deductive inference is infallible, but the infallibility concerns the course of inference and cannot guarantee the truthfulness of premises. Deductive inference makes it certain that the conclusion is a true statement if all premises are true statements. The problem, though, is that a true conclusion might be drawn from false premises. That is why the crucial experiment was of such vital importance: it enabled the elimination of all causes except one from the set of all those that were possible. A true conclusion could be inferred from a true premise then. It is striking that Descartes, who emphasized the heuristic futility of syllogistic (i.e. deduction), at the same time put so much attention to the deductive method of pursuing science, the method which scholastics believed to be the only permissible one. His standpoint becomes understandable when we take into consideration the difference in points of departure. If a conclusion obtained deductively contains in its substance only that which was already contained in the premises, the selection of premises becomes the decisive issue. Descartes' premises were basically different from those of scholastics.

Their point of departure was the external world in the way it appeared to them in their objectively obvious sensory perception. What Descartes started from was the inner reality of the mind. He was convinced that that was the difference which determined the efficiency of his investigative method.

It is true that it is easy to select many separate causes for many effects – Descartes wrote to Morin. However, when one is not concerned with seeking for a direct cause of a single state of affairs, it is not easy to select the one and only cause for many effects. In such cases one needs to select the one cause from which all effects can be deduced and at the same time it is proved to be the true cause of them all. Then Descartes emphasizes the fact that all the causes considered by him are of such a kind. On the grounds of physics efforts had always been made to envisage some causes which would explain the phenomena of nature, yet always to no avail. It was me, Descartes, who broke the deadlock. It is enough to compare the suppositions of scholastics, their real qualities, substantial forms, elements and the like, whose number is almost infinite, with my supposition, i.e. with my only accepted assumption: "all bodies are composed of parts"<sup>7</sup>, which in many cases is visible to the naked eye and in others can be proved by means of an infinite number of reasons.

To this statement Descartes adds that "the parts of certain kinds of bodies are of one shape rather than another"<sup>8</sup>, which can be easily demonstrated to those who recognize the previous supposition. "Compare the deductions I have made from my hypothesis – about vision, salt, winds, clouds, snow, thunder, the rainbow, and so on – with what the others have derived from their hypotheses on the same topics! I hope this will be enough to convince anyone unbiased that the effects which I explain have no other causes than the ones from which I have derived them."<sup>9</sup> In these words Descartes refers directly to what he wrote about in *Discourse on Method* where he explains the consecutive stages of cognition, namely becoming aware of simple ideas and inferring from them basic laws of physics. From them – as from the causes – one can afterwards deduce their effects.<sup>10</sup>

Thus all natural phenomena can be deduced from the basic theses of which the very primary one is the thesis of bodies being composed of parts, viz. that of the infinite divisibility of bodies. Expressing this thesis

<sup>7</sup> AT II, 200; K, 59.

<sup>8</sup> AT II, 200; K, 59.

<sup>9</sup> AT II, 200; K, 59.

<sup>10</sup> See *Discourse on Method*, part VI.

in different words, Descartes propounded it also as the thesis of extension constituting the essence of material bodies. The thesis was put forward as the result of cognitive processes carried out within the mind exclusively. The idea of extension is one of those simple notions which need not even be defined.<sup>11</sup> But it is a natural thesis referring only to the world of natural empiricism. If it is supposed to be the departure for deduction, then a set of natural theses will be the result. From premises one can deduce conclusions of the same character that they are, hence **only natural theses can logically follow from natural theses**. In investigative procedure we deal with the problem of selecting statements expressing unknown causes for statements expressing known effects. In order that such a procedure be methodologically valid, the thesis that extension is the essence of bodies should be understood in the sense of the thesis of the material homogeneity of nature. And that is how Descartes understands it when he writes in the *Principles of Philosophy*: "There is therefore but one matter in the whole universe, and we know this by the simple fact of its being extended. All the properties which we clearly perceive in it may be reduced to the one, viz. that it can be divided or moved according to its parts, and consequently is capable of all these affections which we perceive can arise from the motion of its parts. For its partition by thought alone makes no difference to it; but all the variation in matter, or diversity in its form, depends on motion."<sup>12</sup>

The above fragment is, among others, a reflection of the mechanistic view of the universe held by Descartes. However, its significance, particularly in view of what has been said before, is much greater. After all, the mechanistic theory as such need not assume the material homogeneity of nature. Furthermore, Descartes proclaims something else: all the constitutive elements of the universe are identical in terms of their material homogeneity and it cannot be otherwise even in the very order of thought: "... the earth and heavens are formed of the same matter, and (...) even were there an infinitude of worlds, they would all be formed of this matter, from which it follows that there cannot be a plurality of worlds because we clearly perceive that the matter whose nature consists in its being an extended substance only, now occupies all the imaginable spaces where these other worlds could alone be, and we cannot find in ourselves the idea of any other matter."<sup>13</sup>

<sup>11</sup> AT II, 597; K, 65-66.

<sup>12</sup> AT VIII-1, 52; HR I, 265.

<sup>13</sup> AT VIII-1, 52; HR I, 265.

As can be seen, Descartes' idealistic epistemological standpoint leads him consequently to the materialistic ontological one. It is a standpoint of materialistic monism with reference to the world of nature, contrary to the scholastic standpoint of the plurality of being which holds that in a definite single being there are real qualities of various kinds, substantial forms, etc. Explaining the nature in terms of the plurality of being is believed to be empty rhetoric resulting from the uncritical confidence put in sensory perception. If we recognize sensory data as primary cognitive data, the only departure beyond them simply must consist in multiplying notions without the possibility of checking whether anything in extra-linguistic reality corresponds with them. In Descartes' system the inner reality of the mind is the point from which one should proceed to the reality of nature. The ontological thesis of the material homogeneity of nature is the consequence of the idealistic epistemological thesis of the intramental character of cognition, but this is why it constitutes the methodological foundation of modern science. After Descartes one should no longer pursue science in a way other than that of explaining the material phenomena of nature by means of their causes equally material in their essence. The conviction of the existence of complete isomorphism of the structures of the mind and the universe accounted for the use of mathematics as the only language adequate for describing the realm of nature.

However, the thesis of the material homogeneity of nature, which is fundamental to science, leads on to another fundamental thesis in philosophy. In order to come to it we shall analyse the third of the indicated logical relations between cause and effect. The effect can be deduced from the cause and the cause explains the effect. The relation of proving occurs in the opposite direction: the effect proves the cause. From the thesis of the material homogeneity of nature follows the thesis of the materiality of the relation of causal determination in the realm of nature. Thus, if the effect proves the existence of the cause, it proves the existence of exclusively material causes. It is a conclusion of great philosophical importance. In the philosophical sense this conclusion should be understood as the thesis that nature does not lead outside itself, viz. **rational inquiry into nature cannot lead beyond it**. In other words: all efforts proceeding from the natural realm to reach the transcendental realm are invalid. After Descartes one could no longer pursue philosophy in the way characteristic of Aristotelian scholasticism, viz. trying to reach the transcendental realm by proceeding from the natural realm. The philosophical explanation of nature cannot lead beyond it because natural reality as such does not demand that the reason of its existence should be set forth since nature itself contains

the principles explaining its character. This thesis underlies all materialistic trends in modern and contemporary philosophy.

Descartes' great thesis on nature – the thesis that nature does not lead beyond itself in rational inquiry – was not the ultimate one in his philosophy. He assumed the existence of the transcendental realm and was convinced that one could reach it by the use of reason, only that one should not proceed from the reality of nature but from the inner reality of the mind. That is the aspect in which he formulated his second great thesis: that **the mind leads outside itself demanding that the cause of its existence be given**. This thesis underlines all idealistic trends in modern and contemporary philosophy. Of course, it needs separate explanation and discussion.

Anna Zalewska

## A CRITERION OF DECIDABILITY OF SOME ALGORITHMIC FORMULAS

**Abstract.** In the paper a certain criterion of decidability for proof system of propositional algorithmic logic is presented. The criterion allows to prove validity of the algorithmic formulas in automatic way suitable for computer realization. Two examples of computer experiments are included.

### 1. Introduction

It was Leibniz [3] who first claimed *It is unworthy of excellent [persons] to lose hours like slaves in the labor of calculation which could be safely relegated to anyone else if machines were used*. Now modern computer techniques give us possibility for realization of automated reasoning. In order to prove validity of formulas (in a given logic) in automatic way we should have at our disposal a proof system which is sound, complete and decidable. The property of decidability of the proof system is very important from automated proving point of view. It is necessary to have a procedure that in finite number of steps gives us the answer to the question about validity of a given formula. But it is not enough: the procedure should be suitable for computer realization.

In [4] the proof system for propositional algorithmic logic was described. Soundness and completeness of the system were described there, too. The propositional algorithmic logic is decidable (see [1]) but the procedure proposed by Chlebus is not appropriate from computer realization point of view. In the paper a certain criterion of decidability for the proof system is presented. The criterion gives us a procedure that checks validity of the algorithmic formulas in automatic way suitable for implementation.

In Chapter 2 the basic notions connected with the proof system are reminded. In Chapter 3 we present the basic rules of our proof system. In Chapter 4 a criterion of decidability of some algorithmic formulas is described. In appendix two examples of computer experiments are included.

## 2. Basic notions

Let us remind that the alphabet of the language of propositional algorithmic logic

$$A_{PAL} = V_0 \cup V_p \cup L_c \cup L_{con} \cup I_q \cup P_c \cup A_s$$

where  $V_0$  and  $V_p$  are respectively an enumerable set of propositional variables, and an enumerable set of program variables,  $L_c$  is the set of *logical constants* ( $\{\mathbf{true}, \mathbf{false}\}$ ),  $L_{con}$  denotes the set of *logical connectives* ( $\{\neg, \vee, \wedge, \Rightarrow\}$ ) and  $I_q$  denotes the set of *iteration quantifiers* ( $\{\cup, \cap\}$ ),  $P_c$  is the set of *program connectives* ( $\{\mathbf{begin} \ -; \ - \ \mathbf{end}, \mathbf{if} \ - \ \mathbf{then} \ - \ \mathbf{else} \ - \ \mathbf{fi}, \mathbf{while} \ - \ \mathbf{do} \ - \ \mathbf{od}\}$ ) and  $A_s$  is the set of *auxiliary signs* ( $\{(, )\}$ ).

The set of all *programs* is the least extension of the set  $V_p$  and program constant  $Id$  such that if  $\gamma$  is a classical propositional formula and  $M, M'$  are programs then the expressions **begin**  $M; M'$  **end**, **if**  $\gamma$  **then**  $M$  **else**  $M'$  **fi**, **while**  $\gamma$  **do**  $M$  **od** are programs.

The set of all *formulas* is the least extension of the set of classical propositional formulas such that if  $M$  is a program and  $\alpha$  and  $\beta$  are formulas then  $(\alpha \vee \beta)$ ,  $(\alpha \wedge \beta)$ ,  $(\alpha \Rightarrow \beta)$ ,  $\neg \alpha$   $M\alpha$ ,  $\cup M\alpha$ ,  $\cap M\alpha$  are formulas too.

Let  $pref^o$  stand for *prefix* i.e. finite sequence, perhaps empty, of the form  $\mp K_1 \mp K_2 \dots \mp K_l$  where  $K_i \in V_p$  and  $\mp K_i$  is  $\neg K_i$  when " $\mp$ " = " $\neg$ " or  $K_i$  when " $\mp$ " = " $\varepsilon$ ", and " $o$ " = " $+$ " if the number of programs with negation occurring in prefix is even, otherwise " $o$ " = " $+$ ". An *absolute value* of  $\mp K_i$  is defined in the following way

$$|\mp K_i| = \begin{cases} \neg(\mp K_i) & \text{if } \mp = \neg \\ (\mp K_i) & \text{if } \mp = \varepsilon \end{cases}$$

Let  $pref^o$  ( $pref'^o$ ) will be of the form  $\mp K_1 \mp K_2 \dots \mp K_l$  ( $\mp L_1 \mp L_2 \dots \mp L_l$ ). The absolute value of the prefix  $pref^o$  is equal to the absolute value of the prefix  $pref'^o$  if for all indexes  $i = 1, \dots, n$   $|\mp K_i| = |\mp L_i|$  (we will denote the equality writing  $|pref| = |pref'|$ ). We will write  $pref_k^-$  ( $pref_k^+$ ) in order to mark the fact that the first  $k$  elements of the prefix  $pref^o$  set up the subprefix in which the number of programs with negation occurring in it is even (uneven).

Let  $\Pi$  be a finite sequence of formulas. A prefix  $pref$  is said to be *removable in the sequence*  $\Pi$  if the following conditions hold

$$(*) \exists \beta \in \Pi (\beta = pref^o \beta')$$

$$(**) \forall k \in \{1, \dots, l\} \exists \beta \in \Pi (\beta = pref'^o \beta' \wedge |pref'| = |pref_k|)$$

where  $l$  denotes the length of the prefix  $pref$  and  $\beta'$  is an arbitrary formula.

## 3. Basic rules

Here we remind only basic rules (and the others notions) of the proof system (it is called PS) that are needed in order to understand the next chapter (for better understanding see [4,5]).

According to notational conventions, used in the literature, by  $\Delta, \Delta', \Gamma, \Gamma', \Pi, \Pi', \Sigma$  and  $\Sigma'$  (possible with indices) we denote sequents, i.e. finite sequence (maybe empty) of formulas. By  $\{\Gamma, \alpha, \Delta\}$  we shall mean a sequence in which we have first  $\Gamma$ , next  $\alpha$  and lastly  $\Delta$ .

In the PS system conclusion and premises of the rules will be presented as ordered pair of the form

$$(*) \quad \langle \Pi, \mathcal{A} \rangle$$

where  $\Pi$  denotes main sequent,

$\mathcal{A}$  stands for set (maybe empty) of sequents.

Each of the rules of the PS system describes relation between its conclusion (written over a line) and its premise or premises (written under the line). Then schemes of the decomposition rules are in the following form

$$\frac{\langle \Pi, \mathcal{A} \rangle}{\langle \Pi', \mathcal{A}' \rangle} \quad \text{or} \quad \frac{\langle \Pi, \mathcal{A} \rangle}{\langle \Pi', \mathcal{A}' \rangle ; \langle \Pi'', \mathcal{A}'' \rangle}$$

In all the below schemes  $\Gamma$  denotes a set of indecomposable formulas and  $\Delta$  is an arbitrary set of formulas;  $\alpha, \beta$  are arbitrary formulas;  $\gamma$  denotes an open formula;  $M, M', M''$  denote arbitrary programs;  $p \in V_0$ ;  $o \in \{-, +\}$ ;  $\mp \in \{\neg, \varepsilon\}$ ;  $\square \in \{\wedge, \vee\}$ ;  $\mathbf{Q} \in \{\cup, \cap\}$ .

