IN SEARCH OF SOCIAL ORDER

UNIVERSITY OF BIAŁYSTOK
Białystok 2005

IN SEARCH OF SOCIAL ORDER

Series: STUDIES IN LOGIC, GRAMMAR AND RHETORIC 8(21)

IN SEARCH OF SOCIAL ORDER

Edited by
Halina Święczkowska

UNIVERSITY OF BIAŁYSTOK
Białystok 2005

Series: STUDIES IN LOGIC, GRAMMAR AND RHETORIC
edited by:
The Chair of Logic, Informatics
and Philosophy of Science (Faculty of Mathematics and Physics)
Department of Semiotics (Faculty of Law)
University of Białystok
e-mail: logic@uwb.edu.pl

Editor for the volume: **Halina Święczkowska**
Editorial Assistants: Katarzyna Doliwa, Dariusz Surowik

Refereed: Witold Marciszewski

Cover design: Krzysztof Michałowski
Type-setting: Stanisław Żukowski

ISBN ISBN 83-7431-059-6
ISSN 0860-150X

WYDAWNICTWO UNIWERSYTETU W BIAŁYMSTOKU
15-097 Białystok, ul. Marii Skłodowskiej-Curie 14, tel. 0857457059
http://wydawnictwo.uwb.edu.pl, e-mail: ac-dw@uwb.edu.pl
Nakład 200 egz.

Druk i oprawa: Mazowieckie Zakłady Graficzne s.c.
Wysokie Mazowieckie, ul. Ludowa 89
tel. (086) 2754131, e-mail: drukarnia@mzgraf.pl

CONTENTS

Tom R. Burns[1]
University of Uppsala

Ewa Roszkowska
University of Bialystok

GENERALIZED GAME THEORY: ASSUMPTIONS, PRINCIPLES, AND ELABORATIONS GROUNDED IN SOCIAL THEORY

Abstract. Game theory in its several variants can be viewed as a major contribution to multi-agent modeling. One development of classical game theory, Generalized Game Theory (GGT), entails its extension and generalization through the formulation of **the mathematical theory of rules and rule complexes** and a systematic grounding in **contemporary social science**. Social theory concepts such as norm, value, belief, role, social relationship, and institution as well as game can be defined in a uniform way in terms of such rules and rule complexes.

The paper presents several of the key assumptions, principles, and applications of GGT, among others: (1) GGT provides a cultural/institutional basis for the conceptualization and analysis of games in their social context. Game is reconceptualized as a **social form**, showing precisely the ways in which the rule complexes of social relationships come into play in shaping and regulating game processes. (2) GGT formulates a general theory of judgment on the basis of which actors either construct their actions or make choices among alternative actions through making comparisons and judging similarity (or dissimilarity) between the option or options considered in the game and their salient norms and values in the situation. (3) GGT distinguishes between open and closed games. The structure of a closed game is fixed. Open games are those in which the agents have the capacity to transform game components, either the individual role components of one or more players, or the general "rules of the game". Rule formation and re-formation is, therefore, a function of interaction processes. (4) GGT reconceptualizes the notion of "game solution", stressing above all that any "solution" is from a particular standpoint or perspective, for instance, the perspectives of particular players. Therefore, some "solutions" envisioned or proposed by players with different frameworks and interests are likely to be contradictory or incompatible. Under some conditions, however, players may arrive at "common solutions" which are the basis of game equilibria. (5) GGT reconceptualizes game equilibria, distinguishing different types of game equilibria. Among these is a sociologically important type of equilibrium, namely normative equilibrium, which is the basis of much social order. (6) While the theory readily and systematically incorporates the principle that human actors have bounded factual knowledge and computational capability (Simon, 1969), it emphasizes their extraordinary social knowledge ability and competence: in particular, their knowledge of diverse cultural forms and institutions such as family, market, government, business or work organization, and hospitals, among others, which they bring to bear in their social relationships and game interactions.

The paper concludes with a comparison and contrast between GGT and classical game theory on a number of central theoretical dimensions.

[1] This paper was completed while the author was Visiting Scholar at the Center for Environmental Science and Policy, Stanford University.

ISBN 83-7431-059-6 ISSN 0860-150X

PART ONE: OVERVIEW

1. Introduction

Game theory in its several variants can be viewed as a major contribution to multi-agent modeling. In their classic work, Von Neumann and Morgenstern (1944:49) defined a game as simply the totality of the rules which describe it. They did not, however, elaborate a theory of rules. Other limitations derive from the relatively unrealistic cognitive and social psychological assumptions of the theory and to matters of the weak empirical relevance and applicability of the theory to the analysis of concrete social phenomena. The cumulative critique has been massive and its summary would require a book. Our purpose here is more constructive.

One relevant development of classical game theory, Generalized Game Theory (GGT), entails an extension and generalization, addressing several of the most serious limitations. While critical of the classical approach, the point of departure of GGT has been to explore fruitful ways to extend and develop it (and also rational choice theory). In general, GGT has entailed extending social and cognitive-judgmental as well as mathematical aspects of game theory.

(1) In GGT, games are conceptualized in a uniform and general way as rule complexes in which the rules may be imprecise, possibly inconsistent, and open to a greater or lesser extent to modification and transformation by the participants (Burns and Gomolińska, 1998; 2000, 2001, Burns et al, 2001; Gomolińska, 1999, 2002, 2004, 2005). Rules and rule configurations are mathematical objects (the mathematics is based on contemporary developments at the interface of mathematics, logic, and computer science). GGT has developed the theory of combining, revising, replacing, and transforming rules and rule complexes.

Informally speaking, a rule complex is a set consisting of rules and/or other rule complexes[1]. The notion of rule complex was introduced as a generalization of a set of rules. The motivation behind the development of this concept has been to consider repertoires of rules in all their complexity with complex interdependencies among the rules and, hence, to *not* merely consider them as sets of rules. The organization of rules in rule complexes provides us with a powerful tool to investigate and describe various sorts of rules with respect to their functions such as values, norms, judgment rules, prescriptive rules, and meta-rules as well as more complex objects consisting of rules such as roles, routines, algorithms, models of reality as well as social relationships and institutions.

(2) Classical game theory assumes a social structure where the actors are completely "autonomous" or independent from one another. Each actor judges the situation in terms of her own desires or values. There is no concern with others as such. This is illustrated by the classical rational agent who assigns values or preferences to outcomes and the patterns of interactions in terms of their implications for herself – and only herself – and tries to maximize her own gain or utility.

This extremely narrow conception of social relationships will not do. Actors are not only interdependent in action terms but in social relational, institutional, and cultural-moral terms. Hence, the importance of taking into account and analyzing such factors as the social context of games – which contribute to defining many if not most of the "rules of the game."

GGT can be characterized as a cultural institutional approach to game conceptualization and analysis (Baumgartner et al, 1975a, 1975b; Burns, 1990; Burns, 1994; Burns and Buckley, 1974; Burns et al, 1985; also see Ostrom, 1990; Ostrom et al, 1994; Scharpf, 1997)[2]. A well-specified game in the context or situation S_t at time t, $G(t)$, is an interaction situation where the participating actors typically have defined roles and role relationships (see Figure 1). A social role is a particular rule complex, operating as the basis of the incumbent's values, perceptions,

[1] A **rule complex** is obtained according to the following formation rules: (1) Any finite set of rules is a rule complex; (2) If C_1, C_2 are rule complexes, then $C_1 \cup C_2$ and $P(C_1)$ are rule complexes; (3) If $C_1 \subseteq C_2$ and C_2 is a rule complex, then C_1 is a rule complex. In words, the class of rule complexes contains all finite sets of rules, is closed under the set-theoretical union and the power set, and preserves inclusion. For any rule complexes C_1 and C_2, $C_1 \cap C_2$, $C_1 - C_2$ are also rule complexes. A complex B is a subcomplex of the complex A if $B = A$, or B may be obtained from A by deleting some rules from A and/or redundant parentheses.

[2] Rules and rule systems are key concepts in the new institutionalism (Burns and Flam, 1987; Hodgson, 2002; March and Olsen, 1984; North, 1990; Ostrom, 1990; Powell and DiMaggio, 1991; Scott, 1995), evolutionary sociology (Aldrich, 1979; Burns and Dietz, 1992, 2001; Schmid and Wuketits, 1987), and ethnomethodology (Garfinkel, 1967) and are closely related to important work in philosophy on "language games" (Wittgenstein, 1953) as well as work in linguistics (Chomsky, 1980, 1986; Pinker, 1991). Much contemporary social science research points up that social rule systems – as constituting cultural formations, normative frames, and institutional arrangements – are ubiquitous and partially determinant of social action and interaction. There are cognitive, instrumental, social, aesthetic, and other reasons that human agents introduce, utilize, adhere to, and enforce rules (see later). Of course, some rules are more ephemeral and symbolic than others. Actors may fail (or refuse) to follow (or enforce) some of the rules.

judgments and actions in relation to other actors in their particular roles in the defined game. In general, actors play a number of different roles and are involved in several social relationships and institutional domains.

An actor's role is specified in GGT in terms of a few basic cognitive and normative components (formalized as mathematical objects in (Burns and Gomolińska, 1998, 2000a, 2000b, Burns et al., 2001; Gomolińska, 1999, 2002, 2004, 2005). The role complex includes, among other things: particular beliefs or rules that define the reality of relevant interaction situations; norms and values relating, respectively, to what to do and what not to do and what is good or bad; repertoires of strategies, programs, and routines; and a judgment complex to organize the determination of decisions and actions in the game. GGT has identified and analyzed several types of judgment modalities, for instance: routine or habitual, normative, and instrumental modalities. The rule complex(es) of a game in a particular social context guide and regulate the participants in their actions and interactions at the same time that in "open games" the players may restructure and transform the game and, thereby, the conditions of their actions and interactions.

(3) Game theory makes heroic and largely unrealistic assumptions about actors: complete, shared, and valid knowledge of the game. Also, unrealistic assumptions are made about the abilities of players to compute (for example, payoffs and the maximization of payoffs) and about the consistency of their preferences or utilities. The player is an egoist who at the same time tries to be a strategist, taking into account how other(s) might respond to her and whether or not her own choice or action is the "best response" to others' expected actions (see below). She "takes into account" the other in order to make a best choice for self. Each actor searches through her action space (as in the 2-person game) and finds that action which is the best response to "the best of other(s)".

In GGT, players' knowledge may be only partial, possibly even invalid to varying degrees. It may also differ from player to player. Cognitive and computational capabilities are strictly bounded and, at the same time may vary substantially among players. Judgment and action determinations are also likely to vary, for instance due to the different roles actors play and possibly their different interests in the interaction situation. Their interactions and outcomes depend in part on their beliefs as well as estimates of one another's beliefs, values, and judgement qualities. They operate with models of the situation. These constructions may contain incomplete and imperfect information (and possibly even false information) (Burns and Gomolińska, 2001; Burns and Roszkowska, 2001b). Also, communication processes among players may entail persuasion and deception which influence in game processes beliefs, evaluations, and judgments. GGT thus starts to approach the complexity and peculiarities of actual social games.

In the GGT approach, a well-specified game is a particular **multi-agent interaction structure** in which the participating players have defined roles and role relationships. The general game structure can be represented by a rule complex G (Burns and Gomolińska, 1998; Gomolińska, 1999, 2002, 2004). Such a rule complex may be imprecise, possibly inconsistent, and open to a greater or lesser extent to modification and transformation by the participants[3]. Given an interaction situation S_t in context t (time, space, social and physical environment), some rules and subcomplexes of the general game structure G are activated and implemented or realized. This $G(t)$ complex includes then as sub-complexes of rules the players' social roles vis-à-vis one another along with other relevant norms and rules in the situation S (and time t).

Suppose that a group or population $I = \{1, \ldots, m\}$ of actors is involved in a situationally defined game $G(t)$. $ROLE(i, t, G)$ denotes actor i's role complex in $G(t)$ at moment $t \in T$ (we drop the "G" indexing of the role)[4]:

$$ROLE(i, t) \subseteq_g G(t), \text{ where } t \in T \tag{1}$$

The game structure $G(t)$, in moment $t \in T$, consists then of a configuration of two or more roles together with R, that is, some general rules (and rule complexes) of the game:

$$G(t) = [ROLE(1, t), ROLE(2, t), \ldots, ROLE(k, t); R]. \tag{2}$$

R contains rules and rule complexes which describe and regulate the game such as the general "rules of the game", general norms, practical rules (for instance, initiation and stop rules in a procedure or algorithm) and meta-rules, indicating, for instance, how seriously or strict the roles and rules of the game are to be implemented, and also possibly rules specifying ways to adapt or to adjust the rule complexes to particular situations.

An actor's role is specified in GGT in terms of a few basic cognitive and normative components, that is rule subcomplexes (see Figure 1): (1) the complex of beliefs, $\boldsymbol{MODEL(i,t)}$, frames and defines the situational reality, key interaction conditions, causal mechanisms, and possible scenarios of the interaction situation; (2) there is a complex of values,

[3] Not all games are necessarily well-defined with, for instance, clearly specified and consistent roles and role relationships. Many such situations can be described and analyzed in "open game" terms (Burns, Gomolińska, and Meeker, 2001).

[4] $A \subseteq_g B$ represents that A is a subcomplex of B.

VALUE(i,t), including values and norms relating, respectively, to what is good or bad and what should and should not be done in the situation; (3) there are defined repertoires of possible strategies, programs, and routines in the situation, ***ACT(i,t)***; (4) a judgment complex or function, ***J(i,t)***, is utilized by actor i to organize the determination of decisions and actions in relation to other agents in situation S_t (Burns and Roszkowska, 2005b). The judgment complex consists of rules which enable the agent i to come to conclusions about truth, validity, value, or choice of strategic action(s) in a given situation. In general, judgement is a process of operation on objects. The types of objects on which judgements can operate are: values, norms, beliefs, data, and strategies as well as other rules and rule complexes. There are also different kinds of outputs or *conclusions* of judgment operations such as evaluations, beliefs, data, programs, procedures, and other rules and rule complexes.

In general, $MODEL(i,t)$, $VALUE(i,t)$, $ACT(i,t)$, and $J(i,t)$ are the complexes of rules which are activated in situation S and at moment of time $t \in T$ respectively in complexes $MODEL(i)$, $VALUE(i)$, $ACT(i)$, $J(i)$.

2. The Principle of Action Determination: A Type of Judgment

Judgment is a core concept in GGT (Burns and Gomolińska, 2000, 2002; Burns, Gomolińska, and Meeker, 2001; Burns and Roszkowska, 2004; Burns et al, 2005a). The major basis of judgment is a process of comparing and determining similarity, as stressed earlier. The capacity of actors to judge similarity or likeness (that is, up to some threshold, which is specified by a meta-rule or norm of stringency), plays a major part in the construction, selection, and judgment of action. In this paper, the focus is on similarity of the properties of an object with the properties specified by a value or norm. But there may also be comparison-judgment processes entailing the similarity (or difference) of an actual pattern or figure with a standard or prototypical representation (Sun, 1995).

Several types of judgments are distinguished in GGT, for instance, value judgments, factual judgments, action judgments, among others. For our purposes here, we concentrate on judgments about action.

The action judgment process could be connected with one option, two options, or a set of options. In case of a single option judgment, each actor i estimates the "goodness of fit" of this option in relation to her values in $VALUE(i,t)$. In the case of two options, the actor judges which of them

is better (and possibly how much better). In the case of a set of three or more options, the actor chooses one (or a few) from the set of options as "better than the others". In multiple option judgments, the actors generate preferences over their options.

Let B be a set of possible action alternatives. In making their judgments and decisions about an action b from B, the players activate relevant or appropriate values, norms, and commitments from their value complexes. These are used in the assessments of options through comparison-evaluation processes. In determining or deciding a particular action b, a player(s) compares and judges the similarity between the option b from the set B and the appropriate, primary value or goal v which is to be realized in decisions and performances in $G(t)$, as specified, for instance, in her role complex. More precisely, the actor judges if a finite set of expected or predicted *qualia* or attributes of option b, $Q(b)$ are *sufficiently similar* to the set of those qualia $Q(v)$ which the primary norm or value v (or a vector of values) prescribes.

The principle of action determination states: Given the interaction situation S_t and game $G(t)$, an actor i in Role (i,t) oriented to the value v (or a vector of values) specifying dimensions and standards $Q(v)$, then i tries to construct, or to find and select, an action pattern or option b where $b \in B$, and b is characterized by dimensions and levels $Q(b)$, which satisfy the following rough or approximate equation[5],

$$J(i,t)(Q(b),Q(v)) = \text{sufficiently similar} \qquad (3)$$

Such an action b is a realizer or satisfier of v. The equation implies that the actor i should "enact b" (in other words, the conclusion of the judgment process is to "do b" since $Q(b)$ is judged to sufficiently satisfy $Q(v)$. Or, in the case that there are several options, $Q(b)$ is judged more similar to $Q(v)$ than other options in B).

Action judgment is based then on a comparison of the expected qualia of an action a, $Q(a)$, with the consequences specified by a relevant value or norm v_i, $Q(v_i)$. Each and every actor $i \in I$ in a game $G(t)$ oriented to a specific value or norm v_i tries to construct or find in her repertoire $ACT(i,t)$ an action a^* that satisfies equation (3): thus, $J(i,t)(Q(a^*),Q(v_i))$ = sufficiently similar. She would enact such an action (expecting to realize or satisfy v_i). Whether the actor is successful or not depends, of course, on the interaction conditions and what others do (the principle of interdependency).

[5] Elsewhere (Burns and Roszkowska, 2004; Roszkowska and Burns, 2002) we have elaborated this model using a fuzzy set conceptualization. The general formulation of equation (3) relates to the notion of "satisficing" introduced by Simon (1969).

For $ACT(i,t) = \{a_1, a_2, \ldots, a_p\}$ let the results of judgment of similarity be some expression describing degree of dissimilarity d_j (that is, the **gap** between a particular action performed or to be performed and the norm or value specifications of v_i).

$$J(i,t)(Q(a_k), Q(v_i)) = d_j, \text{ where } a_k \in ACT(i,t). \quad (4)$$

We simplify expression (4): $J(i,t)(Q(a_k), Q(v_i)) = J(i,t)(a_k) = d_j$ where it is understood that the judgment of the action a_k is based on a comparison and assessment with respect to the given value or norm v_i. That is, the desirable qualia of an action $Q(v_i)$ are specified by v_i and are compared to the expected qualia $Q(a_k)$ of the action a_k.

The different degrees of similarity may be compared by means of $>$ (or $\geq$). Given two (or more) alternatives, d_j, d_r, $d_j > d_r$ (or $d_j \geq d_r$) means that the actor judges that action a_k such that $J(i,t)(a_k) = d_j$ better realizes (or, at least not worse in realizing) v_i than does a_s, where $J(i,t)(a_s) = d_r$. She would then prefer a_k to a_s if and only if $J(i,t)(a_k) > J(i,t)(a_s)$ (in the case $J(i,t)(a_k) = J(i,t)(a_s)$ the judgment of the two actors is one of indifference in terms of realizing appropriate values). She would chose to enact a_k rather than a_s (or there is no basis for her to make a choice in the case $J(i,t)(a_k) = J(i,t)(a_s)$. More generally[6], given a repertoire of actions, players are able to rank order (at least, a subset of them) with respect to the capacity of actions to realize the value or norm v_i:

$$J(i,t)(a_{k_1}) > \ldots > \ldots > J(i,t)(a_{k_i}) > \ldots > J(i,t)(a_{k_p}), \quad (5)$$

where $a_{k_i} \in ACT(i,t)$

Given an action repertoire $ACT(i,t)$, the action determination judgment entails finding that action which best fits ("goodness of fit") or is most consonant with v_i. The actor chooses among the given options in her fixed repertoire the action a^* that maximaze d_j, the "goodness of fit" between the anticipated consequences of actions and the consequences prescribed or indicated by the norm. Formally, Actor i selects the action a^* ($a^* \in ACT(i,t)$) for which[7]

$$J(i,t)(a^*) = \mathrm{Max}[J(i,t)(a_k)] \text{ for all } a_k \in ACT(i,t) \quad (6)$$

[6] An action a_k may be cognitively formulated in a complex manner where the qualia associated with a_k, $Q(a_k)$, include such "consequences" as the responses of other agents. Thus, the players in making their judgments may consider and weigh combinations of actions such as cooperation (CC) or non-cooperation ($-C-C$) as well as other patterns in the game, for example, the PD game (see later).

[7] This may be formulated as maximizing goodness of fit (Burns and Gomolińska, 2000).

3. Game Processes: Interaction Patterns and Outcomes

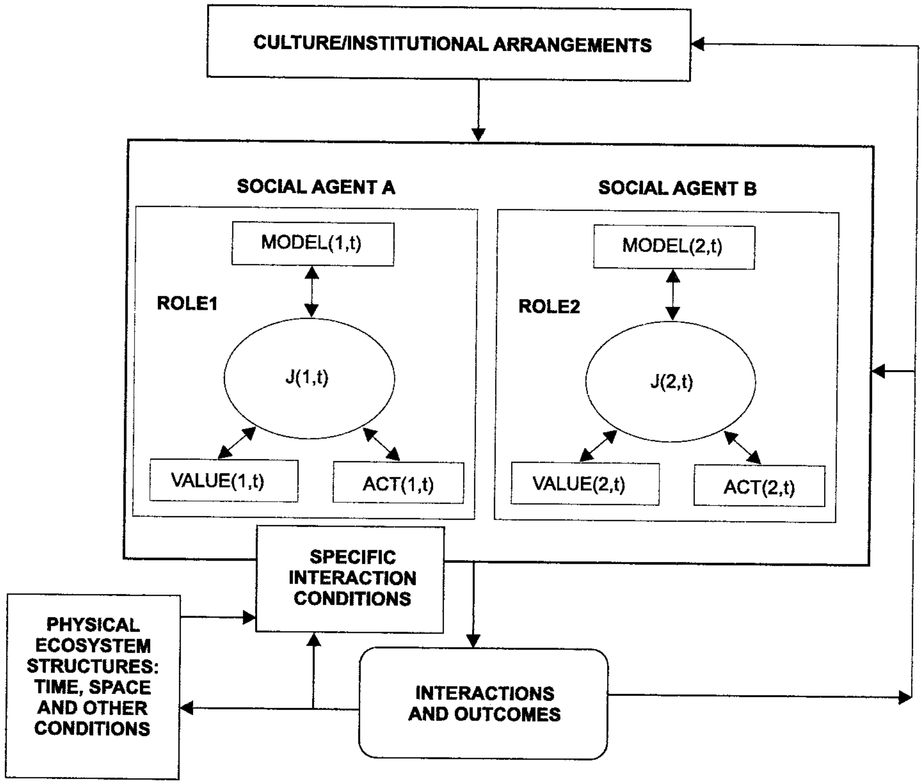
GGT investigates and models multi-agent social systems in which the agents have different roles and role relationships. Most modern social systems of interest can be characterized in this way. That is, there is already a pre-existing social structure or institutional arrangement which defines the general game structure G and shape and regulate interaction among the players (see Figure 1). The G structure is translated into a process whenever the actors defined by G are in an interaction situation S_t in context t (time, space, social and physical environment) such that some rules and subcomplexes of G, $G(t)$, are activated and implemented or realized: $G(t) \subseteq_g G$, where $t \in T$. The $G(t)$ complex includes then as sub-complexes of rules the players' particular social roles vis-à-vis one another along with other relevant norms and rules in the situation S (and time t).

Interactions or games taking place under well-defined conditions entail then the *application and implementation of relevant rules and rule complexes* of game complex $G(t)$. This is usually *not* a mechanical process. Actors conduct situational analyses; they find that rules have to be interpreted, filled in, and adapted to the specific circumstances[8]. Some interaction processes may be interrupted or blocked because of application problems: contradictions among rules, situational constraints, social pressures from actors within $G(t)$ and also pressures originating from agents outside the game situation, that is in the larger social context. In general, not only do human agents apply relevant values and norms specified in their roles vis-a-vis one another in situation S, but they bring to their roles values and norms from other social relationships. For example, their roles as parents may come into play and affect performance in work roles (or vice versa). They also develop personal "interests" in the course of playing their roles, and these may violate the spirit if not the letter of norms and values defining appropriate role behavior. More extremely, they may reject compliance and willfully deviate, for reasons of ideals or even particular interests. Finally, agents may misinterpret, mis-analyze, and, in general, make mistakes in applying

[8] More generally, GGT stresses the process of following or applying a rule in a certain sense (Burns and Gomolińska, 2000a). This may not be a trivial matter, as Wittgenstein (1956) and Winch (1958) pointed out. We limit ourselves to the following observations. Some of the actors in I may allege a violation of the norm. This may not entail a dispute over the norm itself, but over its application, an issue of fact. Related problems may arise: some of the actors have conflicting interpretations of the meanings of the norm or of its particular application in the situation S. Or the participants, while adhering to the common norm, introduce different (and possibly incompatible) rules of other sorts, potentially affecting the scope of a norm and the equilibrium in the situation.

Figure 1

Two Role Model of Interaction Embedded in Cultural-Institutional and Natural Context



and performing rules. In general, role behavior is not fully predictable or reliable.

Given a multi-agent social system, the agents have different roles and role relationships and operate according to the action determination principle (3). Within an already pre-existing institutional arrangement or social structure, agents in two or more roles $(1, 2, 3, \ldots, m)$ vis-à-vis one another interact (or conduct games) generating interaction patterns, outcomes, and developments.To illustrate how games are played, let us consider the role relationship $\{ROLE(1), ROLE(2), R\}$ of players 1 and 2, respectively, in their positions in an institutionalized relationship in which they play a game $G(i, t)$ in the situation t. Such role relationships typically consist of *shared as well as interlocked rule complexes*. The concept of interlocked complementary rule complexes means that given a particular rule in one actor's role complex concerning his or her behavior toward the other, there is

a corresponding rule in the other's actor's complex. For instance, in the case of a superordinate-subordinate role relationship (Burns and Flam, 1987), a rule k in $ROLE(1)$ specifies that actor 1 has the right to ask actor 2 certain questions, or to make particular evaluations, or to direct actions and to sanction 2. In 2's complex there is a rule m, obligating 2 to recognize and respond appropriately to actor 1 asking questions, making particular evaluations, directing certain actions, and sanctioning actor 2.

Human action is determined by means of one or more modalities. A modality may focus on, for instance: (i) the outcomes of the action ("consequentialism" or "instrumental rationality"); (ii) compliance with a norm or law prescribing particular action(s) ("duty theory"); (iii) the emotional qualities of the action ("feel good theory"); (iv) the expressive qualities of the action (action oriented to communication and the reaction of others as in "dramaturgy" theory); (v) or combinations of these. Role incumbents focus on specific qualia in particular contexts, because, among others, (1) such behavior is prescribed by their roles as the "right thing to do", (2) such behavior is institutionalized in the form of routines, (3) the actors lack time, sufficient information, or computational capability to deal with other dimensions (qualia).

Thus, games may be played out in different ways, as actors operate within opportunity structures and constraints and determine their choices and actions (Burns and Roszkowska, 2004, Roszkowska and Burns, 2002; Burns et al, 2005a):

- *routine interactions*, that is, the actors utilize habitual modalities (bureaucratic routines, standard operating procedures (s.o.p.'s), etc.) in their interaction.
- *consequentialist-oriented interactions*. Actors pay attention to the outcomes of their actions, apply values in determining their choices and behavior on the basis of outcomes realizing values.
- *normativist-oriented interactions*. Actors pay attention to, and judge on the basis of norms the qualities or attributes of action and interaction, applying general as well as role specific norms in determining what are right and proper actions.
- *emotional interactions*.
- *symbolic communication and rituals*.

There may be combinations of these, including such mixtures as when some actors orient to outcomes interact with other actors who are oriented instead to qualities of the action. Or, some, following a routine, interact with others who operate according to a "feel good" principle, etc.

4. Applications to the PD and Other Games: Judgment Calculi, Interaction Patterns, and Social Equilibria

For illustrative purposes, let us consider games where the players' social relationships vary. We consider several of the most common social relationships: status or authority relations (hierarchy), solidary relations, rivalry and antagonistic relationships. The values which the players apply and their action determinations in any given interaction situation, for instance, the prisoners' dilemma game, will differ as a function of their established social relationship.

In a symmetrical, solidary relationship, there is a normative order orienting the players to cooperating with one another and assigning high value to mutually satisfying interactions and outcomes. In, for instance, the prisoners' dilemma (PD) game, this action would be one of mutual cooperation. Consider the standard PD game:

Table 1

Outcome Matrix for 2-Actor PD Game[9]

		ACTOR 2	
		Cooperate (C)	Not Cooperate ($-C$)
ACTOR 1	Cooperate (C)	$5, 5$	$-10, 10$
	Not Cooperate ($-C$)	$10, -10$	$-5, -5$

An action a_k may be formulated in a cognitively complex manner where the qualia associated with a_k, $Q(a_k)$, include such "consequences" as the responses of one or more other agents. Thus, the players in making their judgments may consider and weigh combinations of actions such as cooperation (CC) or non-cooperation $(-C-C)$ as well as other patterns in the PD game.

Utilizing interactions patterns in the formalism of equation (5), we obtain the following action judgments for a simple 2×2 PD game for **solidaristic players** 1 and 2:

$$J(1,t)(CC) > J(1,t)(C-C) = J(1,t)(-C-C) = J(1,t)(-CC),$$
$$J(2,t)(CC) > J(2,t)(C-C) = J(2,t)(-C-C) = J(2,t)(-CC).$$

[9] The payoff (numbers) in the matrix are for illustration. Action judgments in GGT are constructed on orderings (partial orderings).

Therefore, player 1 selects C expecting player 2 to select C. Player 2 selects C expecting player 1 to select C. The outcome CC would best satisfy their mutual value orientations in the situation. The other possible interactions, for instance, the asymmetric outcomes fail to satisfy the equality norm which usually applies in their relationship; moreover, $-C-C$ does not satisfy the norm of cooperation (see Table 2).

The players' mutual expectations characterise the relationship and are inherent in each actor's $MODEL(i,t)$ of the interaction situation under the conditions of their solidary relationship.