The basic decomposition rules of PS system are presented below.

$$(1) \quad \frac{\langle \{\Gamma, pref^o \mathbf{true}, \Delta\}, \mathcal{A} \rangle}{\langle \{\Gamma, pref^o \mathbf{false}, \Delta\}, \mathcal{A} \rangle}$$

$$(2) \frac{\langle \{\Gamma, \text{pref}^\circ \neg \text{true}, \Delta\}, \mathcal{A} \rangle}{\langle \{\Gamma, \text{pref}^\circ \text{false}, \Delta\}, \mathcal{A} \rangle}$$

$$(3) \frac{\langle \{\Gamma, \text{pref}^\circ \neg \neg \alpha, \Delta\}, \mathcal{A} \rangle}{\langle \{\Gamma, \text{pref}^\circ \alpha, \Delta\}, \mathcal{A} \rangle}$$

$$(4) \frac{\langle \{\Gamma, \text{pref}^\circ \mp (\alpha \Rightarrow \beta), \Delta\}, \mathcal{A} \rangle}{\langle \{\Gamma, \text{pref}^\circ \mp (\neg \alpha \vee \beta), \Delta\}, \mathcal{A} \rangle}$$

$$(5) \frac{\langle \{\Gamma, \text{pref}^\circ \mp (\alpha \square \beta), \Delta\}, \mathcal{A} \rangle}{\langle \{\Gamma, \text{pref}^\circ \mp \alpha, \Delta\}, \mathcal{A} \rangle; \langle \{\Gamma, \text{pref}^\circ \mp \beta, \Delta\}, \mathcal{A} \rangle}$$

where  $(\circ, \mp, \square) \in \{(-, \varepsilon, \vee), (+, \varepsilon, \wedge), (+, \neg, \vee), (-, \neg, \wedge)\}$

$$(6) \frac{\langle \{\Gamma, \text{pref}^\circ \mp (\alpha \square \beta), \Delta\}, \mathcal{A} \rangle}{\langle \{\Gamma, \text{pref}^\circ \mp \alpha, \text{pref}^\circ \mp \beta, \Delta\}, \mathcal{A} \rangle}$$

where  $(\circ, \mp, \square) \in \{(+, \varepsilon, \vee), (-, \varepsilon, \wedge), (-, \neg, \vee), (+, \neg, \wedge)\}$

$$(7) \frac{\langle \{\Gamma, \text{pref}^\circ \mp Id\alpha, \Delta\}, \mathcal{A} \rangle}{\langle \{\Gamma, \text{pref}^\circ \mp \alpha, \Delta\}, \mathcal{A} \rangle}$$

where  $\alpha \notin \{\text{false}, p\}$

$$(8) \frac{\langle \{\Gamma, \text{pref}^\circ \mp \alpha, \Delta\}, \mathcal{A} \rangle}{\langle \{\Gamma, \text{pref}^\circ \mp Id\alpha, \Delta\}, \mathcal{A} \rangle}$$

where  $\alpha \in \{\text{false}, p\}$  and  $Id$  is not an element of the prefix  $\text{pref}^\circ$

$$(9) \frac{\langle \{\Gamma, \text{pref}^\circ \mp \text{begin } M'; M'' \text{ end}\alpha, \Delta\}, \mathcal{A} \rangle}{\langle \{\Gamma, \text{pref}^\circ \mp M'(M''\alpha), \Delta\}, \mathcal{A} \rangle}$$

$$(10) \frac{\langle \{\Gamma, \text{pref}^\circ \mp \text{if } \gamma \text{ then } M' \text{ else } M'' \text{ fi}\alpha, \Delta\}, \mathcal{A} \rangle}{\langle \{\Gamma, \text{pref}^\circ \mp ((\gamma \wedge M'\alpha) \vee (\neg \gamma \wedge M''\alpha)), \Delta\}, \mathcal{A} \rangle}$$

$$(11) \frac{\langle \{\Gamma, \text{pref}^\circ \mp \text{while } \gamma \text{ do } M \text{ od}\alpha, \Delta\}, \mathcal{A} \rangle}{\langle \{\Gamma, \text{pref}^\circ \mp \bigcup \text{if } \gamma \text{ then } M \text{ fi}(\neg \gamma \wedge \alpha), \Delta\}, \mathcal{A} \rangle}$$

$$(12) \frac{\langle \{\Gamma, \text{pref}^\circ \mp QM\alpha, \Delta\}, \mathcal{A} \rangle}{\langle \{\Gamma, \text{pref}^\circ \mp \alpha, \Delta, \text{pref}^\circ \mp MQM\alpha\}, \mathcal{A} \rangle}$$

The main sequent  $\Pi$  of the ordered pair  $\langle \Pi, \mathcal{A} \rangle$  is said to be

- *indecomposable* iff no *PS* rule can be applied to it,
- *fundamental* iff the formulas  $\text{pref}^- \alpha$  and  $\text{pref}^+ \alpha$  satisfying the conditions (I-II) belong to the sequent  $\Pi$  or the formula  $\text{pref}^- \text{false}$  satisfying the conditions (II) belongs to the sequent  $\Pi$  where
  - (I)  $|\text{pref}| = |\text{pref}'|$ ,
  - (II)  $\text{pref}$  is not empty and removable in the sequent  $\Pi$ ,
- *$\mathcal{A}$ -provable* iff there exists a sequent  $\Sigma \in \mathcal{A}$  such that

$$\forall \alpha \in \Sigma \exists \beta \in \Pi (\beta \text{ is equal to } \alpha)$$

(we will say sometimes that  $\Pi$  is  *$\mathcal{A}$ -provable with respect to the sequent  $\Sigma$* ),

- *$\mathcal{A}^*$ -provable* iff there exists a sequent  $\Sigma \in \mathcal{A}$  for which  $\Sigma^- \neq \emptyset$  (where  $\Sigma^- = \{\beta \in \Sigma \mid \beta = \neg \beta'\}$ ) and there exists a sequence  $L$  of program variables  $L_1 \dots L_m$  such that

$$\forall \alpha \in \Sigma \exists \beta \in \Pi (\beta \text{ is equal to } \alpha_L)$$

where  $\alpha_L = \pm L_1 \dots L_m \alpha'$  if  $\alpha = \pm \alpha'$  and " $\pm$ "  $\in \{\neg, \varepsilon\}$

- *terminal* iff  $\Pi$  is indecomposable but  $\Pi$  is neither fundamental nor  $\mathcal{A}$ -provable nor  $\mathcal{A}^*$ -provable.

A *proof* of the sequent  $\Pi$  is a diagram (diagram is a decomposition tree obtaining by application of decomposition rules to input formula; precise definition of diagram can be found in [5]) of the sequent such that all paths of the diagram are finite and each its leaf is labelled by the ordered pair  $\langle \Pi, \mathcal{A} \rangle$  where  $\Pi$  is fundamental or  $\mathcal{A}$ -provable or  $\mathcal{A}^*$ -provable.

#### 4. A criterion of decidability

In *PS* system its decidability is reduced to solution the problem

*does exist a procedure that allows to check how many times the scheme (12) should be applied to a given formula of the main sequent of the ordered pair (during building the decomposition tree) in order to state that another application of the scheme is useless because it does not lead us to a leaf of the tree.*

The solution above problem is especially useful during the building a counterexample of a given sequent i.e. during checking for a given ordered pair  $\langle \Pi, \mathcal{A} \rangle$  whether another application of the scheme (12) to a given

formula  $\Pi$  of the main sequent do not lead us to state that the sequent is fundamental or  $\mathcal{A}^*$ -provable.

The second case of described above problem is enough simple. In order to check whether the sequent  $\{\Gamma, \text{pref}^\circ \mp \text{QM}\alpha, \Delta\}$ , where  $(\circ, \mp, \text{Q}) \in \{(+, \epsilon, \cup), (-, \epsilon, \cap), (-, \neg, \cup), (+, \neg, \cap)\}$  is  $\mathcal{A}^*$ -provable with respect to the sequent  $\{\Gamma', \text{pref}'^\circ \mp \text{QM}\alpha, \Delta'\} \in \mathcal{A}$  and the sequence of programs  $L_1, \dots, L_n$  it is enough to check whether the length of the prefix  $\text{pref}^\circ$  is greater then sum of the number  $n$  and the length of the prefix  $\text{pref}'^\circ$ . If it is true then next application of the scheme (12) is needless.

The first case of described above problem is more difficult task. However there exists a procedure given by Chlebus ([1]) that allows us to decide the problem. The weak point, from practical point of view, Chlebus's method is it that it requires remembering in computer memory the whole of the decomposition tree what is not convenient during implementation.

Presented below strategy allows us in many cases to eliminate symbols  $\cup, \cap$  from formulas to which the scheme (12) can be applied.

**STRATEGY:**

Let  $\Pi$  be a sequent containing the set of formulas  $S = \{\delta_1, \dots, \delta_n\}$  such that their prefixes are not empty and the scheme (12) can be applied to the formulas. The formulas are in the form

$$\text{pref}^\circ \mp \text{QM}\delta$$

where  $(\circ, \mp, \text{Q}) \in \{(+, \epsilon, \cup), (-, \epsilon, \cap), (-, \neg, \cup), (+, \neg, \cap)\}$ .

1. Separate from  $\Pi$  the set  $\mathcal{P}$  such that its elements are classes of formulas from the sequent  $\Pi$  and for each class  $K_i \in \mathcal{P}$  two following conditions are satisfied:

- at least one formula from the set  $S$  belongs to  $K_i$ ;
- prefixes of formulas belonging to  $K_i$  (from which program constants  $Id$  are removed) can be presented as the following sequence

$$|\text{pref}1| \subseteq |\text{pref}2| \subseteq \dots \subseteq |\text{pref}m|,$$

where  $m$  is the number of formulas in the class  $K_i$ ;

- for each two classes  $K_i, K_j \in \mathcal{P}$  neither  $K_i \subseteq K_j$  nor  $K_j \subseteq K_i$ .
2. Remove from the set  $\mathcal{P}$  these classes that satisfy the condition: there exists the formula with the shortest prefix to which the scheme (12) can be applied such that the prefix is not removable in the class.
3. For each class  $K_i$  such that
- exactly one formula from the set  $S$  belongs to  $K_i$ ;
- check (by after-mentioned procedure  $\text{PROC1}(K_i)$ ) whether application of the scheme (12) to  $K_i$  does not lead us to leaves of the decomposition tree.

4. For each class  $K_i$  such that
- at least two formulas from the set  $S$  belong to  $K_i$  and
  - each application of the scheme (12) to  $K_i$  leads to fixed increment of prefix elements of a given formula
- check (by after-mentioned procedure  $\text{PROC2}(K_i)$ ) whether application of the scheme (12) to  $K_i$  does not lead us to leaves of the decomposition tree.
5. For each class  $K_i$  such that
- $K_i$  does not satisfy conditions of the point 3 or 4
- use Chlebus's method.

Below we give two mentioned before procedures.

**PROC1( $K_i$ )**

**begin**

let  $\alpha = \text{pref}^\circ \mp \text{QM}\delta \in K_i \cap S$ ,

$\beta$  - formula with the greatest length of prefix in  $K_i$   
such that  $\beta \neq \alpha$ ,

$d_1$  - length of the prefix of the formula  $\alpha$ ,

$d_2$  - length of the prefix of the formula  $\beta$  decreased  
by 1,

if  $d_2 \geq d_1$  then

$$\alpha := \text{pref}^\circ \mp \delta \cup \text{pref}^\circ \mp M\delta \cup \dots \cup \text{pref}^\circ \mp M^{(d_2-d_1)+1}\delta \cup \delta'$$

where  $\delta' = \text{pref}^\circ \mp M^{(d_2-d_1)+2}\text{QM}\delta$ ;

build the decomposition tree for  $K_i$ ;

**end**

Let  $\varphi(\alpha)$ , where  $\alpha = \text{pref}^\circ \mp \text{QM}\delta$  i  $(\circ, \mp, \text{Q}) \in \{(+, \epsilon, \cup), (-, \epsilon, \cap), (-, \neg, \cup), (+, \neg, \cap)\}$ , be a partial function defining the fixed increment of prefix elements of a given formula during the application of the scheme (12):

$$\varphi(\alpha) = \begin{cases} 0 & \text{when } M = Id, \\ 1 & \text{when } M \in V_p, \\ \varphi(N) + \varphi(N') & \text{when } M = \mathbf{begin } N; N' \mathbf{end} \text{ and } \varphi(N), \varphi(N') \\ & \text{are defined,} \\ \varphi(N) & \text{when } M = \mathbf{if } \gamma \mathbf{ then } N \mathbf{ else } N' \mathbf{ fi} \text{ and } \varphi(N), \\ & \varphi(N') \text{ are defined and } \varphi(N) = \varphi(N'), \\ \varphi(N) & \text{when } M = \mathbf{while } \gamma \mathbf{ do } N \mathbf{ od} \text{ and } \varphi(N) \text{ is} \\ & \text{defined,} \\ \text{ndef} & \text{otherwise,} \end{cases}$$

**PROC2( $K_i$ )****begin**

let  $\alpha_1, \dots, \alpha_n \in K_I \cap S$ ,

$\beta$  – formula with the greatest length of prefix in  $K_i$  such that

$\beta \neq \alpha_i$  for  $i \in \{1, \dots, n\}$ ,

$d_i = \varphi(\alpha_i)$ ,

$d$  – length of the prefix of the formula  $\beta$  decreased by 1;

if  $d \geq d_i$  then to each formula  $\alpha_i$  apply the scheme (12)  $(d - d_i) + 1$  times;

let  $\alpha'_i = \text{pref}'^{\circ} \mp \text{QM} \delta$  be a formula obtained from  $\alpha_i$  after application of the scheme (12);

$\alpha'_i := \text{pref}'^{\circ} \mp \delta \cup \text{pref}'^{\circ} \mp M \delta \cup \dots$   
 $\dots \cup \text{pref}'^{\circ} \mp M^{(d_1 \dots d_{i-1} \cdot d_{i+1} \dots d_n) + 1} \delta \cup \delta'$

where  $\delta' = \text{pref}^{\circ} \mp M^{(d_1 \dots d_{i-1} \cdot d_{i+1} \dots d_n) + 2} \text{QM} \delta$ ;

build the decomposition tree for  $K_i$ ;

**end****MOTIVATION:**

1. We do not consider formulas with empty prefixes because they can not (if they have not done it up to now) extend the sequent by formulas that could satisfy the definition of fundamental sequent.
2. From the set  $\mathcal{P}$  we remove classes for which there exist formulas with removable prefixes because next applications of the scheme (12) do not extend the classes by formulas that satisfy the condition (ii) of the definition of fundamental sequent.
3. If there exists exactly one element from the set  $S$  in the class  $K_i$ , then we apply the scheme (12) as long as a formula with enough long prefix will be generated (enough long prefix is not comparable with any prefix of another formula from the sequent).
4. If there exist at least two formulas from the set  $S$  in the class  $K_i$  and each application of the scheme (12) to them leads us to fixed increment of prefix elements of a given formula then
  - first apply the scheme (12) to each element from the set  $S$  so many times in order to generate a formula that is not comparable with any formula from the class  $K_i$
  - next apply the scheme (12) to each element from the set  $S$  so many times in order to be sure that these formulas can not generate (during application of the scheme (12)) formulas that satisfy conditions

of the definition of fundamental sequent; proof is by showing semantical equivalence of the formula  $\alpha'_i = \text{pref}'^{\circ} \mp \text{QM} \delta$  and the formula  $\text{pref}'^{\circ} \mp \delta \cup \text{pref}'^{\circ} \mp M \delta \cup \dots \cup \text{pref}'^{\circ} \mp M^{(d_1 \dots d_{i-1} \cdot d_{i+1} \dots d_n) + 1} \delta$ .

**4. Final remark**

In appendix we give two experimental work concerning automated theorem proving with system IPAL (see [5]) that bases on the proof system for algorithmic logic ([4,5]). Experiments are presented in form of computer printings. The computer printings in comparison with these generated by the computer system IPAL have changed a little in order to obtain better readability. The experiments are the following:

**uyes.prn** – the example showing described before criterion of decidability in the case when input formula is tautology,

**uno.prn** – the example showing described before criterion of decidability in the case when input formula is not tautology.