Actors with other types of relationships would reason and judge differently. For example, given an established **relationship of rivalry**, the players would aim for processes that result in maximum difference between outcomes for self and other(s), that is **asymetrical outcome(s) favoring self.** Each actor i has a value v_i directing him or her to find or select an action a_i^* maximizing the difference between self and other, to the advantage of self. Moreover, the "best" for player 1 is clearly not the "best" for player 2: $J(1,t)(Q(a_1^*), Q(v_1)) \neq J(2,t)(Q(a_2^*), Q(v_2))$. They would rank order the options as follows:

$$J(1,t)(-CC) > J(1,t)(-C-C) = J(1,t)(CC) > J(1,t)(C-C),$$
$$J(2,t)(C-C) > J(2,t)(-C-C) = J(2,t)(CC) > J(2,t)(-CC).$$

Aiming (hoping) for the asymmetric outcome, each would choose to enact $-C$ in the game. The likely outcome is the non-cooperative one: $-C-C$.

In an **antagonistic relationship**, the actors would value interactions or outcomes that hurt the other most (possibly at considerable cost to self, maximizing difference is not the point unless this may be interpreted or defined as maximally causing harm).

$$J(1,t)(-CC) > J(1,t)(-C-C) > J(1,t)(CC) = J(1,t)(C-C),$$
$$J(2,t)(C-C) > J(2,t)(-C-C) > J(2,t)(CC) = J(2,t)(-CC).$$

Player 1 selects $-C$ expecting player 2 to choose $-C$; player 2 selects $-C$ expecting player 1 to choose $-C$. In all case(s), $-C$ leads to the best outcome (in terms of each player's value orientation toward the other player) regardless of what the other does[10].

[10] Other norms may come into play, which modify such behavior. For instance, there may be powerful norms of civility limiting extreme actions in the case of some game situations such as this one. Restraints are imposed on the relationship and its instantiations.

Table 2
Expected Patterns of Interaction and Equilibria in a PD Game Situation as a Function of Selected Common Social Relationships

TYPE OF SOCIAL RELATION-SHIP	CHARACTERISTIC VALUE COMPLEX AND RULES: Meta-evaluation and decision rules, specifying appropriate interactions and outcomes (the latter satisfying, for instance, principle(s) of distributive justice)	APPLICATION TO THE PRISONERS' DILEMMA GAME. TYPES OF EQUILIBRIA
SOLIDARY	The actors are governed by the value of solidarity (joint gains or sharing of gains that is, **symmetric distribution**) and norms of cooperation and self-sacrifice.	The norms of the relationship are satisfied by (CC), also the symmetric outcome of (CC) are right and proper. The actors decide jointly on (CC) unless segregated from one another, in which case try to take one another into account). The (CC) pattern provides an optimal outcome, also satisfying the relationship's principle of distributive justice. (CC) is therefore a **normative equilibrium**.
RIVALRY (COMPETITIVE)	Contradictory values. Each is oriented to surpassing the other (maximizing the difference in gains between self and other). The only acceptable outcome for each would be an **asymmetric** one where self gains more (or loses less) than other. But these expectations are mutually contradictory.	$(-CC)$ for actor 1 and $(C\text{-}C)$ for 2 would be judged right and proper, respectively. The likely (and situational) outcome, $-C\text{-}C$, in the game fails to satisfy the distributional rules which motivate them. **Neither normative nor situational equilibrium obtains**. The result is unstable, because each would try to transform the game.
ADVERSARY	The value orientation of each is to cause harm to the other.	The action $-C$ would be judged as right and proper, consistent with the orientation of each. Outcomes when the other suffers $(-C\text{-}C)$, or $(-CC)$ for player 1 or $(C\text{-}C)$ for player 2 would satisfy the normative orientations of both players. Since the non-optimal outcome $(-C\text{-}C)$ satisfies each of their values or goals vis-à-vis the other, namely to harm the other, this would be a type of **equilibrium based on parallel value orientations**.
HIERARCHY	Norm specifying appropriate interaction: player 1 has the right to take initiatives and decide and 2 has the obligation to show deference. Right and proper outcomes are also asymmetric, with 1 receiving more than 2 (which satisfies the relation's principle of **asymmetric** distributive justice).	The asymmetric interaction $(-CC)$ satisfies the norm of assymetric interaction, and the unequal payoff satisfies the principle of distributive justice. $(-CC)$ is therefore a **normative equilibrium**.
RATIONAL EGOISTS (INDIFFERENCE)	Each follows the principle of instrumental rationality (strategies derive value from their accomplishments for self). No interaction pattern or outcomes has collective normative force.	Rational calculation leads to the $(-C\text{-}C)$ pattern of interaction, which is sub-optimal. This would be a **situational equilibrium**, but unsatisfactory and therefore unstable. Rational actors would be predisposed to work out coordinating mechanisms in order to achieve the optimum outcome, that is, a "common solution".

Actors having a **status or authority relationship** operate with a primary norm specifying **asymmetrical** interaction and payoffs. The person of superior status or authority dominates and her subordinate(s) show deference and a readiness to accept leadership or initiatives from the superior person[11]. The principle of distributive justice in the case of such a hierarchical relationship implies asymmetry. Each expects asymmetry in the interaction process and the outcomes.

$$J(1,t)(-CC) > J(1,t)(-C-C) = J(1,t)(CC) > J(1,t)(C-C),$$
$$J(2,t)(-CC) > J(2,t)(-C-C) = J(2,t)(CC) > J(2,t)(C-C).$$

Therefore, player 1 selects $-C$ expecting player 2 to choose C; player 2 selects C expecting player 1 to choose $-C$.

The expected results in other standard games are derivable in a straightforward manner (Burns, 1990). Thus, solidary players in a "zero-sum game"[12] would pursue interactions minimizing their joint losses. In any "positive sum" or coordination game, they would try to select interaction(s) maximizing their joint gains. On the other hand, rivals in a zero-sum game would each pursue options to produce maximum differences between self and other results (favoring of course self). Enemies would look to cause maximal harm to the other (but possibly within some cost limits)[13]. Solidary players in a game of "chicken would choose to avoid confrontation all together. Rational actors in the "game of chicken" would avoid the extreme and risky action to the extent that they are risk-adverse. Enemies would (and do) risk catastrophic play in a game of "chicken" (at least up to the threshold of unacceptable losses to self). Rivals might also risk such catastrophic play. In general, one can identify types of closed games that are problematic for particular social relationships. Players with solidary rela-

[11] On a personal level, the lower status person might want something else but within some limits of acceptance behaves in a way consonant with the relationship.

[12] Games of "total conflict" are those in which what one player gains, the other loses. In a certain sense, this type of game is a **distributional game** rather than one of mutual destruction that characterize the confrontation game (or "game of chícken").

[13] In the case of actors who are hostile to one another (but this applies to rivals as well), there are likely to still be limits to their commitment to "hurt or undo the other". Under extreme conditions, they may experience the dilemma between acting in a manner consistent with their relationship (e.g. causing maximum harm to the other in an adversarial relationship) or restraining self and avoiding the risk of substantial loss to self. The strength of the desire to survive or to avoid "excessive" loss or suffering would be decisive here, but these are assessments exogenous to the logic of their relationship. Such considerations would lead to mutual deterrence. The deterrence may, of course, breakdown under some conditions – where, for instance, one or the other players goes over the limit, either through accident, miscalculation, or brinkmanship, and the other responds in kind, unleashing a process which is difficult, once underway, to curb, because of powerful pressures toward reciprocation. Thus, the conflict tends to escalate.

tionships would find problematic highly asymmetrical zero-sum games and would try to transform them (or avoid playing them)[14]. Rivals, on the other hand, would appreciate such games, and would find games with symmetric outcomes highly problematic; in general, they would want to find or construct games with real differences in outcomes, favoring of course self. Those with adversary relationships would not relish game situations lacking opportunities to cause harm-to-the other.

In an open game, where the players construct their actions and interactions, utilizing appropriate value(s) and norm(s) to guide each of them, processes such as the following would be likely to occur. Each of the players in, for instance, a solidary relationship, finds the action $a(1)^*$ and $a(2)^*$, respectively, which sufficiently realize (or comply with) v_i; these would entail cooperative or helpful type actions[15]. Equation (3) would be satisfied in that the players judge $a(1)^*a(2)^*$ as a right and proper cooperative interaction (it might entail implementing a complex of rules or a program) (recall that the judgment is based on a comparison of the expected properties or attributes of an action a, $Q(a)$, with the attributes specified by the value or norm v_i, $Q(v_i)$) for each actor i). Thus,

$$J(i,t)(Q(a(1)^*a(2)^*), Q(v_i)) = \text{sufficiently consonant} \longrightarrow$$
$$\longrightarrow \text{therefore, enact the } a(1)^*a(2)^* \text{ (“cooperative”) interaction.}$$

$$J(i,t)(Q(\text{non-cooperative}), Q(v_i)) = \text{dissonant} \longrightarrow$$
$$\longrightarrow \text{therefore, avoid such action.}$$

Constructing interactions may take place under conditions of competitive or antagonistic social relationships. Such players would generate new interactions and outcomes, possibly developing or adopting new technologies and strategies, as they strive to outdo or harm one another. While the

---

14 Each player has, however, certain rough limits with respect to the “sacrifices” that she is prepared to make. For instance, actor i has a maximum value above which she is not willing to go for the sake of the relationship (or, if she does, she is intentionally or unintentionally potentially redefining the relationship (as more solidary and entailing a higher level of commitment). The other actor j may accept this limitation, acknowledging such a norm by not pressing i beyond such a threshold. Thus, the maximum value sets a limit for equilibrium interactions between i and j. Of course, the greater the value of a social relationship to the participants, the higher the limit or maximum, and the higher the level of cooperation, self-sacrifice, and commitment. In general, agents in institutionalized solidary relationships are predisposed to make sacrifices up to the value of the relationship. Failure to live up to these implicit mutual obligations or commitments would tend to undermine the relationship.

15 For our purposes here, it is sufficient to consider a general norm such as “the principle of reciprocity” or “cooperativity” applying to both actors. Their roles are likely to prescribe role specific and differing norms for each.

game complex undergoes transformation, the competitive or antagonistic character (or identity) of the relationship – and the interaction patterns – are invariant (or reproduced). This is a type of **dynamic equilibrium** (obviously, in this case there is no normative equilibrium which the players can agree to accept or find collectively meaningful). A mediator may assist in such situations; she helps them establish a new basis for playing the game(s) (for instance, moving from total mistrust and mutual aggressivity to partial trust and cooperativity). Ultimately, a new social relationship is established through such a process.

In general, actors in institutionalized relationships are more or less predictable and understandable to one another through shared characterization and knowledge of their relationship(s). This proposition applies even to open game situations. Participants can thereby take into account in their judgments and calculations **the scope** of what they may “reasonably” request or expect from one another (Burns et al, 2001) (miscalculations and mis-judgments nevertheless occur, of course). Moreover, the knowledge of the principles or meta-rules defining limits and the scope of commitment to a particular value complex (see footnotes 14 and 15) means that the players can to a greater or lesser extent predict some of the likely consequences of adaptations and developments of their relationship.

PART TWO: SYNOPSIS OF MAJOR FEATURES OF GGT

In this part, we identify several of the key features of GGT and indicate their implications for the description and analysis of games and game processes generally.

(1) The contextualized **game structure $G(t)$, that is**, in context $t \in T$, is a rule complex whose subcomplexes are the roles that the different game agents play vis-à-vis one another. The roles are made up of subcomplexes representing key behavioral functions.
For each and every actor $i \in I = \{1, \dots, m\}$

$$\{MODEL(i,t), VALUE(i,t), ACT(i,t), J(i,t)\} \subseteq_g \subseteq_g ROLE(i,t) \subseteq_g ROLE(I,t) \subseteq_g G(t) \tag{7}$$

(2) GGT treats games as **socially embedded** in cultural and institutional contexts (Granovetter, 1985) (see Figure 1). The participants – in defining

and perceiving an interaction situation, assessing it and developments in it, and judging actions and consequences of actions – do so largely from the perspectives of their particular roles and social relationships in the given cultural-institutional context. The role relationships within given institutional arrangements entail contextualized rule complexes including values and norms, the modes for classifying and judging actions and for providing "internal" interpretations and meanings (Burns, 1990, 1994; Burns and Flam, 1987)[16].

(3) GGT provides a systematic theoretical basis on which to represent and analyze **symmetric as well as asymmetric games** (and the social structures in which they are embedded)[17]. Actors are distinguished by their positions and roles in society, by the asymmetries in their relationships (superordinate/subordinate, high status/low status, master/slave), by their endowments, access to resources (including special information, networks, etc.). Such variation implies different action capabilities and repertoires. Expected patterns of interaction and equilibria will vary accordingly. Also, the actors' different information and belief components in their *MODELS*, their diverse values, standards, and goals in *VALUE* complexes, the available repertoires of strategies (*ACT*), and their possibly different judgment complexes (*J*) for action determination are distinguishable and analyzable in GGT. If such variation is specified, taken into account, and analyzed in game investigations, then **empirically diverse interaction patterns and outcomes become more readily described, understandable, and predictable**.

(4) GGT treats the **information** available – the knowledge of the participants – as variables, whereas classical game theory makes heroic claims about the high level and accuracy of players' knowledge (more or less complete). In most interaction situations, information is far from complete,

[16] "Non-cooperation" in, for instance, a prisoners' dilemma (PD is referred to in the classical approach as "defection"). In the GGT perspective on the social contextualization of games, the action is not merely "defection" in the case that the actors are friends or relatives in a solidary relationship. Rather, it is a form of "betrayal" or "disloyalty" and subject to harsh social judgment and sanction. In the case of enemies, "defection" in the PD game would be fully expected and considered "natural" – neither shameful nor contemptible, but "right and proper" harm to one another. Clearly, it is not a matter of "defection". Such a perspective enables us to systematically identify and analyze the symbolic and moral aspects of established social relationships in particular game situations (Burns, 1994).

[17] The structure of game theory limits it to describing and analyzing more or less symmetrical games.

is usually imprecise (or fuzzy), and even contradictory (Burns and Roszkowska, 2004; Roszkowska and Burns, 2002). GGT takes into account such conditions in representing and analyzing games. Moreover, information is typically distributed unequally among players or utilized by them in diverse ways, including even ineffective ways. The level and quality of knowledge of a player i is representable in $MODEL(i,t)$. **This complex may be modified during the course of the game (see later). Some information, which classical game theory would consider essential, may be non-essential in particular GGT games**. For instance, payoffs might not be precisely specified or might be altogether unknown to one or more of the participants. The implications of these conditions differ depending on the established social relationships among the players. Those in solidarity relationships would tend to rely on their inherent **cooperative potential**; that is, they would be inclined to trust in one another's good will in dealing cooperatively with many types of problems. The latter include information problems and the risk of substantial losses. In general, information about individual payoffs would not be essential in many games where the players have strong underlying solidarity relationships, which would predispose them to "correct" ex post unfair results or developments. Such actors are predisposed to focus especially on the characteristics of the action ("cooperativeness") and interactions ("reciprocity"). Moreover, they would expect that in the face of a veil of ignorance (ex ante) or unanticipated consequences (ex post), they can together solve emergent problems (of course, there may be cases where solutions fail to materialize, or "betrayals" occur). In games where agents are alienated from one another, they experience high uncertainty and would want substantially more information not only about outcomes but also about the "character" of other players and their established ways to interpret and enact rules. In cases where such information is unavailable, players tend to rely on standard operating procedures and habitual modalities, requiring much less information, or information of another type than required for instrumental modality. Finally, in open games, there is never full information. Actors generate information as they develop strategies in the game and as the game unfolds, transforming rules and rule complexes.

(5) **The principle of action determination** – corresponding to the principle of maximizing utility in rational choice theory – subsumes several distinct modalities of action determination, each with its own "logic" (Burns and Gomolińska, 2000a, 2000b; Burns, Gomolińska, and Meeker, 2001). The theory encompasses instrumental rationality corresponding in some respects

to the rational choice approach of game theory, but allows for much more variability in the information and calculation conditions. It also encompasses additional modes of action and interaction that are fully intelligible and empirically grounded, but are not reducible to the principle of rational choice. GGT distinguishes modalities such as normatively oriented action, dramaturgical-communicative action, and "play" as well as combinations of these (see earlier). Each modality entails a logic of generating or determining action with a particular **judgment calculus, requiring as inputs specific types of data or information and generating particular evaluative, decisional and action outputs** (Burns and Gomolińska, 2000b). Each modality is a particular way of paying attention and organizing and selecting situational data in the given interaction situation S; it activates certain rule complexes and applyies particular values, norms, and routines in making judgments and determining action.

The modalities of action determination distinguish themselves in part by the prescribed consequences or dimensions, $Q(v)$, specified by the norm or value v. The actor is oriented to, attends to, and tries to regulate actual anticipated or perceived consequences, $Q(a)$ of an action a from A that she constructs or considers for choice. In an instrumental modality, for instance, the value of an action derives from judgments of **action outcomes**, whereas the value of action in the case of normative modality derives from judgments of the **intrinsic qualities of the action itself** (including possibly the intentionality of the actor). In other words, **there are operational differences in cognitive and informational terms between normative and instrumental modalities as well as other modalities.** These differences are particularly noteworthy in the case of open interaction situations where the **players construct their actions and interactions** in an ongoing process. With normative modality, the players construct an action (or actions) which entails or corresponds to prescribed intrinsic properties or qualities of the action (or actions). In the case of instrumental modality, the actors are supposed to produce an outcome or state of the world with prescribed features, that is, they must find or construct an action (or actions) that they believe produces or leads to the prescribed consequences – the properties of the action itself are not prescribed. Of particular importance is the fact that the instrumental modality requires a **model of causality** linking actions to outcomes, or enabling the specification of such linkages.

A narrow focus on outcomes as in the modality of instrumental rationality – ignoring the qualities including ethical qualities of action and interaction – implies that actors behave as if "the ends justify the means." This of

course over-simplifies judgmental computations. But the same one-sidedness and imbalance characterize those who focus only on the intrinsic qualities of actions, ignoring outcomes as in normative or procedural rationality. A narrow focus on the intrinsic properties of action considers action(s) as "right" regardless of outcomes, even catastrophic ones. However, once actors are motivated by and take into account multiple values – for instance, considering ethical qualities of actions as well as their instrumental outcomes – they are likely to be faced with dilemmas and tendencies to blocked or erratic behavior (Burns, Gomolińska, and Meeker, 2001).

(6) **Game transformation** is conceptualized in GGT in terms of the rewriting (updating and revising) as well as restructuring of rules and rule complexes: agents may modify rules, may throw some out, introduce new rules or activate (or deactivate) them; these may also consist of a combination of several such operations. Transformative operations are likely to be taken when one, several, or all players in a game find no game consequences acceptable (for instance, the non-optimal outcome of "rationally" based non-cooperation in the PD game). The game rules that have led to this outcome may be rejected by some of the players; they would try to introduce, for instance, coordination rules – that is, they would take initiatives to establish an institutional arrangement – which increases the likelihood of obtaining the optimal cooperative outcome in the PD game.

Other reasons for transforming games is to make them consistent with core societal values and norms, or with the particular social relationship(s) among the players. For instance, players transform a symmetric game into an asymmetric game more appropriate for actors with differences in status and authority. Or similarly, actors with an egalitarian or democratic type relationship would try to transform an asymmetric game (with differences in action opportunities and payoffs) into a symmetric game more compatible with their estabished social relationship. Such game transformations reflect, of course, not only the players' value orientations but their transformative capabilities and processes.

(7) **Open And Closed Games (that is, open and closed to transformation).** Classical games are closed games with specified, fixed players, fixed value (or preference) structures and judgment complexes (for instance, maxmin or other optimization procedure) as well as fixed action alternatives and outcomes. Such games are analytically distinguishable from *open game situations* (Burns, Gomolińska, and Meeker, 2001). Open and closed games are distinguishable precisely in terms of the degree of fixedness of

the players' role complexes: value, model, action, and judgment complexes for the different players belonging to I at time t in game $G(t)$. In closed game conditions, these are specified and **invariant** for each actor $i \in I$, situation S_t, and game $G(t)$. Such closure is characteristic of classical games (as well as many parlour games), whereas most real human games and interaction processes are open. In open games, the actors participating in $G(t)$ can transform one or more role components as well as the general "rules of the game" R. For instance, one or more players may re-construct or elaborate $ACT(I,t)$ in the course of their interactions. Or, they may change value complexes (including changes in their preferences or utility functions), or modify their models and judgment complexes in such open games. Thus, in a bargaining process, the actors often introduce during the course of the negotiations new options or strategies – or undergo shifts in their values and judgment complexes. In such bargaining processes, the particular social relationships among the actors involved – whether relations of solidarity, anomie, or rivalry – guide the construction of options and the patterning of interaction and outcomes. Thus, each actor i in I reconstructs her repertoire of actions, $ACT(i,t)$ or other role components in the course of her interactions. She tends to do this in accordance with the norms and values relevant to her role or the social relationship appropriate in the situation S_t at time t.

(8) **A Reconceptualization of "Solution".** In classical theory, the theorist or social planner specifies an equilibrium (there may be several) which is the "solution to the game." If an equilibrium is not specified, then some player could gain by changing his or her strategy to something other than what the theorist has specified for her. In the classical theory, equilibrium is a "solution" to the game. And what we refer to as a "common solution" is an equilibrium.

Earlier, we pointed out in the case of the PD game, one or more players may reject some game rules because they prove to be ineffective or to lead to suboptimal (even disastrous) outcomes. They respond to the situation by introducing, for example, particular coordination rules which increase the likelihood of obtaining the optimal (cooperative) outcome. The coordination rules are a "solution" to the "PD problem". The transformed game structure results in one or more "common acceptable solution(s)" to the PD game.

In the GGT perspective, social agents define and understand "solutions" on the basis of the institutional context, their social relationships, roles, value complexes, and cognitive-judgment frames. They have "standpoints"

from which they identify problems and propose solutions[18]. The solutions proposed may or may not converge on one or more outcome(s).

A common or general game solution is a multi-agent strategy or an interaction order that satisfies or realizes the relevant norm(s) or value(s) of the players, resulting in a state that is judged acceptable – or even satisfactory – by the game players. The latter consist of a population of agents or a single collective agent (for instance, a group of people who are organized to make collective decisions such as a "public policy decision"). An "acceptable solution" is the best result attainable under the circumstances; in a certain sense this makes for an "equilibrium" state, although not necessarily a normative equilibrium.

Solution proposals of the actors may diverge. In other words, there is no **common** solution, at least initially; in other words, no multi-agent strategy or outcome which is acceptable to all participants. For instance in a negotiation situation; the positions of the players might be too far apart and no agreement or settlement can be reached. An "equilibrium" in such a game is then the state of **not** bargaining or playing the game.

What is judged a solution for one agent (or several agents) from a particular perspective or perspectives may be judged as a problem from the particular perspective(s) of other players. In other words, any game may entail particular "problems" for one or more players, while others may not experience "problems" in the situation. Realizing a norm or value or achieving a goal is a "solution" to the problem of unrealized goals, values, or norms. The players may have different views on satisfactory or even acceptable "solutions". Or the differences may occur between individual and collective agents. Thus, we distinguish situations where proposed solutions are **convergent** (that multiple actors find it acceptable or even highly satisfactory) as opposed to a situation where the solutions proposed by different agents contradict one another – they are **divergent proposals**. Clearly, not every game has a **common solution** (Roszkowska and Burns, 2002).

(9) **Reconceptualizing Game Equilibrium.** An interaction or game equilibrium is a type of **common solution** where the participants find a particular interaction pattern or outcome as acceptable or even satisfactory. The key to this conception are the judgment processes whereby "problems" are "solved" or partially solved. When there is convergence in the solutions, then an equilibrium state is possible. If there is divergence,

[18] The theorist (as well as arbitrators) also have "standpoints" and can propose "solutions". Whether the players accept such solutions is another question.

however, then no equilibrium obtains (unless "solutions" are imposed, for instance, by a dictator). The players endorse or pursue different, incompatible solutions.

GGT distinguishes different types of game equilibria. One such is the Nash equilibrium. It is a game state from which no actor in the game can improve his or her individual situation by choosing an action or outcome differing from this equilibrium. Elsewhere we have generalized the Nash equilibrium in terms of our conceptualization of players' judgment complexes and their evaluative judgments:

Nash Generalization (Burns and Roszkowska, 2004): Let G be a game, $I = \{1, 2, \ldots, m\}$ set of players, S_i, set of strategies the i-player, where $i \in I$. An m-tuple of strategies $a_I = (a_1, a_2, \ldots, a_i, \ldots, a_m)$, for $a_i \in S_i$, is a Nash equilibrium state in pure strategies in a game G if the inequality below holds for each and every player i and for every strategy $b_i \in S_i$[19]

$$J(i,t)(a_1, a_2, \ldots, a_i, \ldots, a_m) \geq J(i,t)(a_1, a_2, \ldots, b_i, \ldots, a_m)$$

where $J(i,t)(x)$ represents player i's evaluative judgments of the outcomes of the m-tuple of strategies (option) x in situation S_t.

Consider that the game theorist or an arbitrator propose a Nash equilibrium as a "solution". If some reject this "solution" of the game, that is, itis not acceptable to some, many or possibly all players, then the question is whether there is another outcome solution which might be acceptable to all participants. If there is no such solution, then the players might consider the challenge of how and in which direction to change the game ("the rules of the game") or to avoid the game altogether.

Actors are often normatively or cultural interdependent in that they belong to and participate in an established social group or organization, or interact in the context of established normative controls. The agents acting collectively or in an organized way (for example, through a voting procedure or through an authority) judge game patterns and outcomes from the perspective of a common norm applied to the m-tuple, a_I, whose consequences are $Q(a_I)$. The consequences may refer to the action itself (as in performing a ritual properly) or the outcomes (the distribution of goods (or bads)), or both. The production of normatively satisfying patterns or outcomes relates to a major GGT concept, namely **normative equilibrium**. The normative equilibria associated with performances of roles,

[19] The Nash equilibrium entails m individual solutions which aggregate to a type of common solution, which is an equilibrium under some conditions.

norms, and institutional arrangements make for **social facts** and "**focal points**"[20] to which participants orient (Schelling, 1963; Burns and Gomolińska, 2000b)).

In GGT, an activity, program, outcome, condition or state of the world is in a normative equilibrium if it is judged to realize or satisfy appropriate norm(s) or value(s) v_I in the situation S for each and every participant. While the concept of normative equilibria may be applied to role performances and to individuals following a norm, we have especially utilized the concept in terms of **game normative equilibria** in a given institutional and situational context. This means that the game participants judge an m-tuple $a_I = (a_1, a_2, \ldots, a_i, \ldots, a_m)$ on the basis of whether it realizes or satisfies v_I where v_I represents a collective norm, normative procedure, or institutional arrangement. Examples of procedures to produce normative equlibria are democratic processes, adjudication, and negotiation as well as the exercise of legitimate authority; they are particularly relevant as devices to resolve conflict under conditions of contentiousness and conflict (Burns and Roszkowska, 2005).

Normative equilibria are a function of (1) the particular relationship(s) among the actors and the value or norm v_I appropriate or activated in the situation S at a given time t and (2) of the concrete situation S in which rule complexes are applied: the action possibilities found or constructed in the situation and the consequences attributed or associated with the action(s). The participants know (or believe) that others accept or are committed to these equilibria – or to the rules that produce them. This makes for a "social reality" which is more or less predictable; it provides a space for planning and developing complex, individual and collective strategies. Normatively based game equilibria are patterns or sets of consequences generated through actors realizing – or anticipating the realization of – situationally relevant values and norms (or, the collective patterns and consequences are judged in themselves to realize or satisfy shared values). Such interaction patterns and outcomes have normative force and contribute to institutional order(s).

There may also be stable game patterns which are not normative equilibria in that they lack moral force or necessary legitimacy. Game players

[20] Schelling (1963:57–58) refers also to "clues," "coordinators" that have "some kind of prominence or conspicuousness". From a conceptual point of view, his characterization is vague. For instance, "But it is a prominence that depends on time and place and who people are. Ordinary folk lost on a plane circular area may naturally go to the center to meet each other... But in the final analysis we are dealing with imagination as much as with logic... Poets may do better than logicians at this game."

might, nevertheless, accept them because they perceive them to be the best possible options under the circumstances (as in the Nash equilibrium). For instance, in closed games, there are interaction patterns which do not permit the full realization or satisfaction of important values to which participants are oriented. They may accept the patterns pragmatically or conditionally – as long as they are constrained to play the given game. But such equilibria – lacking players' commitments, and confidence or trust in them – cannot be enduring (as is the case of Nash equilibria (Burns and Roszkowska, 2004))[21]. This also applies to equilibria that are imposed, that is, collective solutions imposed, by dictators and dominant groups. Inherently, such solutions are only equilibria to the extent that the dictator or adjudicator can force the participants to comply with the imposition.

Concluding Remarks

GGT has been applied to a wide variety of phenomena: among others:

- formalization of social relationships, roles, and judgment and action modalities (Burns and Gomolińska, 2000; Burns, Gomolińska, and Meeker, 2001; Gomolińska, 1999, 2002, 2004, among others);
- reconceptualization of prisoners dilemma game and other classical games as socially embedded games (Burns, Gomolińska, and Meeker, 2001; Burns and Roszkowska, 2004);
- models of societal conflict resolution and regulation (Burns, Caldas, and Roszkowska, 2005; Burns and Roszkowska, 2005);
- rethinking the Nash equilibrium (Burns and Roszkowska, 2004);
- fuzzy games and equilibria (Burns and Roszkowska, 2004; Roszkowska and Burns, 2002);
- socio-cognitive analysis and belief revision (Burns and Gomolińska, 2001; Roszkowska and Burns, 2002);
- simulation studies in which GGT is applied, for instance, in the formulation of multi-agent simulation models of regulatory processes (Burns et al, 2005a, 2005b).

[21] An outcome that is not Pareto optimal is one where the actors, if they cooperate in restructuring their pattern – or underlying rules – can improve the payoffs for some (or all) of them without reducing the payoffs for others, namely through movement to the cooperative interaction. Pareto optimal points are stable against universal coalitions, because it is not possible to deviate from such states without hurting some players. Thus, this acts as a constraint on collective shifts (Scharpf, 1997; Tsebelis, 1990). The PD game has a Nash equilibrium for rational egoists, namely non-cooperative interaction, which, however, is not Pareto optimal.