**References**

- [1] Chlebus, B.: *On the decidability of propositional algorithmic logic*, "Zeitschr. f. math. Logik und Grundlagen d. Math." Bd 28. pp. 247-261, 1982
- [2] Leibniz, G.W.: *De Principio Individui*, 1663
- [3] Mirkowska, G., Salwicki, A.: *Algorithmic Logic*, D. Reidel Publishing Company, Dordrecht, 1987
- [4] Zalewska, A.: *On a certain deduction system with metainduction*, "Studies in Logic, Grammar and Rhetoric", XII/XIII, pp 95-121, 1993/94
- [5] Zalewska, A.: *Interactive Prover for Programs Properties Expressed in Algorithmic Logic*, Doctoral Dissertation, Warsaw University, 1996



### APPENDIX

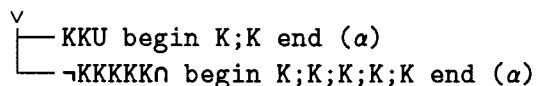
uyes.prn

I P A L - BEGIN OF DOCUMENTATION

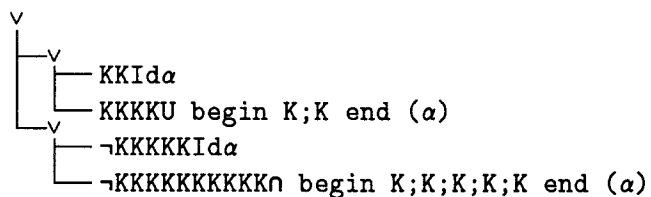
INPUT:

$K(K(U \text{ begin } K;K \text{ end } \alpha)) \vee \neg K(K(K(K(K(\neg \text{ begin } K;K;K;K;K \text{ end } \alpha))))))$

The formula tree:



The formula tree after using IPAL rules:



The number of sequents for the above formula tree: 1

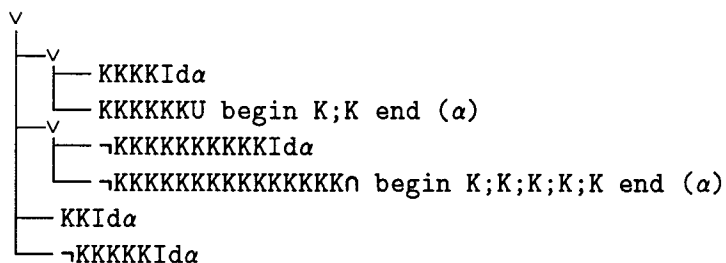
Sequents:

{ ¬KKKKKKKKKK begin K;K;K;K;K end (alpha), KKKKU begin K;K end (alpha), ¬KKKKIda, KKIda } Not\_fund

Let us consider the following pair :

<{ KKKKU begin K;K end (alpha), ¬KKKKKKKKKK begin K;K;K;K;K end (alpha), KKIda, ¬KKKKKIda } , {∅}>

The formula tree after using IPAL rules:



The number of sequents for the above formula tree: 1

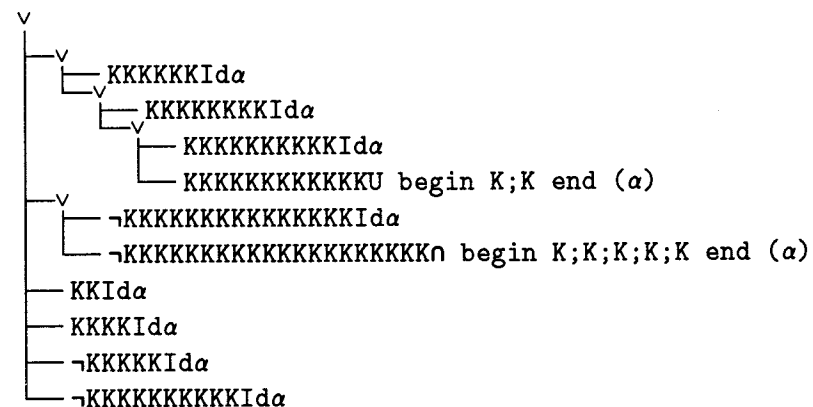
Sequents:

{ ¬KKKKKKKKKKKKKKKKKK begin K;K;K;K;K end (alpha), KKKKKKU begin K;K end (alpha), ¬KKKKKKKKKKIda, KKKKIda, KKIda, ¬KKKKKIda } Not\_fund

Let us consider the following pair:

<{ KKKKKKU begin K;K end (alpha), ¬KKKKKKKKKKKKKKKKKK begin K;K;K;K;K end (alpha), KKIda, KKKKIda, ¬KKKKKIda, ¬KKKKKKKKKKIda } , {∅}>

The formula tree after using IPAL rules:



The number of sequents for the above formula tree: 1

Sequents:

{ ¬KKKKKKKKKKKKKKKKKKKKKKKKKKKK begin K;K;K;K;K end (alpha), KKKKKKKKKKKKKKU begin K;K end (alpha), ¬KKKKKKKKKKKKKKKKKKIda, KKKKKKKKKKKIda, KKKKKKKKKIda, KKKKKKIda, KKIda, KKKKIda, ¬KKKKKIda, ¬KKKKKKKKKKIda } Fund

THE INPUT FORMULA IS ACCEPTED

END OF IPAL DOCUMENTATION

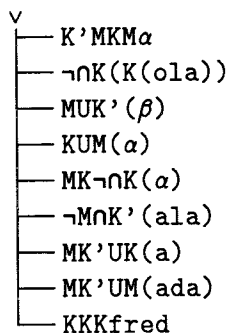
uno.prn

I P A L - BEGIN OF DOCUMENTATION

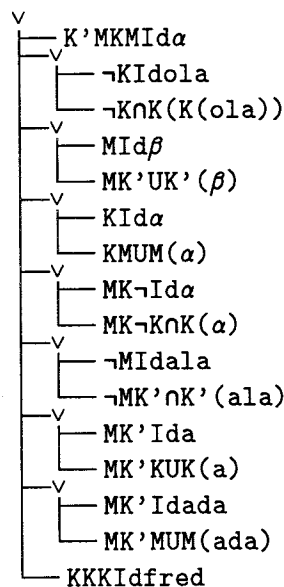
INPUT:

$K'(M(K(M \alpha))) \vee \neg K(K \text{ ol} \alpha) \vee M(U K' \beta) \vee K(U M \alpha) \vee M(K(\neg K \alpha)) \vee \neg M(\neg K' \text{ ala}) \vee M(K'(U K \alpha)) \vee M(K'(U M \text{ ada})) \vee K(K(K(\text{fred})))$

The formula tree:



The formula tree after using IPAL rules:



The number of sequents for the above formula tree: 1

Sequents:

- { MK'MUM(ada), MK'KUK(a), ¬MK'nK'(ala), MK¬KnK(α), KMUM(α), MK'UK'(β), ¬KnK(K(ola)), MK'Idada, MK'Ida, ¬Midala, MK¬Idα, KIdα, Midβ, ¬KIdola, K'MKMIdα, KKKIdfred} Not\_fund

Let us consider the following pair:

- <{ MK'MUM(ada), MK'KUK(a), ¬MK'nK'(ala), MK¬KnK(α), KMUM(α), MK'UK'(β), ¬KnK(K(ola)), MK'Idada, MK'Ida, ¬Midala, MK¬Idα, KIdα, Midβ, ¬KIdola, K'MKMIdα, KKKIdfred}, {∅}>

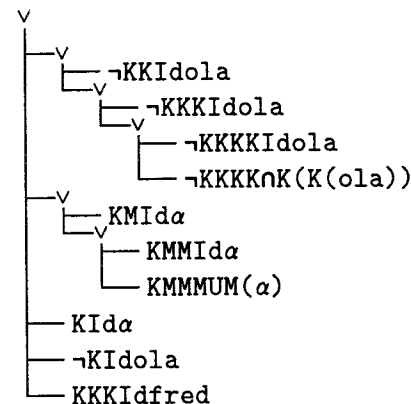
Now we obtain 3 following possible pair(s) to check:

- <{ ¬KnK(K(ola)), KMUM(α), KIdα, ¬KIdola, KKKIdfred}, {∅}>
- <{ MK'UK'(β), ¬MK'nK'(ala), MK'KUK(a), ¬Midala, Midβ, MK'Ida, MK¬Idα, MK'Idada}, {∅}>
- <{ MK'UK'(β), ¬MK'nK'(ala), MK'MUM(ada), ¬Midala, Midβ, MK'Ida, MK¬Idα, MK'Idada}, {∅}>

Let us consider the following pair:

- <{ ¬KnK(K(ola)), KMUM(α), KIdα, ¬KIdola, KKKIdfred}, {∅}>

The formula tree after using IPAL rules:



The number of sequents for the above formula tree: 1

Sequents:

- { KMMMUM(α), ¬KKKKnK(K(ola)), KMMIdα, KMIdα, ¬KKKKIdola, ¬KKKIdola, ¬KKIdola, KIdα, ¬KIdola, KKKIdfred} Not\_fund

Let us consider the following pair:

- <{ KMMMUM(α), ¬KKKKnK(K(ola)), KMMIdα, KMIdα, ¬KKKKIdola, ¬KKKIdola, ¬KKIdola, KIdα, ¬KIdola, KKKIdfred}, {∅}>

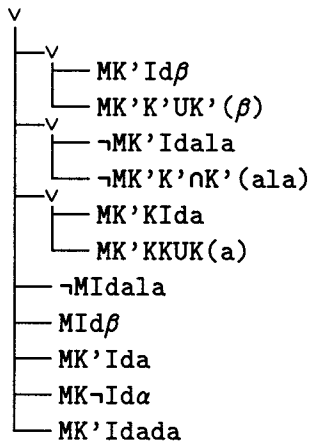
Now we obtain 0 following possible pair(s) to check.

Let us check the next one.

Let us consider the following pair:

- <{ MK'UK'(β), ¬MK'nK'(ala), MK'KUK(a), ¬Midala, Midβ, MK'Ida, MK¬Idα, MK'Idada}, {∅}>

The formula tree after using IPAL rules:



The number of sequents for the above formula tree: 1

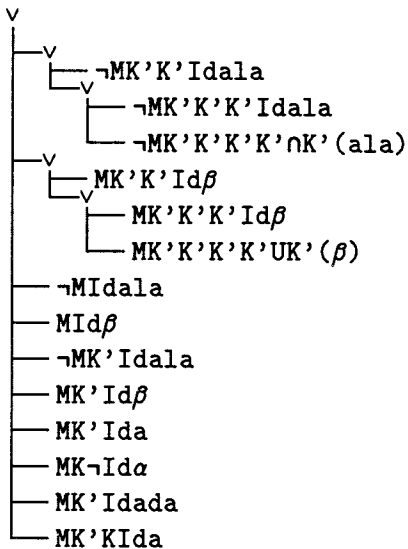
Sequents:

- { MK'KKUK(a), ¬MK'K'∩K'(ala), MK'K'UK'(β), MK'KIda, ¬MK'Idala, MK'Idβ, ¬MIdala, MIdβ, MK'Ida, MK¬Idα, MK'Idada} Not\_fund

Let us consider the following pair:

- <{ ¬MK'K'∩K'(ala), MK'K'UK'(β), ¬MIdala, MIdβ, ¬MK'Idala, MK'Idβ, MK'Ida, MK¬Idα, MK'Idada, MK'KIda}, {∅}>

The formula tree after using IPAL rules:



The number of sequents for the above formula tree: 1

Sequents:

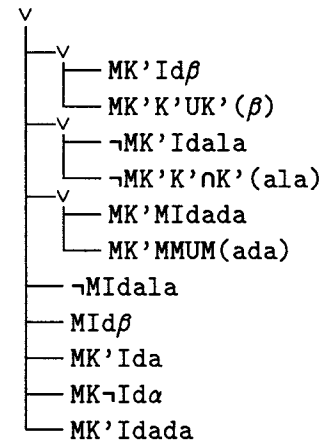
- { MK'K'K'K'UK'(β), ¬MK'K'K'K'∩K'(ala), MK'K'K'Idβ, MK'K'Idβ, ¬MK'K'K'Idala, ¬MK'K'Idala, ¬MIdala, MIdβ, ¬MK'Idala, MK'Idβ, MK'Ida, MK¬Idα, MK'Idada, MK'KIda} Not\_fund

Let us consider the following pair:

- <{ MK'UK'(β), ¬MK'∩K'(ala), MK'MUM(ada), ¬MIdala, MIdβ, MK'Ida, MK¬Idα, MK'Idada}, {∅}>

The main sequent is not accepted

The formula tree after using IPAL rules:



The number of sequents for the above formula tree: 1

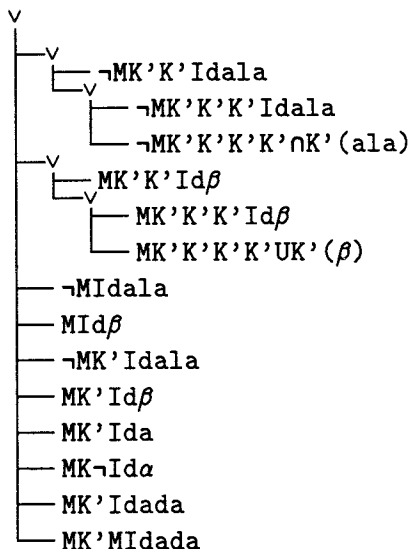
Sequents:

- { MK'MMUM(ada), ¬MK'K'∩K'(ala), MK'K'UK'(β), MK'MIdada, ¬MK'Idala, MK'Idβ, ¬MIdala, MIdβ, MK'Ida, MK¬Idα, MK'Idada} Not\_fund

Let us consider the following pair:

- <{ ¬MK'K'∩K'(ala), MK'K'UK'(β), ¬MIdala, MIdβ, ¬MK'Idala, MK'Idβ, MK'Ida, MK¬Idα, MK'Idada, MK'MIdada}, {∅}>

The formula tree after using IPAL rules:



The number of sequents for the above formula tree: 1

Sequents:

- { MK'K'K'K'UK'(β), ¬MK'K'K'K'∩K'(ala), MK'K'K'Idβ,
- MK'K'Idβ, ¬MK'K'K'Idala, ¬MK'K'Idala, ¬MIdala, MIdβ,
- ¬MK'Idala, MK'Idβ, MK'Ida, MK¬Ida, MK'Idada, MK'MIdada}
- Not\_fund

Let us consider the following pair:

- <{ ¬K∩K(K(ola)), KMUM(α), MK'UK'(β), ¬MK'∩K'(ala),
- ¬MIdala, KIda, MIdβ, ¬KIdola, MK'Ida, MK'KUK(a),
- MK¬Ida, MK'MUM(ada), MK¬K∩K(α), MK'Idada, KKKIdfred,
- K'MKMIda}, {∅}>

The main sequent is not accepted

THE INPUT FORMULA IS NOT ACCEPTED

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Anna Zalewska

## THE NORMS FROM THE POINT OF VIEW OF A CERTAIN LOGIC OF PROGRAMS

**Abstract.** In the paper some logical tools for norms (based on logic of programs) are given. It allows us to express some properties of norms and to state some relations between them.

1

Stig Kanger in his article *Law and Logic* wrote *The combination of law and logic is highly problematic, and the results are few and far between. One of the reasons for this is that very few logicians are interested in law, and very few jurists are interested in logic. Moreover, the purpose of such a combination, as well as suitable approaches to the study of it, is a bit unclear.* Next he mentioned that it appeared suitable to start with problems that are relevant for the creation of well-written systems of law.

The purpose of the paper is to give some logical tools that allow us to express some properties of norms and to state some relations between them in a formal way. We shall discuss norms from the point of view of a certain logic of programs. The general idea is based on combining the following theories

- von Wright's logic of actions (orders and prohibition) ([4]),
- Hoare's logic as the least logic among logic of programs ([1]),
- Wolniewicz's ontology of situation ([3]).

In Hoare's logic we deal with the formulas  $\alpha\{M\}\beta$  that we understand in the following way: if the formula  $\alpha$  is satisfied then the formula  $\beta$  is satisfied after execution of the program  $M$ . If we exchange the program  $M$  for the norm  $N$  then we can read the formulas  $\alpha\{N\}\beta$  in the following way: if the formula  $\alpha$  describes the situation before application of the norm  $N$  then the formula  $\beta$  describes the situation after application of the norm  $N$ . For example:

The child is hungry {Feed the child} The child is fed.

In Hoare's logic beside simple programs there are complex ones constructed

by some program connectives. We will use the program connectives to build more complicated norms. The nature of the norms constructed in this way is algorithmic.

## 2.

The alphabet of the language of norms consists of the following sets:

$V_0$  – an enumerable set of propositional variables such that for some set  $I$

$$V_0 = \bigcup_{i \in I} K_i$$

where  $\{K_i\}_{i \in I}$  is a family of sets of propositional variables satisfying the following conditions:

- (1)  $K_i \cap K_j = \emptyset$  when  $i \neq j$
- (2)  $K_i$  is finite,
- (3)  $K_i$  is at least one element set;

the propositional variables will be denoted by the letter  $p$  with double indexes i.e. for example  $P_{i_2} \in K_{i_1}$ ; if  $j_i$  is a number of elements of  $K_i$  then the set  $\{p_{i_1}, p_{i_2}, \dots, p_{i_{j_i-1}}, p_{i_{j_i}}\} = K_i$ ,

$\{\neg, \vee, \wedge, \Rightarrow\}$  – the set of *logical connectives*,

$V_n$  – an enumerable set of norm variables (intuitively representing simple closes); the norm variables will be denoted by the letter  $N$  with indexes if necessary,

$Id$  – *norm constant* (intuitively representing the action “Do nothing”),

$\{\text{not } -, \text{begin } -, \text{end}, \text{if } - \text{ then } - \text{ else } - \text{ fi}\}$  – the set of *connectives of norms* (prohibition norm, complex norm, condition norm),

$\{(, )\}$  – the set of *auxiliary signs*.

## 3.

The set of all elementary situations  $S_E$  is the least extension of the set  $V_0$  such that if  $\alpha \in S_E$ ,  $p_{ij} \in V_0$  and for each element  $p_{lk}$  occurring in  $\alpha$  holds if  $i \neq l$  then  $\alpha \wedge p_{ij} \in S_E$  ( $S_E$  intuitively represents Wolniewicz's elementary situation).

The set of all *norms*  $\mathcal{N}$  is the least extension of the set  $V_n$  and norm constant  $Id$  such that if  $\gamma$  is a formula  $\gamma \in S_E$  and  $N_1, N_2$  are norms then the expressions *not*  $N_1$ , *begin*  $N_1; N_2$  *end*, *if*  $\gamma$  *then*  $N_1$  *else*  $N_2$  *fi*, are norms.

The set of all *formulas*  $F_{AN}$  is the least extension of the set  $S_E$  such that

- 1) if  $\alpha, \beta \in S_E$  and  $N$  is a norm then  $\alpha\{N\}\beta$  is a formula,
- 2) if  $\alpha, \beta \in F_{AN}$  then  $(\alpha \vee \beta)$ ,  $(\alpha \wedge \beta)$ ,  $(\alpha \Rightarrow \beta)$ ,  $\neg\alpha$  are formulas.