Our generalization of classical game theory implies that there are many game theories or models reflecting or referring to different social relationships and corresponding rationalities or interaction logics. Classical game theory is, therefore, a quite general but nevertheless limited model in its scope. **It is applicable to a particular type of social relationship: namely that between unrelated or anomic agents acting and interacting in accordance with particular "rationality" rules and modalities.** The actors lack sentiments – either for or against – one another. And they are purely egoistic in their relationship. Moreover, their games are closed ones. They may not change the rules such as the number and qualities of participants, the specific action opportunity structures and outcomes, the shared modality of action, their value complexes and models of the interaction situation. The creative aspect of action, as exhibited in open games, has been acknowledged by Tsebelis (1990), but he recognizes that such problems cannot be addressed systematically within the classical game theory framework.

Table 3 identifies several key dimensions which distinguish game theory (and rational choice theory), on the one hand, and GGT, on the other. While sharing a number of common elements, GGT and game theory exhibit substantial differences in conceptualizing and modeling human action and interaction.

Table 3

Comparison of Generalized Game Theory and Classical Game Theory

THE GENERAL THEORY OF GAMES	CLASSICAL GAME THEORY
Game rule complex, $G(t)$ – together with physical and ecological constraints – structure and regulate action and interaction.	**Game constraints** ("rules") which include physical constraints
Players: **Diverse types of actors in varying roles**; actors as creative, interpreting, and transforming beings	Players: **universal**, **super-rational agents lacking** creativity and transformative capabilities.
Games may be **symmetrical** or **asymmetrical** – actors have different roles, positions of status and power, endowments; also, diversity in role components: value, model, act, judgment/modality, etc. They operate in different social and psychological contexts.	**Mainly symmetry**
Game transformation based on the innovative or creative capabilities of players; exogenous agents may also engage in shaping and reshaping games	**Game structures are fixed**
Open and closed games (this follows from the preceding)	**Closed games**

***VALUE*$(i,G(t))$ complex**: A player's value and evaluative structures derive from the social context of the game (institutional setup, social relationships, and particular roles). Some values belong to a sacred core, grounded in identity, status, role(s), and institutions to which agents may be strongly committed. "Not everything is negotiable"	**Utility function** or preference ordering is given and exogenous to the game.
***MODEL*$(i,G(t))$ complex.** A player's model of the game situation which may be based on highly incomplete, fuzzy, or even false information. Imprecise (or fuzzy/rough) data as well as imprecise rules and norms, strategies, and judgment processes. Reasoning processes may or may not follow standard logic.	**Perfect or minimally imperfect information** about the game, its players, their options, payoffs, and preference structures or utilities. **Crisp** information, strategies, decisions.
***ACT*(i,G) complex.** It represents the repertoire of acts, strategies, routines, programs, and actions available to player i in her particular role and role relationships in the game situation. In classical game theory, a particularly important class of actions (and constraints on action) concern communication. In GGT **communication** conditions and forms are specified by the rules defining action opportunities in a given game. The diverse forms of communication and their uses or functions affect game processes and outcomes: for instance, to provide information or to influence the beliefs and judgments of the other. Communication may even entail deception and fabrication. Moreover, actors may or may not use available opportunities in the interaction situation to communicate with one another or to follow the same rules (degree of asymmetry).	**Set of possible strategies and communication conditions. Communication rules** are axioms at the start of the game and apply to all players. Non-cooperative games do not allow for communication. Cooperative games allow for communication (and the making of binding agreements).
JUDGMENT/MODALITY: $J(i,G(t))$-complex. Multiple modalities of action determination including instrumental, normative, habitual, play, and emotional modes of action determination, among others, which depend on context and definitions of appropriateness. The universal motivational factor is the human drive to realize or achieve particular value(s) or norm(s).	**Singular modality**: Instrumental rationality **or "rational choice". Maximization of expected utility** as a universal choice principle.
Bounded capabilities of cognition, judgment, and choice. Contradiction, incoherence and dilemmas, arise because of multiple values and norms which do not always fit together in a given situation. Consistency and coherence are socially constructed and vulnerable.	**Super-capabilities** of deliberation and choice according to fixed axioms of rationality. Hamlet syndrome is not possible.
Solution concept: "solutions" are defined from a particular standpoint or model of each player. Disagreements among actors about appropriate or satisfactory solutions is expected. A common or general game solution satisfies or realizes the values or goals of the multiple players in the game.	**An "equilibrium" is the solution to the game.**
Different types of equilibria, generalized Nash equilibrium, normative and other social equilibria including equilibria imposed by an authority or dictator.	**Mainly Nash equilibrium** (which conflates different types of socially distinct and meaningful equilibria)

References

Aldrich, H. E. (1979) *Organizations and Environments.* Englewood Cliffs, N.J. Prentice-Hall.

Anderson, Elizabeth (2000) "Beyond Homo Economicus: New Developments in Theories of Social Norms", *Philosophy & Public Affairs*, Vol. 29: 170–200.

Baumgartner, T., W. Buckley, and T. R. Burns (1975a) "Relational Control: The Human Structuring of Cooperation and Conflict." *J. of Conflict Resolution*, Vol. 19: 417–440.

Baumgartner, T., W. Buckley, and T. R. Burns (1975b) "Meta-power and Relational Control in Social Life." *Social Science Information*, 14: 49–78.

Boudon, R. (1996) "The Cognitivist Model: A Generalized 'Rational-Choice Model." *Rationality and Society*, Vol. 8 (2): 123–150.

Boudon, R. (1998b) "Limitations of Rational Choice Theory." *American Journal of Sociology*, vol. 104 (3): 817–828.

Bourdieu, P. (1977) *Outline of a Theory of Practice.* Cambridge: Cambridge University Press.

Buckley, W., Burns T. R., and Meeker D. (1974) "Structural Resolutions of Collective Action Problems." *Behavioral Science*, Vol. 19: 277–297.

Burns, T. R. (1990) "Models of Social and Market Exchange: Toward a Sociological Theory of Games and Social Behavior." In: C. Calhoun, M. W. Meyer, and W. R. Scott (eds.), *Structures of Power and Constraints: Papers in Honor of Peter Blau.* Cambridge, Cambridge University Press.

Burns, T. R. (1994) "Two Conceptions of Human Agency: Rational Choice Theory and the Social Theory of Action." In: P. Sztompka (ed), *Human Agency and the Reorientation of Social Theory.* Amsterdam: Gordon and Breach.

Burns, T. R. and Baumgartner T. (1984) *Transitions to Alternative Energy Systems: Entrepreneurs, Strategies, and Social Change.* Colorado: Westview Press.

Burns, T. R., Baumgartner, T. and DeVillé, P., (1985) *Man, Decisions, Society*, London/New York, Gordon and Breach.

Burns, T. R. and Buckley W. (1974) "The Prisoners' Dilemma Game as a System of Social Domination." *J. of Peace Research*, Vol. 11: 221–228.

Burns, T. R., Caldas J. C. and Roszkowska E. (2005a) "Generalized Game Theory's Contribution to Multi-agent Modelling: Addressing Problems of Social Regulation, Social Order, and Effective Security." In: B. Dunin-Keplicz, A. Jankowski, A. Skowron, and M. Szczuka (eds.), *Monitoring, Security and Rescue Techniques in Multiagent Systems.* Springer Verlag, Berlin/London.

Burns, T. R., Caldas J. C., Roszkowska E., and Wang J. (2005b) "Multi-Agent Modelling of Institutional Mechanisms:The Approach Of Generalized Game Theory." Paper to be presented at the World Congress of the International Institute of Sociology, Stockholm, Sweden, July, 2005

Burns, T. R. and Carson M. (2002) "Actors, Paradigms, and Institutional Dynamics: The Theory of Social Rule Systems Applied to Radical Reforms." In: R. Hollingsworth, K. H. Muller, E. J. Hollingsworth (eds) *Advancing Socio-Economics: An Institutionalist Perspective.* Oxford: Rowman and Littlefield.

Burns, T. R. and Dietz T. (2001) "Revolution: The Perspective of the Theory of Socio-cultural Evolution." *International Sociology,* in preparation.

Burns, T. R. and Dietz T. (1992a) "Cultural Evolution: Social Rule Systems, Selection, and Human Agency." *International Sociology,* Vol. 7: 259–283.

Burns, T. R. and Engdahl E. (1998) "The Social Construction of Consciousness: Collective Consciousness and its Socio-Cultural Foundations"; "Individual Selves. Self-Awareness, and Reflectivity." Parts I and II. *J. of Consciousness Studies,* Vol 5, Numbers 1 and 2.

Burns, T. R. and Flam, H. (1987) *The Shaping of Social Organization: Social Rule System Theory with Applications.* London: Sage Publications.

Burns, T. R. and Gomolińska A. (1998) "Modeling Social Game Systems by Rule Complexes." In: L. Polkowski and A. Skowron (eds.), *Rough Sets and Current Trends in Computing.* Berlin/Heidelberg, Springer-V.

Burns, T. R. and Gomolińska A. (2000a) "The Theory of Socially Embedded Games: The Mathematics of Social Relationships, Rule Complexes, and Action Modalities." *Quality and Quantity: International Journal of Methodology,* Vol. 34 (4): 379–406.

Burns, T. R. and Gomolińska A. (2000b) "The Theory of Social Embedded Games: Norms, Human Judgment, and Social Equilibria." Paper presented at the Joint Conference of the American Sociological Association-International Sociological Association on Rational Choice Theory, Washington, D.C., August 2000.

Burns, T. R. and A. Gomolińska (2001) "Socio-cognitive Mechanisms of Belief Change: Application of Generalized Game Theory to Belief Revision, Social Fabrication, and Self-fulfilling Prophesy." *Cognitive Systems,* Vol 2 (1), 39–54.

Burns, T.R., A. Gomolińska, and L. D. Meeker (2001) "The Theory of Socially Embedded Games: Applications and Extensions to Open and Closed Games." *Quality and Quantity: International Journal of Methodology,* Vol. 35 (1): 1–32.

Burns, T. R. and Meeker D. (1974) "Structural Properties and Resolutions of the Prisoners' Dilemma Game." In: A. Rapaport (ed), *Game Theory as a Theory of Conflict Resolution.* Holland: Reidel.

Burns, T. R. and Meeker D. (1977) "Conflict and Structure in Multi-Level, Multiple Objective Decision-making Systems." In C. A. Hooker (ed), *Foundations and Applications of Decision Theory.* Dordrecht, Holland: Reidel.

Burns, T. R. and Roszkowska E. (2001) "Rethinking the Nash Equilibrium: The Perspective on Normative Equilibria in the General Theory of Games." Paper presented at the Group Processes/Rational Choice Mini-Conference, August 17, 2001, American Sociological Association Annual Meeting, Anaheim, California.

Burns, T. R. and Roszkowska E. (2002) "Fuzzy Judgment in Bargaining Games: Diverse Patterns of Price Determination and Transaction in Buyer-Seller Exchange." Paper presented at the 13th World Congress of Economics, Lisbon, Portugal, September 9–13. Also, appears as Working-Paper No. 338, Institute of Mathematical Economics, University of Bielefeld, 2002 (http://www.wiwi.uni-bielefeld.de/~imw/papers/338)

Burns, T. R and Roszkowska, E. (2004) "Fuzzy Games and Equilibria: The Perspective of the General Theory of Games on Nash and Normative Equilibria" In: Pal S. K., Polkowski L., Skowron A. editors, *Rough-Neural Computing. Techniques for Computing with Words.* Springer-Verlag, 435–470.

Burns, T. R. and Roszkowska E. (2005a) "Conflict and Conflict Resolution: A Societal-Institutional Perspective." In: M. Raith, *Procedural Approaches to Conflict Resolution.* Springer Press, Berlin/London. In press.

Burns, T. R. and E. Roszkowska (2005b) "Social Judgment In Multi-Agent Systems: The Perspective Of Generalized Game Theory." In Ron Sun (ed), *Cognition and Multi-agent Interaction.* Cambridge: Cambridge University Press.

Chomsky, N. (1980) *Rules and Representation.* New York: Columbia Univesrsity Press.

Chomsky, N. (1986) *Knowledge of Language: Its Nature, Origins, and Use.* New York: Praeger.

Coleman, J. S. 1990 *Foundations of Social Theory.* Cambridge, Mass.: Belknap Press.

Fedrizzi, M., Kacprzyk J., Nurmi H. (1993) "Consensus Degrees under Fuzzy Majorities and Fuzzy Preferences using OWA (ordered weighted average) Operators." *Control and Cybernetics*, Vol. 22 (1993): 77–86.

Garfinkel, H. (1967) *Studies in Ethnomethodology.* Englewood Cliffs, N.J.: Prentice-Hall.

Goffman, E. (1959) *Presentation of Self in Everyday Life.* Garden City, NY: Doubleday, Anchor Books.

Goffman, E. (1961) *Encounters.* Indianapolis: Bobbs-Merrill.

Gomolińska, A. (1999) "Rule Complexes for Representing Social Actors and Interactions." *Studies in Logic, Grammar, and Rhetoric*, Vol. 3 (16): 95–108.

Gomolińska, A. (2002) "Derivability of Rules From Rule Complexes". *Logic and Logical Philosophy*, Vol. 10: 21–44.

Gomolińska, A. (2004) "Fundamental Mathematical Notions of the Theory of Socially Embedded Games: A Granular Computing Perspective." In: S. K. Pal, L. Polkowski, and A. Skowron (eds.) *Rough-Neural Computing: Techniques for Computing with Words.* Springer-Verlag, Berlin/London, pp. 411–434.

Gomolińska, A. (2005) "Toward Rough Applicability of Rules." In: B. Dunin-Keplicz, A. Jankowski, A. Skowron, and M. Szczuka (eds.) *Monitoring, Security, and Rescue Techniques in Multiagent Systems.* Springer-Verlag, Berlin/London, pp. 203–214.

Granovetter, M., (1985), "Economic Action and Social Structure: The Problem of Embeddedness", *American Journal of Sociology*, 91, 481–510.

Hardin, R. (1982) *Collective Action.* Baltimore: John Hopkins University Press.

Harre, H. (1979) *Social Being.* Oxford: Blackwell.

Harre, H. and Secord P. F. (1972) *The Explanation of Social Behavior.* Oxford: Blackwell.

Hechter, M. (1987) *Principles of Group Solidarity.* Berkeley: U. of California Press.

Hodgson, Geofrey M. (2002) "The Evolution of Institutions: An Agenda for Future Theoretical Research". *Constitutional Political Economy*, 13, pp. 111–127.

Hollis, M. (1987) *The Cunning of Reason.* Cambridge: Cambridge University Press.

Kacprzyk, J., Fedrizzi M., Nurmi H. (1992) "Group Decision making and Consensus under Fuzzy Preferences and Fuzzy Majority." *Fuzzy Sets and Systems* Vol. 49: 21–31.

Kahan, D. M. (2002) "The Logic of Reciprocity: Trust, Collective Action and Law." *Yale Law and Economic Research Paper* No. 201. Yale Law School, John M. Olin Center for Studies in Lawy, Economic, and Public Policy. New Haven, Conn.: Yale University.

Leijonhufvud, A. "Toward a Not-Too-Rational Macroeconomics." *Southern Economic Journal.* Vol. 60: 227–242.

March, J.R. and Olsen J.P. (1989) *Rediscovering Institutions: The Organizational Basis of Politics.* New York: Free Press.

Montgomery, J. D. (2001) "The Self as a Fuzzy Set of Roles, Role Theory as a Fuzzy System." In: *Sociological Methodology* 2000. Malden, Mass.: Blackwell Publishers.

North, N. C. (1990) *Institutions, Institutional Change, and Economic Performance.* Cambridge: Cambridge University Press.