Let  $\alpha \Leftrightarrow \beta$  will be an abbreviation of the following expression  $(\alpha \Rightarrow \beta) \wedge (\beta \Rightarrow \alpha)$ . Let  $\alpha \bar{\vee} \beta$  will be an abbreviation of the following expression  $(\alpha \wedge \neg\beta) \vee (\neg\alpha \wedge \beta)$ .

## 4.

The axioms given below are taken from Hoare's logic. Some of them (Ax3, Ax5 and Ax6) define norm connectives that are similar to program connectives. The axiom Ax4 is an attempt of describing the connective for prohibition norm. This connective has no counterpart in the logic of programs. The rest of the axioms are useful (in general) in formalization of a system of law.

Ax1.

The schema of tautologies of classical propositional calculus

Ax2.

$p_{i_1} \bar{\vee} p_{i_2} \bar{\vee} \dots \bar{\vee} p_{i_{j_i-1}} \bar{\vee} p_{i_{j_i}}$  where  $j_i$  is a number of elements of the set  $K_i$

Ax3.

$\alpha\{Id\}\alpha$

Ax4.

$\alpha\{N\}\beta \vee \alpha\{\text{not } N\}\beta \Rightarrow (\alpha\{\text{not } N\}\beta \Leftrightarrow \neg(\alpha\{N\}\beta))$

Ax5.

$\alpha\{\text{if } \gamma \text{ then } N_1 \text{ else } N_2 \text{ fi}\}\beta \Leftrightarrow (\gamma \Rightarrow \alpha\{N_1\}\beta) \wedge (\neg\gamma \Rightarrow \alpha\{N_2\}\beta)$

Ax6.

$\alpha\{N_1\}\beta \wedge \beta\{N_2\}\delta \Rightarrow \alpha\{\text{begin } N_1; N_2 \text{ end}\}\delta$

Ax7.

$\alpha\{N\}\beta \wedge \alpha\{N\}\delta \Rightarrow \alpha\{N\}(\beta \wedge \delta)$

In order obtain a certain calculus for algorithmic norms we can add the following two rules to the axioms:

$$\text{R1: } \frac{\alpha, \alpha \Rightarrow \beta}{\beta}$$

$$\text{R2: } \frac{\alpha \Rightarrow \alpha', \alpha'\{N\}\beta', \beta' \Rightarrow \beta}{\alpha\{N\}\beta}$$

## 5.

The above-mentioned axioms and rules can be enriched by additional extensions of the set of norms connectives and notion of the set  $S_E$  in the following way:

(1) Some additional norms connectives:

$$\alpha\{N_1 \text{ or } N_2\}\beta \Leftrightarrow \alpha\{N_1\}\beta \wedge \alpha\{N_2\}\beta$$

$$\alpha\{\text{either } N_1 \text{ or } N_2\}\beta \Leftrightarrow \alpha\{N_1\}\beta \vee \alpha\{N_2\}\beta$$

$$\alpha\{N_1 \text{ and } N_2\}\beta \Leftrightarrow \alpha\{\text{begin } N_1; N_2 \text{ end}\}\beta \vee \alpha\{\text{begin } N_2; N_1 \text{ end}\}\beta$$

(2) An extension of the notion of the set  $S_E$  by special situation *all* (any situation of our choice from the set  $S_E$  no matter which one)  
 $S'_E = S_E \cup \{all\}$

$$\alpha \wedge (all\{N\}\beta) \Rightarrow \alpha\{N\}\beta$$

$$(\alpha\{N\}all) \wedge \beta \Rightarrow \alpha\{N\}\beta$$

## 6.

In the paper some logical tools for norms (based on logic of programs) are given. It allows us to express properties of norms and to state some relations between them. In [2] the explications of some concepts of the theory of law are given. They base on described here concept of norms.

## References

- [1] Hoare, C.A.R.: *An Axiomatic Basis for Computer Programming*, Comm. ACM 12, 1969, pp 576-580, 583  
 [2] Malec, A.: *Norms and Programs*, here  
 [3] Wolniewicz, B.: *On Ontology of Situation*, Warszawa, 1985  
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Andrzej Malec

NORMS AND PROGRAMS<sup>1</sup>

## 1.

This paper is devoted to an explication of some concepts of the theory of law (first of all: concepts of inference) by means taken from logic of programs.

In logic of programs there are expressions of the form:  $A\{N\}B$ , where  $A$  and  $B$  are formulas and  $N$  is a program. There are also some connectives for programs which allow us to construct complex programs. Since norms and programs have the same nature (both are rules of behaviour), it should be possible to adopt the above concepts for a formal theory of norms. In this paper we want to show how some methods taken from logic of programs can contribute to the formal apparatus for studying norms. However, we are not going to construct a system of logic of norms.

## 2.

In the paper we will use expressions of the form:  $A\{M\}B$ , where  $A$  and  $B$  are the descriptions of a fragment of the world – respectively – before and after the transformation prescribed by the norm  $M$ . Also we will use normative connectives *begin...end*, *and...*, and *not...*. The connectives are defined as follows:

$$(A1) \quad (A\{M\}B \ \& \ B\{N\}C) \rightarrow A\{\text{begin } M, N \text{ end}\}C,$$

$$(A2) \quad A\{M \text{ and } N\}B \equiv (A\{\text{begin } M, N \text{ end}\}B \ \& \ A\{\text{begin } N, M \text{ end}\}B),$$

$$(A3) \quad (A\{M\}B \ \vee \ A\{\text{not } M\}B) \rightarrow (A\{\text{not } M\}B \equiv \neg A\{M\}B),$$

for any  $A, B, C, M, N$ .<sup>2</sup>

<sup>1</sup> The paper is a revised version of my talk delivered during *The Prague International Colloquium on The Nature of Argument* (Prague, September 27-30, 1994).

<sup>2</sup> So, we have two kinds of variables: for state descriptions and for norms. We use the quantifiers and the symbols:  $\neg, \rightarrow, \equiv, \&, \vee$  as they are used in the classical quantification theory.

## 3.

A norm is a rule of behaviour accepted by a certain community. To obey legal norms one should recognize them. In continental Europe most of legal norms are derived by certain rules from legal texts (statutes, codes, etc.).

Let us reconstruct the process of derivation. Suppose that a judge wants to find a norm relevant to a certain situation. She or he starts with a set of inscriptions contained in legal texts (e.g. Civil Code of Poland).

Firstly, in the process of interpretation these inscriptions are transformed into a set of rules of behaviour. To construct this set one needs only some relatively simple methods which are usually called "the rules of interpretation". The obtained rules of behaviour are called "primary norms" since they are explicitly contained in legal texts.

Secondly, the judge in question can infer some norms from the primary norms by methods which are usually called "the rules of inference". The inferred norms are called "secondary norms" since they are not explicitly contained in legal texts. However, the inference in question is not the inference in the sense of classical logic: imperative clauses are inferred from imperative clauses.

How can we grasp the idea of such inference? To answer this question we will explicate several relations of this kind in terms of the proposed apparatus.

## 4.

According to the rules of inference accepted by lawyers, the following relations should hold:

- (1) Inf.a. ({Help your parents!}, {Help your mother and father!})<sup>3</sup>  
And vice versa, of course!
- (2) Inf.b. ({If you are a man help Earth!}, {If you are a Pole help Earth!})
- (3) Inf.c. ({If somebody is drowning, give her or him your hand}, {If somebody is drowning, pull her or him out of the water})
- (4) Inf.d. ({Dress Adam and Eve!}, {Dress Adam!}).

<sup>3</sup> We read: the norm "Help your mother and father!" is inferred from the norm "Help your parents!".

These examples represent four of several kinds of basic relations of inference between norms. Moreover, various kinds of relations of inference can be combined. For example:

- (5) Inf.a+d. ({Stay with your parents!}, {Don't leave your mother!})  
should hold, since:
- (6) Inf.a. ({Stay with your parents!}, {Don't leave your parents!})  
and
- (7) Inf.d. ({Don't leave your parents!}, {Don't leave your mother!}).

We propose to explicate the relations of inference illustrated by the above examples in the following way:

- Inf.a. ({M}, {N}) iff  
for any  $A, B: A\{M\}B \equiv A\{N\}B$
- Inf.b. ({M}, {N}) iff  
for any  $A, B: A\{N\}B \rightarrow A\{M\}B$
- Inf.c. ({M}, {N}) iff  
for any  $A: A\{begin\ M, not\ N\ end\}A$
- Inf.d. ({M}, {N}) iff  
there exists  $K$  such that for any  $A, B: A\{M\}B \equiv A\{K\ and\ N\}B$

## 5.

In legal sciences there are so called "the collision rules" which help us to solve conflicts of norms. Suppose, a judge has derived from a legal text two norms  $A$  and  $B$ , both relevant to the same situation. If the norms are in conflict, she or he needs the collision rules to know which norm should be suppressed. Three types of conflicts of norms are discussed in legal theory: logical contradiction, logical opposition and praxiological contradiction of norms. Let us set forth some examples. According to lawyers, the following relations should hold:

- (8) LC({"Help him!"}, {"Don't help him!"})<sup>4</sup>
- (9) LO({"Turn left!"}, {"Turn right!"})<sup>5</sup>

<sup>4</sup> We read: there is a logical contradiction between the norms: "Help him!" and "Don't help him!".

<sup>5</sup> We read: there is a logical opposition between the norms: "Turn left!" and "Turn right!".

(10)  $PC(\{\text{"Give him a lifebelt!"}, \{\text{"Put a shark into the water!"}\})^6$

The above relations can be explicated in the following way:

$LC(\{M\}, \{N\})$  iff for any  $A, B: A\{M\}B \equiv A\{\text{not } N\}B$

$LO(\{M\}, \{N\})$  iff for any  $A, B: A\{M\}B \equiv \neg A\{N\}B$

$PC(\{M\}, \{N\})$  iff for any  $A: A\{\text{begin } M, N \text{ end}\}A$ .

## 6.

Undoubtedly the presented explications are preliminary: we did not construct a system of logic of norms. However, the paper shows that some ideas of logic of programs may prove very useful for the formal theory of norms. In particular, the idea of considering programs in relation to certain input data and output data can be used in that theory: every norm can be treated as the prescription of transforming of a fragment of the world. From that point of view several legal concepts can be explicated. It becomes possible to define inference and contradiction between imperative clauses (compare section 3).

Dariusz Surowik

## SOME REMARKS ABOUT INTUITIONISTIC TENSE LOGIC

### Abstract

In this article I would like to consider the system of intuitionistic tense logic. The axioms of the considered system were presented in the article (See [8]) however the semantics of this system was changed. Suggested semantics is simpler and by using it we can better express philosophical ideas described by the considered system. In further part of this article it is shown that if we impose some conditions upon  $R$  then we get temporal order with corresponding properties.

### Introduction

In this article indeterministic tense logic system is considered. The problem of logical determinism was shown in many scientific papers. Deterministic arguments showed in these papers are following:

Let  $T(p)$  means " $p$  is true",  $F(p)$  means " $p$  is false" and  $N(p)$  means " $p$  is necessary". Then if we accept the principle of the causality and the principle of the bivalency we have that:

- 1)  $T(p) \rightarrow N(p)$  – the principle of the causality
- 2)  $F(p) \rightarrow N(\neg p)$  – the principle of the causality
- 3)  $T(p) \vee F(p)$  – the principle of bivalency
- 4)  $N(p) \vee N(\neg p)$  by 1,2,3 and  $\frac{\alpha \rightarrow \gamma, \beta \rightarrow \delta, \alpha \vee \beta}{\gamma \vee \delta}$

Then if we accept the principle of the causality and the law of the excluded middle we have that:

- 1)  $T(p) \rightarrow N(p)$  – the principle of the causality
- 2)  $T(\neg p) \rightarrow N(\neg p)$  – the principle of the causality

<sup>6</sup> We read: there is a praxiological opposition between the norms: "Give him a lifebelt!" and "Put a shark into the water!"



3)  $T(p) \vee T(\neg p)$  – the law of the excluded middle

4)  $N(p) \vee N(\neg p)$  by 1,2,3 and  $\frac{\alpha \rightarrow \gamma, \beta \rightarrow \delta, \alpha \vee \beta}{\gamma \vee \delta}$

The formula  $N(p) \vee N(\neg p)$  expresses determinism. Łukasiewicz showed (See [5]) that rejection of causal principle did not exclude determinism. Then if we try to create indeterministic logic system we have to reject the principle of bivalency or the excluded middle law. In purpose of solution the problem of determinism Łukasiewicz was created tree-valued logic (rejected the principle of bivalency). The meaning of this logic system to solve the problem of the logical determinism is controversial (Kotarbiński, [4]). To solve the problem of determinism we have to introduce difference between past and future. Past is determined (if something was, that can not be changed), future is not determined. We have asymmetry of time and if we would like to express this asymmetry using logical utils we must introduce time to logic. However if we accept, that time has a linear structure (this structure is used by many scientists – philosophers, physicists) and is valid the excluded middle law, we still have determinism because in that structure some formulas (e.g.  $\alpha \rightarrow HF\alpha$ ) are deterministic. Prior (See [7]) give branching time idea, in which future has alternatives whereas past has not got them.

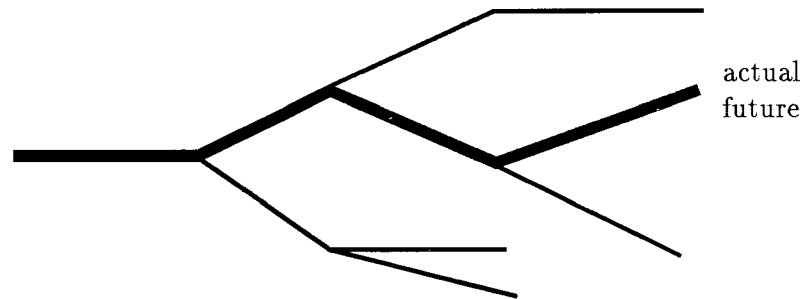


Figure 1:

From all possible future alternatives only one is realized which is called *actual future*. This conception of time as solution of the problem of determinism has opponents. Yourgran maintains (See [9]), that if we choose actual future, then one branch is staying and we return to linear order. Another way to solving this problem can be creation of tense logical system and rejection of the excluded middle law (See [8]). Therefore we can acknowledge that tense logic and intuitionistic propositional logic are proper utils to describe future events from indeterministic point of view.

### The system $T_m$

The system  $T_m$  is the tense logic system based on intuitionistic propositional logic. The tense-logical propositional language  $\mathcal{L}$  consists of: propositional letters ( $p_1, p_2, p_3, \dots$ ), unary connectives ( $\neg, F, G, P, H$ ), binary connectives ( $\wedge, \vee, \rightarrow$ ), parentheses. Tense operators are defined in the usual way:

- $F$  – “at least once in the future”
- $G$  – “it is always going to be that case”
- $P$  – “at least once in the past”
- $H$  – “it has always been that case”

$T_m$  contains an axiom system of intuitionistic propositional logic (A1-A10), tense logical axioms (H1-GD’), rules *Modus Ponens* (MP) and the tense-logical rules (RH and RG)

For all  $\alpha, \beta, \gamma \in \mathcal{L}$ :

- A1)  $\alpha \rightarrow (\beta \rightarrow \alpha)$
- A2)  $(\alpha \rightarrow \beta) \rightarrow \{[\alpha \rightarrow (\beta \rightarrow \gamma)] \rightarrow (\alpha \rightarrow \gamma)\}$
- A3)  $[(\alpha \rightarrow \gamma) \wedge (\beta \rightarrow \gamma)] \rightarrow \{(\alpha \vee \beta) \rightarrow \gamma\}$
- A4)  $(\alpha \wedge \beta) \rightarrow \alpha$
- A5)  $(\alpha \wedge \beta) \rightarrow \beta$
- A6)  $\alpha \rightarrow [\beta \rightarrow (\alpha \wedge \beta)]$
- A7)  $\alpha \rightarrow (\alpha \vee \beta)$
- A8)  $\beta \rightarrow (\alpha \vee \beta)$
- A9)  $(\alpha \wedge \neg \alpha) \rightarrow \beta$
- A10)  $(\alpha \rightarrow \neg \alpha) \rightarrow \neg \alpha$

*Tense-logical axioms:*

- |  |  |
|--|--|
| <b>H1)</b> $H(\alpha \rightarrow \beta) \rightarrow (H\alpha \rightarrow H\beta)$  | <b>G1)</b> $G(\alpha \rightarrow \beta) \rightarrow (G\alpha \rightarrow G\beta)$  |
| <b>H1’)</b> $H(\alpha \rightarrow \beta) \rightarrow (P\alpha \rightarrow P\beta)$ | <b>G1’)</b> $G(\alpha \rightarrow \beta) \rightarrow (F\alpha \rightarrow F\beta)$ |
| <b>H1’’) <math>(H\alpha \rightarrow P\alpha) \vee H\beta</math></b>                | <b>G1’’) <math>(G\alpha \rightarrow F\alpha) \vee G\beta</math></b>                |
| <b>H2)</b> $\alpha \rightarrow HF\alpha$   | <b>G2)</b> $\alpha \rightarrow GP\alpha$   |
| <b>H2’)</b> $PG\alpha \rightarrow \alpha$  | <b>G2’)</b> $FH\alpha \rightarrow \alpha$  |
| <b>HD)</b> $P\alpha \rightarrow \neg H\neg\alpha$                                  | <b>GD)</b> $F\alpha \rightarrow \neg G\neg\alpha$                                  |
| <b>HD’)</b> $\neg P\alpha \rightarrow H\neg\alpha$                                 | <b>GD’)</b> $\neg F\alpha \rightarrow G\neg\alpha$                                 |

*Tense-logical rules:*

- RH:** to infer  $H\alpha$  from  $\alpha$
- RG:** to infer  $G\alpha$  from  $\alpha$

The axioms **H1'**, **H2'**, **G1'**, **G2'** are theorems of minimal tense logic  $K_t$  (based on classical logic) (See [6]). In  $T_m$  system we can not construct proofs of these formulas because the considered system is based on intuitionistic logic (we not accept the principle of the excluded law, double negation law and other law of classical logic).