Nurmi, H. (1976) "On Fuzzy Games." Paper presented at the Third European Meeting on Cybernetics and Systems Research, Vienna, Austria, April 20–23, 1976.

Nurmi, H. (1982) "Imprecise Notions iin Individual and Group Decision Theory: Resolution of Allais' Paradox and Related Problems." *Stochastica*, Vol. 6: 283–303.

Nurmi, H. (2000) "Resolving Group Choice Paradoxes Using Probabilistic and Fuzzy Concepts." *Group Decision and Negotiation*, Vol. 7: 1–21.

Olson, M. (1968) *The Logic of Collective Action.* New York: Schocken.

Ostrom, E. (1990) *Governing the Commons: The Evolution of Institutions for Collective Action.* Cambridge: Cambridge University Press.

Ostrom, E. Gardner R., and Walker J. (1994) *Rules, Games, and Common-Pool Resources.* Ann Arbor: University of Michigan Press.

Parsons, T. 1968 [1937] *The Structure of Social Action.* New York: Free Press.

Pinker, S. (1991) "Rules of Language." *Science* 253: 530–534.

Powell, W. W. and DiMaggio P. J. (eds.) (1991) *The New Institutionalism in Organizational Analysis.* Chicago: University Press.

Roszkowska E., Burns T. R (2002). Fuzzy Judgment in Bargaining Games: Diverse Patterns of Price Determination and Transaction in Buyer-Seller Exchange. Paper presented at *the First World Congress of Game Theory*, Bilbao, Spain, 2000. available at http:www.soc.uu.se/publications/fulltext/tb_>market-pricing-game.doc.

Roszkowska, E., and Burns T. R. (2000) "Market Transaction Games and

Price Determination. The Perspective of the General Theory of Games." Paper presented at Games 2000, the First World Congress of Game Theory, Bilbao, Spain, July 24–28.

Scharpf, F. W. (1997) *Games Real Actors Play: Actor-Centered Institutionalism in Policy Research.* Boulder, Colorado: Westview Press.

Schmid, M. and Wuketits F. M. (eds) (1987) *Evolutionary Theory in the Social Sciences.* Dordrecht: Reidel.

Schelling, T. C. (1963) *The Strategy of Conflict.* Cambridge: Harvard University Press.

Scott, W. R. (1995) *Institutions and Organizations.* London: Sage Publications.

Simon, H. (1969) *The Sciences of the Artificial.* Cambridge: MIT Press.

Tsebelis, G. (1990) *Nested Games: Rational Choice in Comparative Politics.* Berkeley: University of California Press.

von Neumann, J. and O. Morgenstern (1944) *Theory of Games and Economic Behaviour.* Princeton: Princeton University Press.

Winch, P. (1958) *The Idea of a Social Science and Its Relation to Philosophy.* London: Routledge & Kegan.

Wittgenstein, L. (1958) *Remarks on the Foundations of Mathematics.* Oxford: Blackwell.

Wittgenstein, L. (1953) *Philosophical Investigations.* Oxford: Blackwell.

Tom R. Burns
Uppsala Theory Circle, Department of Sociology
University of Uppsala
Box 821, 75108 Uppsala
Sweden
e-mail: tom.burns@soc.uu.se

Ewa Roszkowska
University of Bialystok, Faculty of Economics
15–062 Bialystok, Warszawska 63
Bialystok School of Economics
15–732 Bialystok, Choroszczanska 31
Poland
e-mail:erosz@w3cache.uwb.edu.pl

STUDIES IN LOGIC, GRAMMAR AND RHETORIC 8 (21) 2005

Katarzyna Zbieć
Białystok University

CONFLICT AND COOPERATION IN TERMS OF GAME THEORY – THOMAS SCHELLING'S RESEARCH

Abstract. The Nobel Prize in Economic Sciences 2005 became to Robert Aumann[1] and Thomas Schelling[2] "for having enhanced our understanding of conflict and cooperation through game-theory analysis"[3].

Their work was essential in developing non-cooperative game theory further and bringing it to bear on major questions in the social sciences. Approaching the subject from different angles – Aumann from mathematics and Schelling from economics – they both perceived that the perspective of game theory had the potential to reshape the analysis of human interaction. Schelling showed that many social interactions could be viewed as non-cooperative games that involve both common and conflicting interests. Aumann demonstrated that long-run social interaction could be comprehensively analyzed using formal non-cooperative game theory.

This paper presents a report from Schelling's research.

Introduction

Thomas Schelling began applying game theory methods to one of the era's most important matters – global security and the arms race. He was particularly interested by the ways in which the players negotiating strength could be touched by different factors, such as the initial alternatives at their disposal and their potential to influence their own and each others alterna-

[1] born 1930 in Frankfurt, Germany. PhD in mathematics in 1955 from Massachusetts Institute of Technology (MIT), Cambridge, MA, USA. Professor at the Center for Rationality, Hebrew University of Jerusalem, Israel

[2] born 1921 in Oakland, CA, USA. PhD in economics in 1951 from Harvard University, Cambridge, MA, USA. Distinguished University Professor at the Department of Economics and School of Public Policy, Emeritus, at University of Maryland, College Park, MD, USA and Lucius N. Littauer Professor of Political Economy, Emeritus, at Harvard University

[3] *Press Release*, The Royal Swedish Academy of Sciences, 10/10/2005

ISBN 83-7431-059-6 ISSN 0860-150X

tives during the process. He explained why it could be profitable to limit one's own alternatives or worsen one's own options. He was also interested in the process of establishing a climate of confidence, whereby long-term cooperation could be built up over a period of time, and in the long-run gains a party could achieve by making short-run concessions. The results of his work were published in 1960 in book *The Strategy of Conflict*[4], which became a classic and has influenced generations of strategic thinkers.

Credible deterrence

Schelling's earliest main contribution is his analysis of behavior in bilateral situations of trade negotiations. Trade negotiations be interpreted fundamentally: except clear negotiations – on example between two countries or seller and buyer – trade negotiations are also "when two trucks loaded with dynamite meet on a road wide enough for one"[5]. Trade negotiations always cause some conflict of interest, in that every party usually looks for an agreement which is as favourable as possible. Yet, any agreement is better for both party than no agreement at all. Every player has to ballance search for large the "part of cake" against interest for agreement.

Schelling examines tactician trade negotiations, which player can use in order to, to bend down in him result or her service. He underlines particularly, that this can be profitable, to worsen one's own options in order to to obtain from opponent concession. For example, it can be beneficial for general, to burn bridges for his squads as credible commitment to enemy to does not resign. A politician can gain from announcing promises which they would be to embarrass, to to break. Such tactics work, if the commitment is irreversible or it can to be unfastened in great cost only, while commitments that are cheap to reverse will turn away not obtain large concessions. Yet, if both parties do irreversible and discordant commitment, harmful disagreement may follow.

Let us consider a simple example. Suppose, that two countries do not agree by right to patch of territory. Every country can choose to mobilize military strength or hold back from doing so. If both mobilize there is a high probability of war. The payoff to each country be 0 if both mobilize. If they

[4] T. C. Schelling: *The Strategy of Conflict*, Harvard University Press, Cambridge MA, 1960

[5] T. C. Schelling: "An essay on bargaining", *American Economic Review* 46 (1956), p. 281–306

instead both refrain from mobilization, a peaceful agreement about division of territory has a high probability. In this case, each country receives a payoff b. If only one country mobilizes, then it can take the complete control of the territory without war, and other can't force a military retreat by the occupant. Aggressor receives payoff a, and the loser's payoff is c, where $a > b > c > 0$ (Table 1).

	mobilize	refrain
mobilize	$0, 0$	a, c
refrain	c, a	b, b

Table 1. Payoff matrix

Such games have three Nash equilibria[6]: two pure and one mixed. The pure equilibria cause to exactly mobilization one country; if one country waits the other to mobilize, then it is optimal to hold back from mobilization. The mixed equilibrium causes assured chance variation mobilization by each country and in this way positive probability of war.

The game's mixed equilibrium appears more plausible than the pure equilibria. Each country is then insecure about the other's movement, marking some probability p to event that the other country will mobilize. The Nash equilibrium probability of mobilization is:

$$p = \frac{(a-b)}{(a-b+c)}.$$

It's result from equating expected payoff of mobilize: $(1-p)a$ and payoff of refrain: $pc + (1-p)b$. Notice, the probability of war is decreasing in the loser's payoff c. The key to minimizing the risk of war is not only to contain the winner's profit, but also to improve the loser's payoff.

Mobilizing and threatening to mobilize are not equivalent. Suppose that first Country 1 chooses whether to hold back from mobilization completely or to commit to mobilize if and only if Country 2 mobilizes. Thereafter, Country 2 observes 1's movement and decides whether or not to mobilize. If payoffs are as described in Table 1, the equilibrium result will be that

[6] Nash equilibrium – a set of strategies, one for each player, such that no player has incentive to unilaterally change her action. Players are in equilibrium if a change in strategies by any one of them would lead that player to earn less than if she remained with her current strategy. For games in which players randomize (mixed strategies), the expected or average payoff must be at least as large as that obtainable by any other strategy.

Country 1 makes the mobilization commitment, and both countries refrain from mobilization. Such scaring away in this way guarantees a peaceful outcome.

Judge, moreover, that it Country 1 is uncertain, or Country 2 prefers war to the negotiated result. The question is: should Country 1 still commit to mobilize if country Country 2 mobilizes? Schelling's analysis reveals that the optimal commitment strategy is then often to choose a probability of mobilization that is less than one. Therefore, in the face of an enemy's military escalation, a country should threaten rather than commit to certain retaliation; in Schelling's words, make "threats that leave some things to chance", because a modest probability of war can suffice to hold back enemy's mobilization.

The above analysis suggests, that countries should keep conjecturing enemy about their answer on aggression, simultaneously assuring that strong vengeance is concerned as true option. It's necessary to remember that instability is dangerous. The balance of terror is maintained only if vengeance is sufficiently probable and heavy compared to the profits from occupation. War can be ignited by changes in preferences, in technology and successful attempts at disarmament have to be balanced throughout. Schelling's analysis of credible commitments demonstrated that some Nash equilibria are more plausible than others. His study of credible deterrence takes up a major part of *The Strategy of Conflict*.

Sometimes conflicts of interest may appear so strong as to be insoluble. The best strategy for person can call out bad result for group. The shortrun gains from cheating on an agreement might by far outweigh the short-run losses. Schelling wrote that "What makes many agreements enforceable is only the recognition of future opportunities for agreement that will be eliminated if mutual trust is not created and maintained, and whose value outweighs the momentary gain from cheating in the present instance"[7]. In this way, if parties take long perspective and affect mutually many times in fact, them common interests can be suffciently strong to support cooperation. People can structure their relationships, by extending interaction over time, in such a way as to reduce the incentive to behave opportunistically at each point in time.

The next part of Schelling's work is studying a class of social interactions that involve little or no conflict of interest (pure coordination games). These are games where all players prefer coordination on some common course of action and no player cares about which coordinated course of action is taken. In this case, coordination may be easy, if players can communicate with each other but difficult without communication. By experimenting with his students and colleagues, Schelling discovered that they were often able to coordinate rather well without communicating even in unknown games, which had host Nash equilibria. As an example, consider the game, where two peole would be asked to choose total positive integer each. They both gen an award, if choose the same number, differently no award is given. The majority was inclines to choose the number 1. This number is outstanding distinctive, this is the smallest positive integer. It seems probable that many social conventions and organizational preparation appeared because they facilitate coordination.

Mutual distrust

A final interesting class of social decision problems are interactions in which participants are mutually distrustful. For example, two generals may both agree that war is undesirable, and will hence prepare for peace as long as they both think that the other will do likewise. Yet, if one general suspects that the other is preparing for war, then his best response may be to prepare for war as well – when war is less undesirable than being occupied. This idea had already been clearly formulated by Xenophon (in the fourth century B.C.)[8]. Schelling expressed it in game-theoretic terms and considered explicitly the role of uncertainty in triggering aggression[9]. To illustrate the possibility that war is caused solely by mutual distrust, consider the payoff matrix:

	war	peace
war	2, 2	3, 0
peace	0, 3	4, 4

Table 2. Payoff matrix

[7] T. C. Schelling: "An essay on bargaining", *American Economic Review* 46(1956), p. 301

[8] T. C. Schelling: *Arms and Influence*, Yale University Press, New Haven, 1966, p. 261

[9] T. C. Schelling: *The Strategy of Conflict*, Harvard University Press, Cambridge MA, 1960, p. 207–229

Each player has the choice between going to war and keeping peacefully. The two pure strategy Nash equilibria are (War, War) and (Peace, Peace). If players are rational, conduct their plans perfectly and have not no uncertainty about opponent payoff. Schelling thought that peace would be the most credible result of such a game[10]. However, Schelling also fought, that the small quantity of nervousness about opponent's intentions would can be infectious sufficiently, to make peaceful equilibrium crush. He describe this situation as an attack dilemma: "If I go downstairs to investigate a noise at night, with a gun in my hand, and find myself face to face with a burglar who has a gun in his hand, there is a danger of an outcome that neither of us desires. Even if he prefers just to leave quietly, and I wish him to, there is danger that he may think I want to shoot, and shoot first. Worse, there is danger that he may think that I think he wants to shoot"[11].

The Strategy of Conflict has had a durable influence on the economics profession as well as on other social sciences. It has inspired, the detailed analysis of negotiating in historical crisis situations. The book and its draughts *Strategy and Arms Control*, (coauthored with Morton Halperin) and *Arms and Influence*, also had a profound impact on military theorists and practitioners in the cold war era, played a major role in establishing strategic studies as an academic field of study, and may well have contributed significantly to deterrence and disarmament among the superpowers.

Segregation

Schelling also consider what it happens when individual plans and forehead the examples of behaviour are confronted in social arena. His book *Micromotives and Macrobehavior*[12] total discloses this subject. Schelling formulated a simple model where he put, that all individuals are tolerant in the sense, that they live on place of work willingly the men's closeness with different culture, the religion or the colour of skin, but that they want to have at least several neighbour neighbors this part their own features. If not, they move to neighbourhood then they can find more people as them. Schelling

---

[10] T. C. Schelling: *The Strategy of Conflict*, Harvard University Press, Cambridge MA, 1960, p. 210

[11] T. C. Schelling: *The Strategy of Conflict*, Harvard University Press, Cambridge MA, 1960, p. 207

[12] T. C. Schelling: *Micromotives and Macrobehavior*, Harvard University Press, Cambridge MA, 1978

was visible, that even rather weak preferences regarding part person's like in neighbourhood can result in strongly he sorted the life of examples. Differently saying, no extreme preferences on the part of individuals are required in order to for a social problem to arrise.

References

Schelling T. C.: *The Strategy of Conflict*, Harvard University Press, Cambridge MA, 1960

Schelling T. C.: "An essay on bargaining", *American Economic Review* 46 (1956)

Schelling T. C.: *Arms and Influence*, Yale University Press, New Haven, 1966

Schelling: *Micromotives and Macrobehavior*, Harvard University Press, Cambridge MA, 1978

STUDIES IN LOGIC, GRAMMAR AND RHETORIC 8 (21) 2005

Honorata Sosnowska
Warsaw School of Economics

A CHOICE OF A PENSION FUND AS AN EXAMPLE OF BOUNDED RATIONALITY

Abstract. The paper describes an experiment concerned a choice of a pension fund in Poland, in 1999. That time the reform of a pension system was made in Poland. Citizens at first chose a pension fund. Many causes influenced a choice of a pension fund. People were unaware of some of them. Experiment presents some not logical causes which may be treated as examples of bounded rationality.