**Semantics**

Let  $T$  be any nonempty set. A time  $\mathcal{T}$  is an ordered couple  $\langle T, R \rangle$ , where  $R$  is a binary relation (earlier-later) on  $T$ .

Let  $V$  be a function mapping points  $t \in T$  to subsets  $V(t)$  of the set of propositional letters.

Let  $\mathcal{F}$  be nonempty class of such functions.  $\mathfrak{M}_{(\mathcal{T}, \mathcal{F})}$  denotes  $\{ \langle T, R, V \rangle : V \in \mathcal{F} \}$ .

Let  $\leq$  be a relation between elements of  $\mathfrak{M}_{(\mathcal{T}, \mathcal{F})}$  defined as follows:

$\langle T, R, V_1 \rangle \leq \langle T, R, V_2 \rangle$  iff for each  $t \in T : V_1(t) \subseteq V_2(t)$ . The relation  $\leq$  is reflexive and transitive.

Let us consider three-element time structure showed on Figure 2.

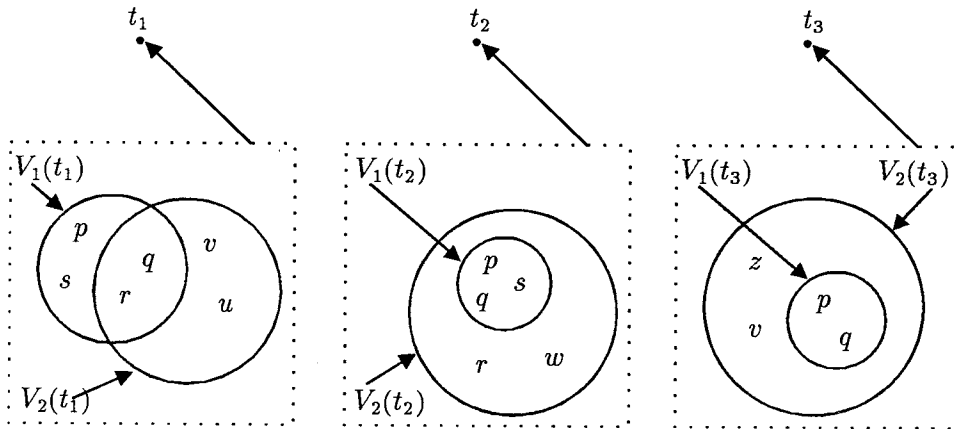


Figure 2:

Let  $m_1 = \langle T, R, V_1 \rangle, m_2 = \langle T, R, V_2 \rangle$ . Analysing the situation showed on **Figure 2**. we conclude that in this case (accordingly to definition of relation  $\leq$ ) it does not holds  $m_1 \leq m_2$ , because for point  $t_1$  we have that  $V_1(t_1)$  is not included in  $V_2(t_1)$ .

For three-element time structure showed on **Figure 3** it holds  $m_1 \leq m_2$ .

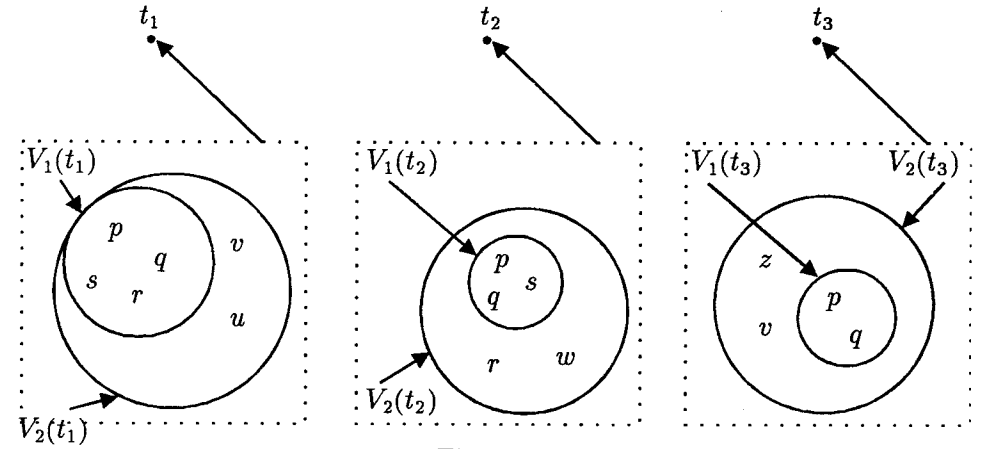


Figure 3:

**Remark**

For  $m \in \mathfrak{M}_{(\mathcal{T}, \mathcal{F})}$  by  $m^*$  we shall mean any  $m_1 \in \mathfrak{M}_{(\mathcal{T}, \mathcal{F})}$  such that  $m \leq m_1$ .

$\mathfrak{M}_{(\mathcal{T}, \mathcal{F})}$  is called a model based on time  $\mathcal{T}$ . For a model  $\mathfrak{M}_{(\mathcal{T}, \mathcal{F})}$ , a point  $t \in T$  and  $m \in \mathfrak{M}_{(\mathcal{T}, \mathcal{F})}$ , a tense-logical formula  $\alpha$ , the so-called 'truth definition' explains what it means for  $\alpha$  to be true in  $\mathfrak{M}_{(\mathcal{T}, \mathcal{F})}$  at  $[t, m]$

**Definition:**

$\mathfrak{M}_{(\mathcal{T}, \mathcal{F})} \models \alpha[t, m]$  is defined through the clauses:

- 1)  $\mathfrak{M}_{(\mathcal{T}, \mathcal{F})} \models p[t, m]$  iff  $p \in V(t)$ .
- 2)  $\mathfrak{M}_{(\mathcal{T}, \mathcal{F})} \models \neg \alpha[t, m]$  iff for any  $m^* \in \mathfrak{M}_{(\mathcal{T}, \mathcal{F})}$  holds  $\mathfrak{M}_{(\mathcal{T}, \mathcal{F})} \not\models \alpha[t, m^*]$
- 3)  $\mathfrak{M}_{(\mathcal{T}, \mathcal{F})} \models (\alpha \vee \beta)[t, m]$  iff  $\mathfrak{M}_{(\mathcal{T}, \mathcal{F})} \models \alpha[t, m]$  or  $\mathfrak{M}_{(\mathcal{T}, \mathcal{F})} \models \beta[t, m]$
- 4)  $\mathfrak{M}_{(\mathcal{T}, \mathcal{F})} \models (\alpha \wedge \beta)[t, m]$  iff  $\mathfrak{M}_{(\mathcal{T}, \mathcal{F})} \models \alpha[t, m]$  and  $\mathfrak{M}_{(\mathcal{T}, \mathcal{F})} \models \beta[t, m]$
- 5)  $\mathfrak{M}_{(\mathcal{T}, \mathcal{F})} \models (\alpha \rightarrow \beta)[t, m]$  iff for any  $m^* \in \mathfrak{M}_{(\mathcal{T}, \mathcal{F})}$  holds  $\mathfrak{M}_{(\mathcal{T}, \mathcal{F})} \not\models \alpha[t, m^*]$  or  $\mathfrak{M}_{(\mathcal{T}, \mathcal{F})} \models \beta[t, m^*]$
- 6)  $\mathfrak{M}_{(\mathcal{T}, \mathcal{F})} \models F\alpha[t, m]$  iff  $\exists t_1, t_1 R t$ , such that  $\mathfrak{M}_{(\mathcal{T}, \mathcal{F})} \models \alpha[t_1, m]$
- 7)  $\mathfrak{M}_{(\mathcal{T}, \mathcal{F})} \models G\alpha[t, m]$  iff  $\forall t_1, t_1 R t$  holds  $\mathfrak{M}_{(\mathcal{T}, \mathcal{F})} \models \alpha[t_1, m]$
- 8)  $\mathfrak{M}_{(\mathcal{T}, \mathcal{F})} \models P\alpha[t, m]$  iff  $\exists t_1, t_1 R t$ , such that  $\mathfrak{M}_{(\mathcal{T}, \mathcal{F})} \models \alpha[t_1, m]$
- 9)  $\mathfrak{M}_{(\mathcal{T}, \mathcal{F})} \models H\alpha[t, m]$  iff  $\forall t_1, t_1 R t$  holds  $\mathfrak{M}_{(\mathcal{T}, \mathcal{F})} \models \alpha[t_1, m]$

$\mathfrak{M}_{(\mathcal{T}, \mathcal{F})} \models \alpha[t, m]$  ( $\alpha$  is true in  $\mathfrak{M}_{(\mathcal{T}, \mathcal{F})}$ ) iff for any  $t \in T$  and any  $m \in \mathfrak{M}_{(\mathcal{T}, \mathcal{F})} : \mathfrak{M}_{(\mathcal{T}, \mathcal{F})} \models \alpha[t, m]$ .

$T \models \alpha$  ( $\alpha$  is true in time  $T$ ) iff it is true in  $\mathfrak{M}_{(T, \mathcal{F})}$  for any  $\mathcal{F}$ .

$\models \alpha$  ( $\alpha$  is a tautology of the minimal intuitionistic tense logic) iff it is true in  $T$ , for any  $T$ .

It is easy to check that holds:

**Lemma 1**

If for some  $t$  and  $m$  holds  $\mathfrak{M}_{(T, \mathcal{F})} \models \alpha[t, m]$  then for any  $m^*$  holds  $\mathfrak{M}_{(T, \mathcal{F})} \models \alpha[t, m^*]$

**Lemma 2**

If for some  $t$  and  $m$  holds  $\mathfrak{M}_{(T, \mathcal{F})} \models P\alpha[t, m]$  then for any  $m^*$  holds  $\mathfrak{M}_{(T, \mathcal{F})} \models P\alpha[t, m^*]$

**proof**

Let us assume that for some  $t$  and  $m$  (where  $m = \langle T, R, V \rangle$ ) holds  $\mathfrak{M}_{(T, \mathcal{F})} \models P\alpha[t, m]$ . Then there is  $t_1$  such that  $t_1 R t$  and holds  $\mathfrak{M}_{(T, \mathcal{F})} \models \alpha[t_1, m]$ . Hence  $\alpha \in V(t_1)$ . Take any  $m^*$  (where  $m = \langle T, R, V^* \rangle$ ). Because (from definition  $\leq$  and Remark) for any  $m^*$  holds  $V(t_1) \subseteq V^*(t_1)$  then we have, that  $\alpha \in V^*(t_1)$ . Then  $\mathfrak{M}_{(T, \mathcal{F})} \models \alpha[t_1, m^*]$ . Because  $t_1 R t$  then  $\mathfrak{M}_{(T, \mathcal{F})} \models P\alpha[t, m^*]$ .  $\square$

**Lemma 3**

If for some  $t$  and  $m$   $\mathfrak{M}_{(T, \mathcal{F})} \models H\alpha[t, m]$  then  $\mathfrak{M}_{(T, \mathcal{F})} \models H\alpha[t, m^*]$

**proof**

Let us assume that for some  $t$  and  $m$  (where  $m = \langle T, R, V \rangle$ ) holds  $\mathfrak{M}_{(T, \mathcal{F})} \models H\alpha[t, m]$ . Then for any  $t_1$  such that  $t_1 R t$  holds  $\mathfrak{M}_{(T, \mathcal{F})} \models \alpha[t_1, m]$ . Let us take any  $t_1$  such that  $t_1 R t$ . Then  $\alpha \in V(t_1)$ . Because for any  $m^*$  we have that for any  $t$  holds  $V(t) \subseteq V^*(t)$ , then from definition of include relation  $\alpha \in V^*(t_1)$ . However  $t_1$  was any such that  $t_1 R t$ , then for any  $t_1$  such that  $t_1 R t$  holds  $\mathfrak{M}_{(T, \mathcal{F})} \models \alpha[t_1, m^*]$ . Hence  $\mathfrak{M}_{(T, \mathcal{F})} \models H\alpha[t, m^*]$ .  $\square$

The following facts showing that all the axioms are true in any model  $\mathfrak{M}_{(T, \mathcal{F})}$  and the rules preserve validity.

**F\_H1)** For any  $\mathfrak{M}, t$  and  $m$  holds  $\mathfrak{M}_{(T, \mathcal{F})} \models H(\alpha \rightarrow \beta) \rightarrow (H\alpha \rightarrow H\beta)[t, m]$

**proof**

Let us assume that for a certain  $\mathfrak{M}, t$  and  $m$ :  $\mathfrak{M}_{(T, \mathcal{F})} \not\models H(\alpha \rightarrow \beta) \rightarrow (H\alpha \rightarrow H\beta)[t, m]$ . Then there is  $m_1, m \leq m_1$  such that  $\mathfrak{M}_{(T, \mathcal{F})} \models H(\alpha \rightarrow \beta)[t, m_1]$  and  $\mathfrak{M}_{(T, \mathcal{F})} \not\models (H\alpha \rightarrow H\beta)[t, m_1]$ . Hence for some  $m_2$  such

that  $m_1 \leq m_2$  holds  $\mathfrak{M}_{(T, \mathcal{F})} \models H\alpha[t, m_2]$  and  $\mathfrak{M}_{(T, \mathcal{F})} \not\models H\beta[t, m_2]$ . Then for any  $t_1$  such that  $t_1 R t$  we have that  $\mathfrak{M}_{(T, \mathcal{F})} \models \alpha[t_1, m_2]$  and there is  $t_2, t_2 R t$  that we have  $\mathfrak{M}_{(T, \mathcal{F})} \not\models \beta[t_2, m_2]$ . At  $t_2$  we have that (1)  $\mathfrak{M}_{(T, \mathcal{F})} \models \alpha[t_2, m_2]$  and (2)  $\mathfrak{M}_{(T, \mathcal{F})} \not\models \beta[t_2, m_2]$ . From  $\mathfrak{M}_{(T, \mathcal{F})} \models H(\alpha \rightarrow \beta)[t, m_1]$  we have that for any  $t_3$  such that  $t_3 R t$  holds  $\mathfrak{M}_{(T, \mathcal{F})} \models (\alpha \rightarrow \beta)[t_3, m_1]$ . Hence from the definition of implicitity we have that for any  $m_1^*$  holds  $\mathfrak{M}_{(T, \mathcal{F})} \not\models \alpha[t_3, m_1^*]$  or  $\mathfrak{M}_{(T, \mathcal{F})} \models \beta[t_3, m_1^*]$ . Thus it is in contradiction with (1) and (2).  $\square$

**F\_H1')** For any  $\mathfrak{M}, t$  and  $m$  holds  $\mathfrak{M}_{(T, \mathcal{F})} \models H(\alpha \rightarrow \beta) \rightarrow (P\alpha \rightarrow P\beta)[t, m]$