1. Introduction

An economic system changed in 1989 in Poland. In a previous, communist system, there were no many economic institutions, which work in market economies. People started to live in new economic system without an experience in a market economy and knowledge about its laws. In such situations people made decisions without sufficient knowledge about results of their decisions. Moreover, such knowledge was not attainable. In this paper we shall deal with a problem of a choice of a pension fund. Before 2000 year a system "pay as you go" was obligatory in Poland. The reform of a pension system introduced pension funds. A choice of a fund was obligatory for all people before 30 and optional for people between 30 and 50. It was a beginning of an activity of pension funds in Poland. No people nor funds have experience. It was very difficult to define criterions of a choice of a pension fund. A very intensive advertising was conducted. Many irrational choices were observed. In this paper we deal with a situation in which people are sure that their choices are rational. The other reasons causes that that it is a bounded rationality.

People had to choose a pension fund to the end of 1999. In the autumn on 1999 most of them chose the fund. The experiment was conducted in the autumn 1999 on a group students of MBA studies (mostly managers, 30–50 years old) in one of non public Polish high economic schools.

ISBN 83-7431-059-6 ISSN 0860-150X

2. Procedural decision making

Rubinstein (1998) in his book presents procedural decision making. We use one of his examples as a basis of our experiment. The example follows Huber, Payne and Puto (1982).

Let us consider a situation in which a travel bureau offers a holiday packages (x, y), where x denotes number of days in London, y – number of days in Paris, all at the same price. The packages $A = (7, 4)$, $B = (4, 7)$, $C = (6, 3)$, $D = (3, 6)$ are considered. It is observed that the people have a tendency to choose A from packages A, B, C and B from A, B, D. "Rational man" should choose the same element from both triples because A and B are the best packages. So, in both cases he really chooses from the same set of packages $\{A, B\}$. The difference of results is caused by procedure of decision making ("dominating another alternative") and is an example of bounded rationality.

In the above example procedure of decision making influences the choice. In the next sections we analyze an experiment based on a choice of a pension fund. People made their decision at first and results are very important in their life. It appears that a procedure of decision making influences the choice.

3. A choice of a pension fund – description of experiment

Polish people should choose their first pension fund in 1999. In autumn 1999 they have many information about pension funds. Most of people chose their fund. The experiment of a choice of a pension fund was conducted in autumn 1999 as part of exercises on bounded rationality at MBA studies in one of non public high economic school. Students were after lectures of economics and game theory.

We considered 19 pension funds. The following data were known for students. Stock capital (in millions of zlotys), commission of pension premium in percentages and annual management fee in percentages. The data based on a publication "A prospective pensioner's guide" published in weekly "Polityka" from September 4, 1999. The names of funds were not known, funds were numbered. The form with basic list of pension funds is presented in the Table 1. There is not the fund no. 3 on the basic list of funds. The reason of this lack was to make the form like the other forms used in the experiment. The fund no. 3 was not popular and it was forecasted that it would not be chosen.

Table 1

The basic list of pension funds (forms 2 and 4)

Pension Fund	Stock capital in mln zł	Commission o pension premium in %	Annual management Feein %
1.	68,988	8,5	0,6
2.	72	6,5 in 1999–2000, 6 – 2001–2002, 5 – 2003–2004, 4 – after 2004	0,6
4.	120	7,9	0,6
5.	130	10 – two first years, 4 – third year	0,6
6.	175	8,9 – two firs years, 4 – third year	0,6
7.	65	7,9 – two first years, 6,5 – till 15th year, 2,5 – since 16th year	0,6
8.	45	9,5 – first year, 9 – second year, 8,5 – third year, 5,8 – since 4th year	0,6
9.	40	8 – till third year, 6 – till 10th, 5,8 – since 10th	0,6
10.	29	8,7 – two first years, 5,8 – third year	0,6
11.	208,4	8	0,6
12.	29,167	8,9 – two first years, 8,4 – 3th, 4th years, 7,9 – since 5th	0,6
13.	17,1	7,8 – two first years, 8 – third, 5,5 – 4th, 5 – 5th, 4,5 – since 6th+	0,6
14.	120	9,9 – till 20 months, next – 8,9, after 3 years – 8,7, after 4 – 8,4, after 10 – 7,4, after 15 – 5,4, after 20 – 2,5, after 25 – 0	Till end of 1999 – 0
15.	18,77	8,9 – first two years, 7,5 – till third to 15th, 2,9 since 16th	0,6
16.	48,571	8,8 – till second year, 8,2 – after	0,6
17.	18	8,5	0,6
18.	20	9 – first two years, 7,5 – since third till 15th, 3 – after	0,6
19.	100	9 two first years, 5,25 – since third to 10th, 5 since 11th till 20th, 3,5 – after	0,6
20.	140,5	9 – first year, 7 – second year	0,6

Students chose a fund four times. Each time they got a special form. First time they got the form, where some funds (no. 2, 5, 6) were removed from the basic list. Second time they chose from the basic list. Third time they got the form where some funds were removed (no. 10, 13) but another than first time. Fourth time again the basic list was presented on the same

Table 2

Results of the experiment and real choices

Pension fund	I choice	II choice	III choice	IV choice	Real Choices
1. AIG	1	1	2	2	2
2. Allianz	0	6	7	6	0
3. –	–	–	–	–	0
4. BIG Bank	7	6	6	5	0
5. Commercial Union	x	1	7	7	7
6. Dom	x	0	2	2	1
7. Epoka	3	2	2	2	0
8. Rodzina	0	0	0	0	0
9. Kredyt Bank PBI	0	0	0	0	0
10. Nationale Nederlanden	0	0	x	0	18
11. Norwich	10	9	7	7	0
12. Orzeł	0	0	0	0	0
13. Pekao Alliance	0	0	x	0	0
14. Bankowy	3	3	6	4	2
15. Pocztylion	0	0	0	0	0
16. Pioneer	0	0	0	0	0
17. Polsat	1	0	0	0	0
18. PZU	0	0	0	0	3
19. Winthertur	7	6	0	4	1
20. Zurich – Solidarni	5	3	1	2	0

form as in second choice. At the end of experiment students wrote which fund they had chose in their real life (anonymously, of course). The results of experiment are presented in the Table 2. Results of the first choice are in the column I, the second – II, the third – III, the fourth – IV. The real choice is presented in the last column. Funds are numbered, but their names are also given.

There are the following reasons of removing just these funds in forms for first and third choice. First time, the funds of high stock capital were removed, second time (in the third form) – of low capital stock. Both times one of the most popular funds was removed (the Commercial Union, no. 5 – first form and the Nationale Nederlanden, no. 10 – third form). We supposed that the essential observations would be concerned these funds.

The number of answers are different in each choice. It yields from the fact that some students were absent in the some parts of experiment. The system of studies in this high school is such that it is almost impossible to bring together all students. Independently of absences all students understood the rules of the experiment.

4. Analysis of the results of the experiment

Let us consider the second and the fourth choice. Both time the same form with the basic list of funds was given. So, if students chose rationally, results would differ slightly. But in case of fund no. 5 (the Commercial Union, one of most popular funds) there is the essential difference. In first form the fund was removed, in second choice one student chose the fund from the basic list of fund, in third form, where other funds were removed, 7 students chose the fund, 7 students also chose the fund from the basic list in fourth form. In real life also 7 student chose the fund. So, there is a difference between second choice (1 person) and four choice (7 persons). Both time the forms was the same. We suppose that this essential difference is caused by a procedure of choice. The fund no. 5 was in the form first time only at second choice. The situation is not repeated in case of the fund no. 10 (the Nationale Nederlanden, one of most popular funds). No one chose this fund in any choice. So, the data were not persuasive for student. Most of them chose this fund in real life. They said that the reason was reputation of the fund. The reputation could not be taken into account in the experiment because the names of funds were not known.

The observed difference between number of choices in case of fund no. 5 we interpret as a result of procedural decision making and case of bounded rationality. After the experiment, but before announcement of results, students discussed their strategies of choices. All were sure that their strategy is convincing one and that they chose in a rational way.

References

Huber J., J. Payne, C. Puto, 182, "Adding asymmetrically dominated alternatives. Violations of regularity and the similarity hypothesis", "Journal of Consumer Research" 9, pp. 90–98.

Rubinstein A., 1998, "Modeling Bounded Rationality", MIT Press, Cambridge Mass.

Honorata Sosnowska
Warsaw School of Economics
Institute of Econometrics
02–554 Warsaw
al. Niepodległości 162
Poland
honorata@sgh.waw.pl

Kazimierz Trzęsicki
University of Białystok

MEDICAL INFORMATICS ETHICS (SUBJECT AND MAJOR ISSUES)

Abstract. Application of information and communication technology (ICT) in medicine and healthcare is a source of ethical questions of practical importance. We argue that medical informatics ethics (MIE) is not a new branch of applied ethics. It is rather a name under which some problems of medical (ME) and computer ethics (CE) are gathered. Some questions of application of ICT in medicine belong to CE and others to ME. In MIE medical ethics meets computer ethics. The borderline between them is neither clear nor easy to draw.

Introduction

Computer ethics (CE) has its beginnings in works of Norbert Wiener, the father of cybernetics. Around 1948 he started considering the impacts of information and communication technology (ICT) upon human values like peace, knowledge, health, education, justice. Published in 1950, his book *The Human Use of Human Beings* [16] established his position as the creator of CE. Since the middle of the sixties when Don Parker started an investigation of unethical and illegitimate use of computers, CE is still under development. Creation of the natural-language processing system ELIZA by Weizenbaum was the next important event for CE. ELIZA imitated a psychologist. Weizenbaum was appalled when psychiatrists suggested that the program might be an acceptable substitute for human therapy. Horrified, Weizenbaum began work on the philosophical problem presented by the mechanization of human characteristics and talents. His book *Computer Power and Human Reason* [15] published in 1976, is Weizenbaum's exploration of his own misgivings about technology and Artificial Intelligence (AI).

Healthcare is a very important area of application of ICT since the technology has been developed. ICT has many advantages and can deliver great hopes for better healthcare. Advances in ICT provide users with new capabilities without ethical policies having been formulated to guide those users in their conduct. The concern about ethical implications of the use of ICT in

ISBN 83-7431-059-6 ISSN 0860-150X

medicine/healthcare is ongoing. Today it is the subject of conference papers (e.g., ETHICOMP), publications (e.g., *Ethics, Computing, and Medicine: "Informatics and the Transformation of Health Care"* [3]), teaching (the course *"Ethical, Legal and Social Issues in Medical Informatics"* MINF 515 – 2 credits – is offered by Department of Medical Informatics & Clinical Epidemiology, Oregon Health Sciences University[1]: Medical College of Wisconsin[2] offers the course in "Ethics in Medical Informatics" MI-13201[3]). There are established organizations with the aim of setting and observing ethical standards of using ICT in medicine, e.g., The Health On the Net Foundation (HON)[4]. This non-governmental organization was created in 1995 under the aegis of the Direction générale de la santé Département de l'Action Sociale et de Santé: République et canton de Genève, Switzerland). Porfirio Barroso Asenjo at the conference ETHICOMP 95 presented "Health Informaticians' Deontology Code" (HIDEC) which had already been accepted in Greece. In 2000 the eHealth Ethics Initiative introduced an international code of ethics for healthcare sites and services on the Internet[5]. There are working groups dedicated to investigations of ethical, and legal issues of medical informatics, e.g., the Working Group "Ethical, Legal, and Social Issues" (ELSI-WG) of American Medical Informatics Association[6]. In 2002

---

1 See http://www.ohsu.edu/dnice/courses/offering.shtml. The course comprises the following topics: The protection of confidentiality and privacy in an electronic environment; Implications of the use of telemedicine and decision-support tools in diagnosis and treatment; The implications of electronic communication for the physician-patient relationship; Principles for design and functionality of consumer-oriented Web sites.

2 See http://www.mcw.edu/display/Home.asp.

3 See http://www.journeyofhearts.org/jofh/jofh_old/minf_528/intro.htm. The following topics are included in the course: privacy, security, confidentiality, encryption, coding, reimbursement, conflicts of interests, reporting, protecting information.

4 HON's mission is to guide lay persons or non-medical users and medical practitioners to useful and reliable online medical and health information. HON provides leadership in setting ethical standards for Web site developers. More on the site: http://www.hon.ch/ The Code of Conduct for Medication and Health Web Sites is translated in Polish by Piotr Kasztelowicz (http://www.hon.ch/HONcode/Polish) and presented at the II Conference of Medical Internet in 1997.

5 http://www.ihealthcoalition.org/ethics/ehcode.html.

6 The mission of the Group is:

- To draw attention to and raise awareness of ethical, legal, and social issues (ELSI-WG) in health informatics.
- To serve as a resource to help AMIA members and others address ethical, legal and social issues in professional and academic endeavors.
- To identify additional resources and develop educational programs and curricular materials for AMIA members and others.
- To conduct and support scholarly research aimed at identifying ethical, legal, and social issues in health informatics and at expanding discussions and analyses of these issues.

See http://www.amia.org/working/elsi/main.html.

at Taipei the International Medical Informatics Association (IMIA)[7] endorsed "The Ethical Code of Practice"[8]. A working group "Ethical, Legal, and Social Issues" of IMIA is represented by Peter Winkelstein (AMIA)[9].

In this paper the methodological status and major questions of medical informatics (computer) ethics (MIE) will be discussed.

There are at least two sets of issues. One set concerns existing problems of medical ethics (ME) which are exacerbated by the employment of ICT in medicine, e.g., the problem of privacy and anonymity. The second set concerns potentially new problems, problems which as yet have not arisen, at least not in any significant way, e.g., the existence of cyborg.

MIE comprises problems of CE that are related to health and healthcare and questions of ME that arise from applications of ICT. It means that MIE is not a separate scientific discipline. It is rather a conglomerate of CE and ME. For example, the question of sale of drugs via the Web belongs to CE, but the questions concerning treatment supported by an expert program belong to ME. We will try to establish a demarcation line between problems of MIE that are considered by CE and problems of MIE that are subject of ME. As we will see the borderline is neither clear nor easy to observe. The frontiers are fuzzy. Our coverage of the main themes is by no means intended to be exhaustive, and several of the issues raised here need further consideration.

1. Subject of medical informatics ethics

We will try to determine the scope of MIE. We will argue that it comprises several subjects of study.

In order to achieve our aim first of all we have to distinguish between ethical problems of impact of ICT on health and ethical problems of application of ICT in healthcare and medicine.

Technology enhances productivity, expands functionality and improves quality of life. This statement is especially true about ICT. But it is only one side of technology. Technology has also another side. It is potentially harmful to the natural environment and in particular to health, physiologically and

---

7 http://www.imia.org.

8 IMIA Code of Ethics for Health Information Professionals: httpwww.imia.org/English_code_of_ethics.html.

9 See http://www.imia.org/2002_scientific_map.html.

psychologically[10]. Long hours spent at the computer can cause problems with sight, spine, wrist. Health may be threatened by radiation emanating from computer monitors. It is possible, for example, that users will feel stressed trying to keep up with high speed computerized devices. Addiction to computers and Internet is already a social issue. For example, in 2004 a center to help addicted young people was established in Ełk, a Polish town of 60 000 inhabitants. Problems of ICT effects on individual and on public health are subjects of work safety. Their ethical aspects are being considered by CE.