**proof**

Let us assume that for a certain  $\mathfrak{M}, t$  and  $m$ :  $\mathfrak{M}_{(T, \mathcal{F})} \not\models H(\alpha \rightarrow \beta) \rightarrow (P\alpha \rightarrow P\beta)[t, m]$ . Then there is  $m_1, m \leq m_1$  such that  $\mathfrak{M}_{(T, \mathcal{F})} \models H(\alpha \rightarrow \beta)[t, m_1]$  and  $\mathfrak{M}_{(T, \mathcal{F})} \not\models (P\alpha \rightarrow P\beta)[t, m_1]$ . Hence for some  $m_2$  such that  $m_1 \leq m_2$  holds  $\mathfrak{M}_{(T, \mathcal{F})} \models P\alpha[t, m_2]$  and  $\mathfrak{M}_{(T, \mathcal{F})} \not\models P\beta[t, m_2]$ . Thus (1) there is  $t_1, t_1 R t$  that we have  $\mathfrak{M}_{(T, \mathcal{F})} \models \alpha[t_1, m_2]$  and (2) there is not  $t_2, t_2 R t$  such that  $\mathfrak{M}_{(T, \mathcal{F})} \models \beta[t_2, m_2]$ . From  $\mathfrak{M}_{(T, \mathcal{F})} \models H(\alpha \rightarrow \beta)[t, m_1]$  we have that for any  $t_3$  such that  $t_3 R t$  holds  $\mathfrak{M}_{(T, \mathcal{F})} \models (\alpha \rightarrow \beta)[t_3, m_1]$ . From definition of implicitity we have that for any  $t_3$  such that  $t_3 R t$  and for any  $m_1^*$  holds  $\mathfrak{M}_{(T, \mathcal{F})} \not\models \alpha[t_3, m_1^*]$  or  $\mathfrak{M}_{(T, \mathcal{F})} \models \beta[t_3, m_1^*]$ . Thus it is in contradiction with (1) and (2).  $\square$

**F\_H1'')** For any  $\mathfrak{M}, t$  and  $m$  holds  $\mathfrak{M}_{(T, \mathcal{F})} \models [(H\alpha \rightarrow P\alpha) \vee H\beta][t, m]$

**proof**

Suppose that for a certain  $\mathfrak{M}, t$  and  $m$ :  $\mathfrak{M}_{(T, \mathcal{F})} \not\models [(H\alpha \rightarrow P\alpha) \vee H\beta][t, m]$ . Then (1)  $\mathfrak{M}_{(T, \mathcal{F})} \not\models (H\alpha \rightarrow P\alpha)[t, m]$  and (2)  $\mathfrak{M}_{(T, \mathcal{F})} \not\models H\beta[t, m]$ . From (1) we have that there is  $m^*$ , such that (3)  $\mathfrak{M}_{(T, \mathcal{F})} \models H\alpha[t, m^*]$  and (4)  $\mathfrak{M}_{(T, \mathcal{F})} \not\models P\alpha[t, m^*]$ . Then we have from (4) that there is not element  $t_1, t_1 R t$  such that  $\mathfrak{M}_{(T, \mathcal{F})} \models \alpha[t_1, m^*]$ . If there is  $t_1, t_1 R t$  then by (3) we have  $\mathfrak{M}_{(T, \mathcal{F})} \models \alpha[t_1, m^*]$  and we have a contradiction (3) and (4). Hence there is not element  $t_1$  such that  $t_1 R t$  ( $t$  is the first element of the set  $T$ ). Then we have that  $\mathfrak{M}_{(T, \mathcal{F})} \models H\beta[t, m]$ . Thus it is in contradiction with (2).  $\square$

**F\_H2)** For any  $\mathfrak{M}, t$  and  $m$  holds  $\mathfrak{M}_{(T, \mathcal{F})} \models \alpha \rightarrow HF\alpha[t, m]$

**proof**

Assume that for a certain  $\mathfrak{M}, t$  and  $m$  we have:  $\mathfrak{M}_{(T, \mathcal{F})} \not\models \alpha \rightarrow HF\alpha[t, m]$ . Then there is  $m^*$  such that (1)  $\mathfrak{M}_{(T, \mathcal{F})} \models \alpha[t, m^*]$  and (2)  $\mathfrak{M}_{(T, \mathcal{F})} \not\models HF\alpha[t, m^*]$ . From (2) we have that there is  $t_1, t_1 R t$  such that

$\mathfrak{M}_{(\mathcal{T}, \mathcal{F})} \not\models F\alpha [t_1, m^*]$ . Thus it is in contradiction with (1) because from (1) we have that for any  $t_1$ ,  $t_1 R t$  holds  $\mathfrak{M}_{(\mathcal{T}, \mathcal{F})} \models F\alpha [t_1, m^*]$ .  $\square$

**F\_H2')** For any  $\mathfrak{M}, t$  and  $m$  holds  $\mathfrak{M}_{(\mathcal{T}, \mathcal{F})} \models PG\alpha \rightarrow \alpha [t, m]$

**proof**

Suppose that for a certain  $\mathfrak{M}, t$  and  $m$  we have:  $\mathfrak{M}_{(\mathcal{T}, \mathcal{F})} \not\models PG\alpha \rightarrow \alpha [t, m]$ . Then there is  $m^*$  such that (1)  $\mathfrak{M}_{(\mathcal{T}, \mathcal{F})} \models PG\alpha [t, m^*]$  and (2)  $\mathfrak{M}_{(\mathcal{T}, \mathcal{F})} \not\models \alpha [t, m^*]$ . From (1) we have that (3) there is  $t_1$ ,  $t_1 R t$  such that  $\mathfrak{M}_{(\mathcal{T}, \mathcal{F})} \models G\alpha [t_1, m^*]$ . Then we have that for any  $t_2$ ,  $t_1 R t_2$  holds  $\mathfrak{M}_{(\mathcal{T}, \mathcal{F})} \models \alpha [t_2, m^*]$ . Thus it is in contradiction with (2).  $\square$

**F\_HD)** For any  $\mathfrak{M}, t$  and  $m$  holds  $\mathfrak{M}_{(\mathcal{T}, \mathcal{F})} \models P\alpha \rightarrow \neg H\neg\alpha [t, m]$

**proof**

Suppose that for a certain  $\mathfrak{M}, t$  and  $m$  we have:  $\mathfrak{M}_{(\mathcal{T}, \mathcal{F})} \not\models P\alpha \rightarrow \neg H\neg\alpha [t, m]$ . Then there is  $m_1, m \leq m_1$  such that (1)  $\mathfrak{M}_{(\mathcal{T}, \mathcal{F})} \models P\alpha [t, m_1]$  and (2)  $\mathfrak{M}_{(\mathcal{T}, \mathcal{F})} \not\models \neg H\neg\alpha [t, m_1]$ . From (1) we have that there is  $t_1$ ,  $t_1 R t$  such that (3)  $\mathfrak{M}_{(\mathcal{T}, \mathcal{F})} \models \alpha [t_1, m_1]$  and from (2) we have that for some  $m_1^*$  holds (4)  $\mathfrak{M}_{(\mathcal{T}, \mathcal{F})} \models H\neg\alpha [t, m_1^*]$ . From (3) and **Lemma 1** we have that for any  $m_1^*$  holds (5)  $\mathfrak{M}_{(\mathcal{T}, \mathcal{F})} \models \alpha [t_1, m_1^*]$ . From (4) we have that (6) for any  $t_2$ ,  $t_2 R t$  holds  $\mathfrak{M}_{(\mathcal{T}, \mathcal{F})} \models \neg\alpha [t_2, m_1^*]$ . Thus is contradiction with (5) and (6).  $\square$

**F\_HD')** For any  $\mathfrak{M}, t$  and  $m$  holds  $\mathfrak{M}_{(\mathcal{T}, \mathcal{F})} \models \neg P\alpha \rightarrow H\neg\alpha [t, m]$

**proof**

(The proof is similar to **F\_HD**)

Tense logical rule **RH** written in other words preserves validity.

**F\_RH)** If for any  $t$  and  $m$  holds  $\mathfrak{M}_{(\mathcal{T}, \mathcal{F})} \models \alpha [t, m]$  then holds  $\mathfrak{M}_{(\mathcal{T}, \mathcal{F})} \models H\alpha [t, m]$

**proof**

Suppose that for any  $t$  and  $m$  holds  $\mathfrak{M}_{(\mathcal{T}, \mathcal{F})} \models \alpha [t, m]$ . Then for any  $t_1 R t$  and  $m$  holds  $\mathfrak{M}_{(\mathcal{T}, \mathcal{F})} \models \alpha [t_1, m]$ . Hence for any  $t$  and  $m$  we have  $\mathfrak{M}_{(\mathcal{T}, \mathcal{F})} \models H\alpha [t, m]$ .  $\square$

The proofs for the axioms **G1-GD'** and rule **RG** are analogous.

Some theorems can be proved in  $K_t$  (See [6]) and  $T_m$ :

**T1:**  $(G\alpha \vee G\beta) \rightarrow G(\alpha \vee \beta)$

**proof**

- 1)  $\alpha \rightarrow (\alpha \vee \beta) - A7$
- 2)  $\beta \rightarrow (\alpha \vee \beta) - A8$
- 3)  $G[\alpha \rightarrow (\alpha \vee \beta)] - RG, 1$
- 4)  $G[\beta \rightarrow (\alpha \vee \beta)] - RG, 2$
- 5)  $G[\alpha \rightarrow (\alpha \vee \beta)] \rightarrow [G\alpha \rightarrow G(\alpha \vee \beta)] - G1$
- 6)  $G[\beta \rightarrow (\alpha \vee \beta)] \rightarrow [G\beta \rightarrow G(\alpha \vee \beta)] - G1$
- 7)  $G\alpha \rightarrow G(\alpha \vee \beta) - 3, 5, MP$
- 8)  $G\beta \rightarrow G(\alpha \vee \beta) - 4, 6, MP$
- 9)  $\{[G\alpha \rightarrow G(\alpha \vee \beta)] \wedge [G\beta \rightarrow G(\alpha \vee \beta)]\} \rightarrow [(G\alpha \vee G\beta) \rightarrow G(\alpha \vee \beta)] - A3$
- 10)  $(G\alpha \vee G\beta) \rightarrow G(\alpha \vee \beta) - 7, 8, 9, MP. \square$

**T2:**  $(H\alpha \vee H\beta) \rightarrow H(\alpha \vee \beta)$

**proof** (analogous to T1)

**T3:**  $(F\alpha \vee F\beta) \rightarrow F(\alpha \vee \beta)$

**proof**

- 1)  $\alpha \rightarrow (\alpha \vee \beta) - A7$
- 2)  $\beta \rightarrow (\alpha \vee \beta) - A8$
- 3)  $G[\alpha \rightarrow (\alpha \vee \beta)] - RG, 1$
- 4)  $G[\beta \rightarrow (\alpha \vee \beta)] - RG, 2$
- 5)  $G[\alpha \rightarrow (\alpha \vee \beta)] \rightarrow [F\alpha \rightarrow F(\alpha \vee \beta)] - G1'$
- 6)  $G[\beta \rightarrow (\alpha \vee \beta)] \rightarrow [F\beta \rightarrow F(\alpha \vee \beta)] - G1'$
- 7)  $F\alpha \rightarrow F(\alpha \vee \beta) - 3, 5, MP$
- 8)  $F\beta \rightarrow F(\alpha \vee \beta) - 4, 6, MP$
- 9)  $\{[F\alpha \rightarrow F(\alpha \vee \beta)] \wedge [F\beta \rightarrow F(\alpha \vee \beta)]\} \rightarrow [(F\alpha \vee F\beta) \rightarrow F(\alpha \vee \beta)] - A3$
- 10)  $(F\alpha \vee F\beta) \rightarrow F(\alpha \vee \beta) - 7, 8, 9, MP. \square$

**T4:**  $(P\alpha \vee P\beta) \rightarrow P(\alpha \vee \beta)$

**proof** (analogous to T3)

In  $T_m$  we can prove the following theorems in a similar way, too:

$$\mathbf{T5}: G(\alpha \wedge \beta) \leftrightarrow (G\alpha \wedge G\beta)$$

$$\mathbf{T6}: H(\alpha \wedge \beta) \leftrightarrow (H\alpha \wedge H\beta)$$

$$\mathbf{T7}: F(\alpha \wedge \beta) \rightarrow (F\alpha \wedge F\beta)$$

$$\mathbf{T7}: P(\alpha \wedge \beta) \rightarrow (P\alpha \wedge P\beta)$$

### Time with different temporal order

If we would like to describe time with different temporal order from intuitionistic point of view we have to add to the set of axioms a formulas which will be express corresponding properties of relation  $R$ . The considered system is a minimal system of intuitionistic tense logic (no condition was imposed upon  $R$ ).

### Reflexive time

If we wish to describe reflexive time we must add the axioms

$$\mathbf{R1)} \quad G\alpha \rightarrow \alpha$$

$$\mathbf{R1')} \quad \alpha \rightarrow F\alpha$$

to the axioms for  $T_m$ . We call the new system  $T_m^R$  and we show, that the new axioms are valid in all tense structures, where relation  $R$  has a property reflexivity. Let us consider  $\mathbf{R1}$  axiom. Let us take any  $\mathfrak{M}, t$  and  $m$  such that  $R$  is reflexive. Suppose that holds  $\mathfrak{M}_{(\mathcal{T}, \mathcal{F})} \models G\alpha[t, m]$ . Then we have that (1) for any  $t_1$  such that  $tRt_1$  holds  $\mathfrak{M}_{(\mathcal{T}, \mathcal{F})} \models \alpha[t_1, m]$ . From reflexivity of  $R$  we have that  $tRt$ . Hence by (1)  $\mathfrak{M}_{(\mathcal{T}, \mathcal{F})} \models \alpha[t, m]$ .  $\square$

It is easy to check that if the new axiom is a tautology then relation  $R$  has a property reflexivity. Suppose that  $G\alpha \rightarrow \alpha$  is a tautology and  $R$  is not reflexive. Take  $\mathfrak{M}, t$  and  $m$  such that  $\mathfrak{M}_{(\mathcal{T}, \mathcal{F})} \not\models \alpha[t, m]$  and for any  $t_1, tRt_1$  holds  $\mathfrak{M}_{(\mathcal{T}, \mathcal{F})} \models \alpha[t_1, m]$ . Let  $t$  be element such that not  $tRt$ . Then we have that  $\mathfrak{M}_{(\mathcal{T}, \mathcal{F})} \models G\alpha[t, m]$  and  $\mathfrak{M}_{(\mathcal{T}, \mathcal{F})} \not\models \alpha[t, m]$ . Hence  $\mathfrak{M}_{(\mathcal{T}, \mathcal{F})} \not\models (G\alpha \rightarrow \alpha)[t, m]$ . Thus is contradiction that the formula  $G\alpha \rightarrow \alpha$  is a tautology.  $\square$

### Symmetry time

Similarly to reflexive time if we wish to make our temporal order symmetry we add the axiom

$$\mathbf{R2)} \quad G\alpha \rightarrow H\alpha$$

to the axioms of  $T_m$ . We call the new system  $T_m^S$ . The new axiom is valid in all tense structures, where relation  $R$  has a property symmetry.

Let us take any  $\mathfrak{M}, t$  and  $m$  such that  $R$  is symmetry. Suppose that holds  $\mathfrak{M}_{(\mathcal{T}, \mathcal{F})} \models G\alpha[t, m]$ . Then we have that for any  $t_1$  such that  $tRt_1$  holds  $\mathfrak{M}_{(\mathcal{T}, \mathcal{F})} \models \alpha[t_1, m]$ . From property of symmetry of  $R$  we have that  $t_1Rt$ . Then for any  $t_1$  such that  $t_1Rt$  holds  $\mathfrak{M}_{(\mathcal{T}, \mathcal{F})} \models \alpha[t_1, m]$ . Hence  $\mathfrak{M}_{(\mathcal{T}, \mathcal{F})} \models H\alpha[t, m]$ .  $\square$

If the new axiom is valid in all tense structures, then the relation  $R$  has a property of symmetry. Suppose that  $G\alpha \rightarrow H\alpha$  is a tautology and  $R$  is not symmetry. Take  $\mathfrak{M}, t, t_1$  and  $m$  such that (1)  $tRt_1$ , (2) not  $t_1Rt$ , (3)  $\mathfrak{M}_{(\mathcal{T}, \mathcal{F})} \models \alpha[t_1, m]$ , (4) for any  $t_2, tRt_2$  holds  $\mathfrak{M}_{(\mathcal{T}, \mathcal{F})} \models \alpha[t_2, m]$  and (5) for any  $t_3, t_3Rt$  holds  $\mathfrak{M}_{(\mathcal{T}, \mathcal{F})} \not\models \alpha[t_3, m]$ .

Then (6)  $\mathfrak{M}_{(\mathcal{T}, \mathcal{F})} \models G\alpha[t, m]$  by (4). On the other hand from (5) we have (7)  $\mathfrak{M}_{(\mathcal{T}, \mathcal{F})} \not\models H\alpha[t, m]$ . Thus we have  $\mathfrak{M}_{(\mathcal{T}, \mathcal{F})} \not\models (G\alpha \rightarrow H\alpha)[t, m]$  by (6) and (7).

Thus is contradiction that the formula  $G\alpha \rightarrow H\alpha$  is a tautology.  $\square$

### Transitive time

The real time seems to be transitive. Let us consider a situation: X was born earlier that Y and Y was born earlier that Z. For all it is obvious, that X was born earlier than Z. If we wish to make our tense logic system to description of time with transitive temporal order we add the axiom

$$\mathbf{R3)} \quad G\alpha \rightarrow GG\alpha \quad \text{or} \quad \mathbf{R3')} \quad FF\alpha \rightarrow F\alpha$$

to the axioms of  $T_m$ . We call the new system  $T_m^{Tr}$ . The new axioms are valid in all tense structures in which relation  $R$  has a property transitivity. Let us consider  $\mathbf{R3'}$  axiom.

Let us take any  $\mathfrak{M}, t$  and  $m$  such that  $R$  is transitive. Suppose that holds  $\mathfrak{M}_{(\mathcal{T}, \mathcal{F})} \models FF\alpha[t, m]$ . Then we have that (1) there is  $t_1, tRt_1$  such that holds  $\mathfrak{M}_{(\mathcal{T}, \mathcal{F})} \models F\alpha[t_1, m]$ . Then there is  $t_2, t_1Rt_2$  such that holds (2)  $\mathfrak{M}_{(\mathcal{T}, \mathcal{F})} \models \alpha[t_2, m]$ . Because  $R$  is transitive we have that (3)  $tRt_2$ . Then  $\mathfrak{M}_{(\mathcal{T}, \mathcal{F})} \models F\alpha[t, m]$  by (2) and (3).  $\square$

If the axiom  $\mathbf{R3'}$  is valid in all tense structures, then the relation  $R$  is transitive.

Suppose that  $FF\alpha \rightarrow F\alpha$  is a tautology and  $R$  is not transitive. Take  $\mathfrak{M}, t_0, t_1, t_2$  and  $m$  such that (1)  $t_0Rt_1$ , (2)  $t_1Rt_2$ , (3) not  $t_0Rt_2$ , (4)  $\mathfrak{M}_{(\mathcal{T}, \mathcal{F})} \models \alpha[t_2, m]$ , (5) for any  $t, t_0Rt$  holds  $\mathfrak{M}_{(\mathcal{T}, \mathcal{F})} \not\models \alpha[t, m]$ . Then  $\mathfrak{M}_{(\mathcal{T}, \mathcal{F})} \models FF\alpha[t_0, m]$  by (1),(2) and (4). On the other hand from (5) we have  $\mathfrak{M}_{(\mathcal{T}, \mathcal{F})} \not\models F\alpha[t_0, m]$ . In consequence we have that  $\mathfrak{M}_{(\mathcal{T}, \mathcal{F})} \not\models (FF\alpha \rightarrow F\alpha)[t_0, m]$ . Thus it is in contradiction with the assumption.  $\square$

## Dense time

If our system is to describe time in with dense temporal order, we add the axiom

$$\mathbf{R4)} \quad GG\alpha \rightarrow G\alpha \quad \text{or} \quad \mathbf{R4')} \quad F\alpha \rightarrow FF\alpha$$

to the axioms of  $T_m$ . We call the new system  $T_m^D$ . The new axioms are valid in all tense structures in which relation  $R$  is dense. Let us consider **R4'** axiom.

Suppose that for some  $\mathfrak{M}, t$  and  $m$  holds  $\mathfrak{M}_{(\mathcal{T}, \mathcal{F})} \not\models F\alpha \rightarrow FF\alpha [t, m]$  and  $R$  is dense. Then there is  $m_1, m \leq m_1$  such that (1)  $\mathfrak{M}_{(\mathcal{T}, \mathcal{F})} \models F\alpha [t, m_1]$  and (2)  $\mathfrak{M}_{(\mathcal{T}, \mathcal{F})} \not\models FF\alpha [t, m_1]$ . Then (3) there is  $t_1, t_1 R t$  such that (4)  $\mathfrak{M}_{(\mathcal{T}, \mathcal{F})} \models \alpha [t_1, m_1]$ . From (2) we have that (5) there is not  $t_2, t R t_2$   $\mathfrak{M}_{(\mathcal{T}, \mathcal{F})} \models F\alpha [t_2, m_1]$ . Since  $R$  is dense we have that (6) there is  $t_3$  such that  $t R t_3 \wedge t_3 R t_1$ . Let us take  $t_3$  satisfying (6). Because  $t_3 R t_1$  we have (7)  $\mathfrak{M}_{(\mathcal{T}, \mathcal{F})} \models F\alpha [t_3, m_1]$  from (4). Thus since  $t_3$  has a property  $t R t_3$ , we have a contradiction by (5) and (7).

If the axiom **R4'** is valid in all tense structures, then the relation  $R$  is dense.

Suppose that  $F\alpha \rightarrow FF\alpha$  is a tautology and  $R$  is not dense. Take  $\mathfrak{M}, t_0$  and  $m$  such that (1)  $t_0 R t_1$ , (2) there is not  $t_2$  such that  $t_0 R t_2$  and  $t_2 R t_1$ , (3)  $\mathfrak{M}_{(\mathcal{T}, \mathcal{F})} \models \alpha [t_1, m]$ , (4) for any  $t, t_1 R t$  holds  $\mathfrak{M}_{(\mathcal{T}, \mathcal{F})} \not\models \alpha [t, m]$ . Then  $\mathfrak{M}_{(\mathcal{T}, \mathcal{F})} \models F\alpha [t_0, m]$  by (1), and (3). From (4) and (2) we have  $\mathfrak{M}_{(\mathcal{T}, \mathcal{F})} \not\models FF\alpha [t_0, m]$ . In consequence we have that  $\mathfrak{M}_{(\mathcal{T}, \mathcal{F})} \not\models (FF\alpha \rightarrow F\alpha) [t_0, m]$ . Thus it is in contradiction with the assumption.  $\square$

## Time without start point

We make our temporal order without start point by adding the axiom

$$\mathbf{R5)} \quad H\alpha \rightarrow P\alpha$$

to the axioms of  $T_m$  instead of **H1'** axiom. We call the new system  $T_m^{wS}$ . The new axiom is valid in all tense structures in which relation  $R$  has not the minimal element.

Suppose that  $R$  has not the minimal element and assume that for some  $\mathfrak{M}, t$  and  $m$  holds  $\mathfrak{M}_{(\mathcal{T}, \mathcal{F})} \not\models (H\alpha \rightarrow P\alpha) [t, m]$ . Then there is  $m_1, m \leq m_1$  such that (1)  $\mathfrak{M}_{(\mathcal{T}, \mathcal{F})} \models H\alpha [t, m_1]$  and (2)  $\mathfrak{M}_{(\mathcal{T}, \mathcal{F})} \not\models P\alpha [t, m_1]$ . Then from (1) we have that (3) for any  $t_1, t_1 R t$  holds  $\mathfrak{M}_{(\mathcal{T}, \mathcal{F})} \models \alpha [t_1, m_1]$ . From (2) we have that (4) there is not  $t_2, t_2 R t$  such that  $\mathfrak{M}_{(\mathcal{T}, \mathcal{F})} \models \alpha [t_2, m_1]$ . Since  $R$  has not the minimal element we have that for any  $t$  there is  $t_3$  such

that (5)  $t_3 R t$ . Let us take  $t_3$  satisfying condition (5). Then by (3) we have  $\mathfrak{M}_{(\mathcal{T}, \mathcal{F})} \models \alpha [t_3, m_1]$ . Hence we have a contradiction by (4).  $\square$

If the axiom **R5** is valid in all tense structures, then the relation  $R$  has not the minimal element.

Suppose that  $H\alpha \rightarrow P\alpha$  is a tautology of and the relation  $R$  has the minimal element. Let us take  $\mathfrak{M}, t_0$  and  $m$  and such that (1)  $t_0$  is the minimal element. Then for any  $t_1$  such that  $t_1 R t_0$  holds  $\mathfrak{M}_{(\mathcal{T}, \mathcal{F})} \models \alpha [t_1, m]$ . Then we have  $\mathfrak{M}_{(\mathcal{T}, \mathcal{F})} \models H\alpha [t_0, m]$  and from (1) we have that  $\mathfrak{M}_{(\mathcal{T}, \mathcal{F})} \not\models P\alpha [t_0, m]$ . Hence we have that  $\mathfrak{M}_{(\mathcal{T}, \mathcal{F})} \not\models (H\alpha \rightarrow P\alpha) [t_0, m]$ . Thus is contradiction with the assumption.  $\square$

## Time without end point

We make our temporal order without end point by adding the axiom

$$\mathbf{R6)} \quad G\alpha \rightarrow F\alpha$$

to the axioms of  $T_m$  instead of **G1'** axiom. We call the new system  $T_m^{wE}$ . The new axiom is valid in all tense structures in which relation  $R$  has not the maximal element.

Suppose that  $R$  has not the maximal element and assume that for some  $\mathfrak{M}, t$  and  $m$  holds  $\mathfrak{M}_{(\mathcal{T}, \mathcal{F})} \not\models (G\alpha \rightarrow F\alpha) [t, m]$ . Then there is  $m_1, m \leq m_1$  such that (1)  $\mathfrak{M}_{(\mathcal{T}, \mathcal{F})} \models G\alpha [t, m_1]$  and (2)  $\mathfrak{M}_{(\mathcal{T}, \mathcal{F})} \not\models F\alpha [t, m_1]$ . Then from (1) we have that (3) for any  $t_1, t R t_1$  holds  $\mathfrak{M}_{(\mathcal{T}, \mathcal{F})} \models \alpha [t_1, m_1]$ . From (2) we have that (4) there is not  $t_2, t R t_2$  such that  $\mathfrak{M}_{(\mathcal{T}, \mathcal{F})} \models \alpha [t_2, m_1]$ . Since  $R$  has not the maximal element we have that for any  $t$  there is  $t_3$  such that (5)  $t R t_3$ . Let us take  $t_3$  satisfying condition (5). Then by (3) we have  $\mathfrak{M}_{(\mathcal{T}, \mathcal{F})} \models \alpha [t_3, m_1]$ . Hence we have contradiction by (4).  $\square$

If the axiom **R6** is valid in all tense structures, then the relation  $R$  has not the maximal element.

Suppose that  $G\alpha \rightarrow F\alpha$  is a tautology of and the relation  $R$  has the maximal element. Let us take  $\mathfrak{M}, t_0$  and  $m$  and such that (1)  $t_0$  is the maximal element. Then for any  $t_1$  such that  $t_0 R t_1$  holds  $\mathfrak{M}_{(\mathcal{T}, \mathcal{F})} \models \alpha [t_1, m]$ . Then we have  $\mathfrak{M}_{(\mathcal{T}, \mathcal{F})} \models G\alpha [t_0, m]$  and from (1) we have that  $\mathfrak{M}_{(\mathcal{T}, \mathcal{F})} \not\models F\alpha [t_0, m]$ . Hence we have that  $\mathfrak{M}_{(\mathcal{T}, \mathcal{F})} \not\models (G\alpha \rightarrow F\alpha) [t_0, m]$ . Thus it is in contradiction with the assumption.  $\square$

It is easy to check in a similar way, that if we wish to make our temporal orders linear, we add the axioms:

$$(F\alpha \wedge F\beta) \rightarrow [F(\alpha \wedge \beta) \vee F(F\alpha \wedge \beta) \vee F(\alpha \wedge F\beta)] \quad (\text{linear future})$$

$$(P\alpha \wedge P\beta) \rightarrow [P(\alpha \wedge \beta) \vee P(P\alpha \wedge \beta) \vee P(\alpha \wedge P\beta)] \quad (\text{linear past})$$

to the axioms of  $T_m$ .

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Kazimierz Trzęsicki

## OMNISCIENCE, OMNIPOTENCE AND RELATED NOTIONS

The theological notions of *omniscience* and *omnipotence* play an important role in philosophy. They are connected with the notion of *freewill*. The main question concerns the compatibility of freewill with existence of omniscient and omnipotent being. Here the notion of *time* has a crucial function – the answer to this question depends on the structure of time. The main aim of this discussion is to show possible logical interrelations between notions of *omniscience*, *omnipotence* and *time* using the language of formal logic, in particular, tense logic. The subject has a long history and is a matter of interest of contemporary philosophical logic.

### 1. Omniscience

The notions of *omniscience* and *omnipotence* could be defined as attributes of an atemporal being<sup>1</sup> or – as it will be done here – something that is in time, in other words, as attributes of a temporal being.

A Cantorian argument against a set of all truths is raised to show that there is not possible an omniscient being, as a being that knows all and

<sup>1</sup> Divine atemporality was offered by Boethius in the sixth century as solution to the problem of theological fatalism. There is no question of God foreknowing human actions because God's knowing cannot be located at any point in time. God is 'outside' time, nowhere on the line of time, but with exactly the same epistemic access to each moment of time. Unchanging 'presence' which on this view all things have to God, is in some way less like our own present than our past. The view is held by many very reputable philosophers, e.g. St. Thomas Aquinas.

God knows those truths, if any, which are themselves timeless. God's knowledge is in some way right outside of time, in which case presumably the verb 'knows' in translation would have to be thought of as tenseless. The idea of omniscient atemporal being is questioned by Nicholas Wolterstorff (*God Everlasting* [in:] *Contemporary Philosophy of Religion*, edited by David Shatz and Steven M. Cahn. New York: Oxford University Press, 1982). Eternal being could know only tensed statement. Tenseless statements are not translatable into tensed ones. If God knows every true statement then he cannot be timeless.

only true propositions.<sup>2</sup> This and other problems depend on the notion of omniscient being. Following A. N. Prior an omniscient being  $\mathcal{G}$  (for "God") could be defined as follows:<sup>3</sup>

(Os)  $\mathcal{G}$  is omniscient iff it is, always has been, and always will be the case that for all  $\alpha$  ( $\in \mathcal{L}$ ): if it is true that  $\alpha$ , then  $\mathcal{G}$  knows that  $\alpha$  is true, where  $\mathcal{L}$  is a language (a set of sentences).

Let us add that:

(I)  $\mathcal{G}$  is infallible iff it is, always has been, and always will be the case that for all  $\alpha$  ( $\in \mathcal{L}$ ): if  $\mathcal{G}$  knows that  $\alpha$  is true, then it is true that  $\alpha$ .

If we suppose that:

1. If it is true that  $\alpha$ , then it has always been true that it would be the case that  $\alpha$ ,<sup>4</sup> then by the definition of omniscience (Os) we obtain:
2. If it is true that  $\alpha$ , then  $\mathcal{G}$  has always known (as true) that it would be the case that  $\alpha$ .

$\mathcal{G}$  possesses an infallible knowledge of man's future actions. How is this provision possible, if man's future acts are not necessary?

According to the Arystotelian principle that what is true is necessary. In consequence of that (2) expresses the theological fatalism. The rejection of (1) is a necessary condition avoiding of (2). This is possible if there are some future contingencies, i.e.:

(FC) for some  $\alpha$  ( $\in \mathcal{L}$ ) and  $t$  ( $\in T$  in the future, after the present moment) neither at present it is true that  $\alpha$  at the moment  $t$  nor at present it is true that *not*- $\alpha$  at the moment  $t$ .

Only in such a world in which FC holds, there are alternative futures between which choice is possible. This is a necessary ontological condition of possibility of free deeds. The contingent, considered as future (*ut futurum*), cannot be the object of any sort of knowledge of a temporal being – with the exception of almighty and omniscient  $\mathcal{G}$  – which cannot fall into falsehood.  $\mathcal{G}$  is omniscient, thus  $\mathcal{G}$  knows all the contingent deeds. Because  $\mathcal{G}$  is omnipotent,  $\mathcal{G}$  is able to decide about any of such deeds. The infallible

<sup>2</sup> See Grim Patrick, *Logic and Limits of Knowledge and Truth*, "Philosophical Studies", 22 (1988), 341–367; Plantinga A. and Grim P., *Truth, Omniscience, and Cantorian Arguments: An Exchange*, "Philosophical Studies", 71 (1993), 267–306; Mar G., *Why "Cantorian" Arguments Against the Existence of God Don't Work*, <http://w.w.w.sunysb.edu/philosophy/faculty/gmar/cantor.txt>

<sup>3</sup> Prior A. N., *The Formalities of Omniscience* [in:] *Papers on Time and Tense* Oxford 1968.

<sup>4</sup> *Id quod est verum in praesenti, semper fuit verum esse futurum.*

knowledge of  $\mathcal{G}$  has the source in its taken in advance irrevocable decisions concerning all the contingent deeds.

The idea of 'closed' past: *quod fuit, non potest non fuisse* – what has been, cannot now not have been – has the main stock source in the "Nicomachean Ethics" (vi, 1139b).<sup>5</sup> Agaton says that even God is not able to change what has been done (1139b5-10; 2, T.4). For C. S. Peirce the past is the region of 'brute fact'. Some writers to support the idea of 'open' future maintain that also the past is a subject of some kind of change, namely some past facts are falling into abyss. For Karneades even Apollo is not able to know past facts if there is no trace of them, thus, moreover, future facts that are not decided yet could not be known by him. In antiquity, the same view was maintained by Cicero. Łukasiewicz to avoid fatalistic consequences of the principle of bivalence – already discussed by Arystoteles in the famous see-fight passage of "De Interpretatione" – invented many-valued logics which principles would have to govern our thinking not only about the future but also about the past. He writes:

We should not treat the past differently from the future. If the only part of the future that is now real is causally determined by the present instant, and if causal chains commencing in the future belong to the realm of possibility, then only those parts of the past are at present real which still continue to act by their effect today. Facts whose effects have disappeared altogether, and which even an omniscient mind could not infer from those now occurring, belong to the realm of possibility. One cannot say about them that they took place, but only that they were possible. It is well that it should be so. There are hard moments of suffering and still harder ones of guilt in everyone's life. We should be glad to be able to erase them not only from our memory but also from existence. We may believe that when all effects of those fateful moments are exhausted, even should that happen after our death, then their causes too will be effaced from the world of actuality and pass into the realm of possibility. Time calms our cares and brings us forgiveness (Łukasiewicz "On Determinism", pp. 127-128).

This does not mean that for Karneades, Cicero or Łukasiewicz the past is 'open'. On the places of 'forgotten' facts no new 'facts' occur. The places remain 'empty'.

The 'closed' future seems to be a consequence of 'closed' past. The past truths belong to the realm of necessity thus each sentence in past tense form, if true, is necessarily true. But some sentences are essentially future in sense though past in form. The rule that true past sentences are necessarily true has to be restricted to sentences past in sense – this is Occamists' answer<sup>6</sup> –

<sup>5</sup> See also *De Caelo* i, 283b13.

<sup>6</sup> See Prior, *The Formalities of Omniscience* [in:] *Papers on Time and Tense*, Oxford 1968, pp. 26–44.



or – such is Perceian answer – the future tense operator has to be conceived as symmetrical to the past tense operator, i.e. it has to mean “necessarily it will be that”.

## 2. Omnipotence

We can consider the notion of an omnipotent atemporal or – what will be done here – a temporal being. Questions like “is an omnipotent being able to change something in the past?”, or “will an omnipotent being able to do that-and-that?” have sense in the case of temporal being and are senseless in the case of atemporal being.

What does it mean that a temporal being is omnipotent? It is clear that even an omnipotent being is not able to create an absolutely immovable boulder that the being can move. Generally, the omnipotent being is not able to do something that is not possible for logical reasons. This answer bears new questions. First of all: “what does it mean ‘possible’?” and “what logic we are talking about?”. Moreover, we can ask if the omnipotent being is able to change the logic.

The possibility should be conceived in such a way that only these deeds are possible that are not necessary and that are not impossible, i.e. these deeds which are contingent. Such a notion was searched for by Łukasiewicz.<sup>7</sup> If we assume that what is true is necessary, then we have to admit other logical value than truth and falsehood, namely possibility.

Omnipotence could be defined as follows:

**(Om)**  $\mathcal{G}$  is omnipotent iff for each  $\alpha (\in \mathcal{L})$ : if  $C\alpha$ , then  $\mathcal{G}$  is able to do  $\alpha$ , where „ $C$ ” stands for “contingent”.

Let us remark that if it is true that  $not-\alpha$ , then  $\alpha$  is not contingent. Thus:  $C\alpha$  iff  $Cnot-\alpha$ .

$\mathcal{G}$  is omniscient, thus its knowledge is complete: for any  $\alpha (\in \mathcal{L})$ ,  $\mathcal{G}$  knows whether  $\alpha$  (it is true that  $\alpha$ ) or  $\alpha$  is contingent (it is true that  $C\alpha$ ) or  $\alpha$  is false (it is true that  $not-\alpha$ ).  $\mathcal{G}$  is omnipotent, thus for any contingent  $\alpha (C\alpha)$   $\mathcal{G}$  is able to decide if  $\alpha$  or  $not-\alpha$ . But in a world in that all possible deeds are done by  $\mathcal{G}$ , another free agent – if there is any – has no possibility to choose anything; there is no contingent  $\alpha$  left for it.  $\mathcal{G}$ 's omnipotent providence exercises a complete and perfect control over all events that happen, or will happen, in the universe. How is this secured without infringement of man's

<sup>7</sup> We mean here his three- and four-valued modal logics.

freedom? Our answer is:  $\mathcal{G}$  is not obliged to decide about any possible  $\alpha$ .  $\mathcal{G}$  is free to leave it to other free agents.

## 3. Logic of free world

Let us consider the possibility of a world in which there are some free deeds, i.e. a free world. In the case of “closed” world there is no place for free deeds of a free agent. Even if there are some free agents there is no possibility to act for them. The existence of a free world does not prove the existence of some free agents.

Openness of the future is expressed by **FC**. In a slightly modified form it says:

**(oF)** The future is open iff there are  $\alpha (\in \mathcal{L})$  and  $t (\in T)$  such that neither it is true that *it will be the case at the moment  $t$  that  $\alpha$*  nor it is true that *it will be the case at the moment  $t$  that  $not-\alpha$* .

Instead of **oF** we can consider a stronger condition:

**(OF)** The future is open iff there is  $\alpha (\in \mathcal{L})$  such that neither it is true that *it will be the case that  $\alpha$*  nor it is true that *it will be the case that  $not-\alpha$* .

The condition **oF** or **OF** does not imply or contradicts to:

**(OA)** There is  $\alpha (\in \mathcal{L})$  such that neither it is true that  $\alpha$  nor it is true that  $not-\alpha$ .

**(OP)** There is  $\alpha (\in \mathcal{L})$  such that neither it is true that *it has been the case that  $\alpha$*  nor it is true that *it has been the case that  $not-\alpha$* .

The same is true for the past analogue to **oF**.

Supposing that everything what is necessary is just, evil is possible only as a result of some free deeds. The omniscient and omnipotent free agent  $\mathcal{G}$  can decide about any possible deed. If  $\mathcal{G}$  does it and if it is absolutely just, no evil is possible. But we see a lot of evil. Thus evil is done by free agents that are not omniscient or absolutely just. We can ask why  $\mathcal{G}$ , the omnipotent and omniscient free agent – if  $\mathcal{G}$  exists – does not complete all the empty places, all the places open for a free agent.<sup>8</sup> The “empty

<sup>8</sup> The problem of evil concerns the contradiction, or apparent contradiction, in holding the following pair of propositions

1. God exists and is almighty, omniscient, and perfectly just;
2. Evil exists.

Cf. Mar, G., On not Multiplying Depravity Beyond Necessity, [www.sunysb.edu/philosophy/faculty/gmar/evil.txt](http://www.sunysb.edu/philosophy/faculty/gmar/evil.txt). In our setting of the problem there is a question why  $\mathcal{G}$  left uncompleted some places for other free agents. Even  $\mathcal{G}$  does not know in advance how free

places" not completed by some free agents give the opportunity to other free agents.

In the case of **OA**,  $\alpha$  can be present in sense, i.e. no tense operators occur in it. The proposition in **OP** and **oP** can also be past in sense, e.g. tense operators do not occur in  $\alpha$ . Such a situation can take place if some past facts are forgotten. The past can be changed only due to some future facts (that are actualized in the presence).

The language of tense logic consists of an infinite set  $S$  of propositional letters, logical connectives (negation:  $\neg$ , disjunction:  $\vee$ , conjunction:  $\wedge$ , implication:  $\rightarrow$ , biconditional:  $\leftrightarrow$ ), unary tense operators:  $P$  (past tense operator – *it has been the case that*),  $F$  (future tense operator – *it will be the case that*),  $H$  (*it has always been the case that*),  $G$  (*it will always be the case that*) and parenthesis as punctuation marks. Usual formation rules are applied.

A time-frame (a time) is a structure  $\mathcal{T} [= (T, R)]$ , where  $T$  is a non-empty set (of moments of time) and  $R (\subseteq T \times T)$  is a binary relation of precedence (earlier-later) on it.

A valuation  $V$  is a function:  $V(t) \in 2^S$ , i.e. a subset of the set of propositional letters is assigned to any  $t$ . Each valuation can be uniquely extended for any formula  $\alpha (\in \mathcal{L})$  and  $t (\in T)$ .

A model  $\mathcal{M}$  is a pair  $(\mathcal{T}, V)$ .

$\mathcal{M}, t \models \alpha$  is intended to mean that  $\alpha$  is true in the model  $\mathcal{M}$  at the moment  $t$ .

$\mathcal{T} \models \alpha$  iff for any  $t (\in T)$  and any  $V$ :  $\mathcal{M}, t \models \alpha$ .

Let  $\mathcal{C}$  be a class of time-frames.

$\mathcal{C} \models \alpha$  iff for any  $\mathcal{T} (\in \mathcal{C})$ :  $\mathcal{T} \models \alpha$ .

Time  $(T, R)$  is linear in the past (branching in the future) iff for any  $t, t_1, t_2 (\in T)$ : if  $t_1 R t$  and  $t_2 R t$ , then  $t_1 = t_2$  or  $t_1 R t_2$  or  $t_2 R t_1$ .

Time  $(T, R)$  is linear in the future (branching in the past) iff for any  $t, t_1, t_2 (\in T)$ : if  $t R t_1$  and  $t R t_2$ , then  $t_1 = t_2$  or  $t_1 R t_2$  or  $t_2 R t_1$ .

Time  $(T, R)$  is linear iff it is linear in the past and in the future.

Linear time does branch neither in the past nor in the future.

A branch  $b_t$  is a maximal linear subset of  $T$  such that:  $t \in b_t$ .

The tense operators can be defined as follows:

( $F_c$ )  $\mathcal{M}, t \models F\alpha$  iff there is  $t_1 (\in T)$ ,  $t R t_1$ :  $\mathcal{M}, t_1 \models \alpha$ .

( $P_c$ )  $\mathcal{M}, t \models P\alpha$  iff there is  $t_1 (\in T)$ ,  $t_1 R t$ :  $\mathcal{M}, t_1 \models \alpha$ .

deeds will be done by other free agents. To prevent evil  $\mathcal{G}$  as omniscient and omnipotent could complete all the places in that evil is possible. But would it be in such a world any possibility of free acts? Thus we can only wonder why the omnipotent  $\mathcal{G}$  left something to do to others.

( $G_c$ )  $\mathcal{M}, t \models G\alpha$  iff for each  $t_1 (\in T)$ ,  $t R t_1$ :  $\mathcal{M}, t_1 \models \alpha$ .

( $H_c$ )  $\mathcal{M}, t \models H\alpha$  iff for each  $t_1 (\in T)$ ,  $t_1 R t$ :  $\mathcal{M}, t_1 \models \alpha$ .

For the above defined tense operators:

1'.  $\alpha \rightarrow H F \alpha$

holds. The formula expresses the intuitive meaning of (1), thus it should be rejected to avoid the theological fatalism. Moreover, in such a case for non-ending time [if for any  $t (\in T)$  there is  $t_1 (\in T)$  such that:  $t R t_1$ ]:

3.  $F\alpha \vee F\neg\alpha$

holds, too. It means that in such a world there is no place for free deeds. It contradicts to **FC** (and **oF**, **OF**).

If we base on the classical logic to avoid both the consequences, (1') and (3), the future tense operator  $F$  has to be defined in another way.

( $F_p$ )  $\mathcal{M}, t \models F\alpha$  iff for any branch  $b_t (\in T)$  there is  $t_1 (\in b_t)$ ,  $t R t_1$ , such that:  $\mathcal{M}, t_1 \models \alpha$ .

The operator  $F$  is modalized and means: *it necessarily will be the case that*. For this Perceian notions if the time is branching in the future,

4.  $F(\alpha \vee \beta) \rightarrow (F\alpha \vee F\beta)$

does not hold. In particular, for some  $\alpha$  it could be that: *neither  $F\alpha$  nor  $F\neg\alpha$* .

The modal operator can be separated from the tense operator. In order to have the Occamists' solution one branch has to be distinguished (*prima facie*). Let it be  $b'_t$ . Now:

( $F_o$ )  $\mathcal{M}, t \models F\alpha$  iff for  $b'_t$  there is  $t_1 (\in b'_t)$ ,  $t R t_1$ , such that:  $\mathcal{M}, t_1 \models \alpha$ .

( $N$ )  $\mathcal{M}, t \models N F \alpha$  iff for any branch  $b_t (\in T)$  there is  $t_1 (\in b_t)$   $t R t_1$ , such that:  $\mathcal{M}, t_1 \models \alpha$ .

Now for non-ending time:

5.  $F(\alpha \vee \beta) \rightarrow (F\alpha \vee F\beta)$

holds, but

6.  $N F(\alpha \vee \beta) \rightarrow (N F \alpha \vee N F \beta)$

and

7.  $F N \alpha \rightarrow N F \alpha$

do not hold.

In any of the both solutions, the tense logic is based on the classical logic. It is possible to construct a tense logic based on the intuitionistic logic. We have to modify semantics according to intuitionistic requirements. Instead

of one structure  $(T, R)$  we have a partially-ordered  $(\leq)$  set of times  $(T_\gamma, R_\gamma)$ ,  $\gamma \in \Gamma$ . It is supposed that:

if  $\gamma \leq \phi$ , then  $T_\gamma \subseteq T_\phi$  and  $R_\gamma \subseteq R_\phi$ .

The valuation is such that:

if  $\gamma \leq \phi$ , then  $V_\gamma(t) \subseteq V_\phi(t)$ .

The semantics has the following intuitive motivation. At any moment  $t$  there is given partially-ordered set of "states of knowledge". Within each state-of-knowledge there is a set of times and a temporal ordering. The information in a lesser states-of-knowledge is retained in a greater states-of-knowledge.

The function  $V$  can be uniquely extended for any formula. The definition of the future and past tense operators are essentially the same as  $F_c$  and  $P_c$ , respectively.

$(F_i)$   $\mathcal{M}_\gamma, t \models F\alpha$  iff there is  $t_1(\in T)$ ,  $tR_\gamma t_1$ , such that:  $\mathcal{M}_\gamma, t_1 \models \alpha$ .

$(P_i)$   $\mathcal{M}_\gamma, t \models P\alpha$  iff there is  $t_1(\in T)$ ,  $tR_\gamma t_1$ , such that:  $\mathcal{M}_\gamma, t_1 \models \alpha$ .

There is a difference in the case of  $G$  and  $H$ :

$(G_i)$   $\mathcal{M}_\gamma, t \models G\alpha$  iff for each  $\phi$ ,  $\gamma \leq \phi$ , for each  $t_1(\in T_\phi)$ ,  $tR_\phi t_1$ :  $\mathcal{M}_\phi, t_1 \models \alpha$ .

$(H_i)$   $\mathcal{M}_\gamma, t \models H\alpha$  iff for each  $\phi$ ,  $\gamma \leq \phi$ , for each  $t_1(\in T_\phi)$ ,  $t_1R_\phi t$ :  $\mathcal{M}_\phi, t_1 \models \alpha$ .

In the case of intuitionistic tense logic:

1'.  $\alpha \rightarrow HF\alpha$

holds. It means that the arguments for theological fatalism remains. But, because:

3.  $F\alpha \vee F\neg\alpha$

does not hold, it is a logic of a world in which there are possible some free deeds. The logical structure of the world allows some free deeds, but these deeds could be done as free only by the omniscient being  $\mathcal{G}$ .

So far we have showed logical compatibility of worlds in that:

- I. the argument for theological fatalism is valid and no free deeds are possible,
- II. the argument for theological fatalism is not valid and there are possible some free deeds;
- III. the argument for theological fatalism is valid and there are possible some free deeds – in this case the free deeds could be done only by the omniscient being.

It remains the fourth combinatorial possibility:

IV. the argument for theological fatalism is not valid and no free deeds are possible.

In both the logics in which  $F\alpha \vee F\neg\alpha$  does not hold – tense logic of time branching in the future and intuitionistic tense logic – if we suppose  $F\alpha \vee F\neg\alpha$ , in consequence we receive:  $\alpha \rightarrow HF\alpha$ . It means that in the case of completely determined world, the argument for theological fatalism is valid.

The fact that there are possible some free deeds does not contradict to the possibility that any contingent  $\alpha$  sooner or later is actualized (*The principle of plenitude*). This kind of fatalism was already considered in antiquity. The famous *Master Argument* of Diodorus Cronus is based on the definition of possibility as something that is or will be. The question of this kind of fatalism does not have direct connection with problems considered in this paper.



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