Medicine in the broadest sense comprises organization and administration of health services, prophylaxis, treatment and rehabilitation, manufacturing and distribution of medical equipment and drugs, study and education. Ethical problems both in medicine and in application of ICT are common to all these domains. It seems that as in ME as well as in MIE the problems should be divided according to human values. If so the MIE has to be focused on human and his/her health or – quite generally speaking – on human life.

The Hippocratic Oath expresses the principal precepts of ME. It is not only the oldest professional code but a pattern of professional codes at all. In 1976 one of the creators of CE, Walter Maner,

> while teaching a medical ethics course, noticed that, often, when computers are involved in medical ethics cases, new ethically important considerations arise. Further examination of this phenomenon convinced Maner that there is a need for a separate branch of applied ethics, which he dubbed 'computer ethics' (Wiener had not used this term, nor was it in common use before Maner.). ... By the early 1980s, the name 'computer ethics' had caught on, and other scholars began to develop this 'new' field of applied ethics. [1]

Maner noticed that some old ethical problems are made worse by computers, while others are wholly new because of information technology. He[11] defined CE as a branch of applied ethics which studies ethical problems "aggravated, transformed or created by computer technology." For Deborah Johnson, CE studies the way in which computers

> pose new versions of standard moral problems and moral dilemmas, exacerbating the old problems, and forcing us to apply ordinary moral norms in uncharted realms. [7, p. 1]

James Moor taking into account that ICT provides us new capabilities and these in turn give us new choices for action, maintains that:

> a typical problem in computer ethics arises because there is a policy vacuum about how computer technology should be used. [11, p. 266]

For Terrell Ward Bynum it is the best available definition of the field[12]. Krystyna Górniak-Kocikowska predicts that due to globalization of ICT, computer ethics will disappear. "Local" ethical theories will eventually be superceded by a global ethics evolving from today's CE. "Computer" ethics, then, will become the "ordinary" ethics of the information age[13]. Deborah Johnson maintains that in information age CE will become ordinary ethics and ordinary ethics will become CE[14]. On Johnson's view, in information age ICT will permeate all aspects of our everyday life. Its presence will no longer be noticed. Thus there will be no special CE problems. In all the ethical issues the questions of CE will be involved.

For the discussed concepts of CE, MIE is a part of CE that concerns ethical questions raised by application of ICT in medicine. ME is not proper to examine ethical problems of medicine implied by ICT technology. Moreover, in the future ME will be only a branch of applied CE (albeit – according to Górniak-Kocikowska and Johnson – the name CE may not be in usage).

Different approach to defining the field of CE is advocated by Donald Gotterbarn. For him CE is a branch of professional ethics. It concerns

> the values that guide the day-to-day activities of computing professionals in their role as professionals. By computing professional I mean anyone involved in the design and development of computer artefacts. The ethical decisions made during the development of these artefacts have a direct relationship to many of the issues discussed under the broader concept of computer ethics. [5]

For this concept of CE we may maintain that as ICT engineers and medical doctors are different professions as CE and ME are different ethics. As long

[10] The same is true about medicine. The famous Hippocratic aphorism *primum non nocere* (first do no harm) reminds a physician that he or she must consider the possible harm that any intervention might do. It is most often mentioned when debating use of an intervention with an obvious chance of harm but a less certain chance of benefit.

[11] Maner contributed not only to the theory of CE. Traveling around America he gave speeches and conducted workshops at conferences. He self-published *A Starter Kit in Computer Ethics* [9]. *Computer Ethics* [7], the first textbook – and for more than a decade, the defining textbook – in the field was published by Deborah Johnson of Rensselaer Polytechnic Institute.

[12] Cf http://plato.stanford.edu/entries/ethics-computer.

[13] Cf [4].

[14] Cf [8].

as there is no such a profession as ICT medical doctor or medical ICT engineer, there is no MIE branch of professional ethics.

2. Major issues of medical informatics ethics

Even if we agree that MIE is not a separate branch of ethics, we should not negate that there are some issues involved as well in CE as in ME. The application of ICT in medicine posed a new set of ethical problems. These problems have been compounded by the increasing use of ICT for supporting clinical decisions, record keeping etc. These ethical problems have two dimensions, one of them is related to CE and the second one related to ME. MIE comprises such questions that need to be discussed as well by computer ethicists as by medical ethicists.

2.1 Code of ethics in medical informatics

In contemporary world to be professional means to have an ethical code. Medical doctors have the oldest code, rooted in antiquity, i.e., Hippocratic Oath. ICT engineers as any other engineers are challenged to be professional. In 1968 Donn Parker, whose work is the next important milestone in the history of computer ethics after Wiener, published *Rules of Ethics in Information Processing* [12]. The first code of professional conduct[15] adopted by the Association for Computing Machinery (ACM)[16] in 1973 was the upshot of his work. The IEEE[17] Computer Society[18] (IEEE-CS), the largest of so-

[15] See http://www.acm.org/constitution/code.html. Its last version was adopted by ACM Council 92.10.16.

[16] See http://www.acm.org "ACM is the world's oldest and largest educational and scientific computing society. Since 1947 ACM has provided a vital forum for the exchange of information, ideas, and discoveries. Today, ACM serves a membership of computing professionals and students in more than 100 countries in all areas of industry, academia, and government." "ACM is dedicated to advancing the arts, sciences, and applications of information technology. With a world-wide membership ACM is a leading resource for computing professionals and students working in the various fields of Information Technology, and for interpreting the impact of information technology on society."

[17] The IEEE (Eye-triple-E) is a non-profit, technical professional association of more than 360,000 individual members in approximately 175 countries. The full name is the Institute of Electrical and Electronics Engineers, Inc., although the organization is most popularly known and referred to by the letters I-E-E-E.

[18] http://www.computer.org/ "With nearly 100,000 members, the IEEE Computer Society is the world's leading organization of computer professionals. Founded in 1946, it is the largest of the 37 societies of the Institute of Electrical and Electronics Engineers (IEEE). The Computer Society's vision is to be the leading provider of technical information and services to the world's computing professionals. The Society is dedicated to advancing the theory, practice, and application of computer and information processing technology."

cieties of IEEE, is engaged in several activities to advance the professionalism of, e.g., software engineering. "Software Engineering Code of Ethics and Professional Practice"[19] is a result of joint efforts of the ACM and IEEE-CS. The Code has been adopted by both the ACM and the IEEE-Computer Society in the official unanimous approval by the leadership of both professional organizations. In the preamble to the code we read:

> Computers have a central and growing role in commerce, industry, government, medicine, education, entertainment and society at large. Software engineers, those who contribute by direct participation or by teaching, to the analysis, specification, design, development, operation, certification, maintenance and testing of software systems, have significant opportunities to do good or cause harm, to enable others to do good or cause harm, or to influence others to do good or cause harm. To ensure, as much as possible, that their efforts will be used for good, software engineers must commit themselves to making software engineering a beneficial and respected profession.

At ETHICOMP 95 Porfirio Barroso Asenjo presented "Health Informaticians' Deontology Code" (HIDEC). The HIDEC is designed for the community of ICT engineers and users of the computers in the health sector. "The HIDEC will refer to the practices and behavior according to which health informaticians are expected to exercise their profession, offering their services, and also to the practice and behavior expected from the users."[20] The primacy of wellbeing and quality of life of the public in all decisions is emphasized throughout the codes. What concerns the health, safety and welfare of the public is primary. "Public interest" is central to the codes.

2.2. Privacy and anonymity

In the Hippocratic Oath there is a promise:

> That whatsoever you shall see or hear of the lives of men or women which is not fitting to be spoken, you will keep inviolably secret.

One of the earliest CE topics to arouse public interest was privacy. In the comparative analysis of the codes of CE, we observe that "privacy" and "intimacy" are most frequently repeated. If we see on helmets of soldiers in Iraq the information about their blood group, we wonder where the borders

[19] The code was distributed widely via *Communications* of the ACM and IEEE's *Computer magazine*.

[20] See http://www.ccsr.cse.dmu.ac.uk/journal/abstract/barroso_p_hidec.html.

are between private and public information. It is a truism that the more relevant information about the patient the more effective treatment. Nowadays, more and more applications use sensitive and personal information. In the foreword to [10] Paul Davies writes that in the future:

> there could be detecting devices so sensitive that they could pick up the equivalent of the drop of a pin on the other side of the world.

Monitoring and surveillance will become very easy. All the information does not have to be on "helmet". It could be in an implanted chip. In such a case will our right to privacy be preserved? Will there be a place for intimacy, i.e., to that which is most reserved, most deeply felt by the human being? What does privacy mean in an information society? How our right to be alone, without being subjected to unsolicited and unwanted publicity should be conceived? What does intimacy mean in the information era? How the right to live in seclusion and anonymity can be understood? Respecting citizens' privacy is becoming extremely important. Since the middle of the sixties the theory of privacy conceived as "control over personal information" has been elaborated. A stronger notion of privacy is defined in terms of restricted access. The concept of intimacy and private life in traditional social media focuses on the respect and the absence of interference in private lives of individuals. Since informaticians work with information, not with people who are the object of information, CE focuses more on personal intimacy, on private life, on anonymity and the confidentiality of information and data. Advances of ICT in compiling, storing, accessing and analyzing information, rapid growth of the Internet and the rise of world-wide-web have led to new privacy issues, such as data-mining, data matching, recording of "click trails" on the web and are forcing continual debate about the meaning of privacy.

Anonymity can provide many of the same benefits as privacy, e.g., in obtaining medical or psychological counselling, or to discuss sensitive topics. We need anonymity to preserve values such as security, mental health, self-fulfillment and peace of mind.

For medical reasons all the information about us should be as complete as possible if it concerns our health and it should allow at least medical personnel full and easy access to our records. In 1964 electronic devices were used to monitor the location of patients with mental health problems. Due to technology, e.g., VeriChip[21], our health can be steadily monitored and the proper help may be offered. In the future, people with microscopic implants will be able to be tracked using Geographic Positioning Systems (GPSs) just as cars can be now, only more efficiently. One need never be lost again! This technology is already adapted for noncustodial penalties, e.g., in USA, UK, Germany and France. The appropriate bill is under discussion in the Polish parliament, Sejm.

The ease and efficiency to gather, store, search, compare, retrieve and share personal information make ICT especially threatening to anyone who wishes to keep various kinds of "sensitive" information (e.g., medical records) out of the public domain or out of the hands of those who are perceived as potential threats. Security and integrity of data is a serious problem. There are people who are interested in our privacy and intimacy not only just for amusement as it is in the case of "big brother" TV series but also to work against our interest and even life. Privacy and anonymity can be exploited to facilitate unwanted and undesirable activities in cyberspace. Hackers use ICT to break into data bases, to change and even to damage some data. New kinds of computer viruses are produced. In 2004 the number of viruses was about 50% higher than in 2003. Professor Kevin Warick from Reading University expects that new kind of "wireless" viruses can be used to attack implanted chips and the same, similarly as biological ones, they can directly threaten to the health. We are astonished that computers can be used to do harm. Though computer and Internet users seem not to be ignorant because they are able to use the most advanced technology, some of them are unimaginably irresponsible and dangerous. While from all over the world the relief was sent to people undergone by the Tsunami disaster some internauts prepared bogus sites to collect relief money for themselves. They made profits. But what about these internauts who used e-mail to disseminate false information about allegedly dead people?

Confidentiality of electronically stored patient information is a crucial issue of MIE. Complete information about health of an individual is important for care institutions, health agencies, and insurance corporations. Electronic records differently from paper records allow unprecedented in-

[21] VeriChip is an inert, encapsulated microchip that is energized and transmits its information when activated by a VeriChip reader. VeriChip is about the size of a pen point hence it is virtually undetectable and practically indestructible once inserted under skin. The chip has no battery and never "runs down"; its expected life is up to 20 years. VeriChip was originally intended to function in much the same way a medical alert bracelet does by giving medical personnel life-saving information about a patient's history. It is now being used for security and automated data collection, as well as medical, purposes. In 2004 the Food and Drug Administration (FDA) approved an implantable computer chip that can pass a patient's medical details to doctors. – see http://www.adsx.com/prodservpart/verichip.html.

sight into patient's health profile. The ethical implications of preparation of such records need special consideration. The rights and duties of physicians, health information professionals, as well as hospitals and other institutions, have to be regulated by appropriate ethical codes and laws based on it.

2.3. Intellectual property

One of the more controversial areas of CE concerns the intellectual property rights connected with software ownership. In any country healthcare expenses are growing faster than economy. ICT has borne hopes for lowering these costs. The costs of hardware drops according to famous Moore law[22]. The price of Rolls Royce would be $1 if the prices in car manufacturing were decreasing similarly. For the implementation of ICT, the costs of software are crucial. It is similar in pharmacy: the costs of manufacturing are relatively low compared to the costs of formulating drugs. Hence generic medication can be many times cheaper. Since the beginning of the computer era mainly young people have claimed that in order to develop ICT "information wants to be free", in particular all programs should be available for copying, studying and modifying. Richard Stallman's "Free Software Foundation" is based on this ideology. Due to these who believe that all people have the right to access to the Internet and to use computers there are free and open source programs as, e.g., Linux created by Linus Torvalds. In the discussion of intellectual property rights it is already a custom to point out section 8 of the first article of the United States Constitution (adopted by Congress: 17 September 1787, put into effect: 4 March 1789) that empowers the Congress to legislate:

> To promote the Progress of Science and useful Arts, by securing for limited Times to Authors and Inventors the exclusive Right to their respective Writings and Discoveries[23].

Many believe that this together with a free-market economy and adherence to it, also competition and diversity have decided about the role USA plays in science, technology and economy of the world. Creativity in the form of

[22] Moore Law says that the number of transistors the industry would be able to place on a computer chip would double every year. In 1995, Moore updated his prediction to once every two years. While originally intended as a rule of thumb in 1965, it has become the guiding principle for the industry to deliver ever-more-powerful semi-conductor chips at proportionate decreases in cost.

[23] See http://www.whatreallyhappened.com/RANCHO/POLITICS/DOCUMENTS//US_Constitution.txt.html.

ideas, innovations and inventions, has replaced gold, colonies and raw materials as the new wealth of nations. Nevertheless, no right is absolute. The purpose of intellectual property rights is to promote economic efficiency, reward investment, access to information, for the benefit of all. The protection of intellectual property rights must be achieved in a constitutional setting which upholds other values for public good. Health is one of the most important values. Thus this value has to be crucial in conceiving of intellectual property rights in medical informatics.

Ownership of software is a complex matter. There are distinguished three types of ownership:

- copyrights,
- trade secrets,
- patents.

Several aspects of software can be owned:

- source code
- object code
- algorithm
- the way the program appears on the screen and interfaces with users.

In 2004 the Polish Government rejected a proposal of legislation that would make possible owning a patent on software. Many people (about 26 000) wrote letters of thanks. "Thank you, Poland" was put on the site of "Free Software Foundation". Klaus Knopper, maker of Linuks distribution Knoppix, said that he would have to finish work since he had no possibility of paying for about 900 patents in Linux. According to Linus Torvalds if software were patented it would be the threat for future development of software that distribution is based on GNU[24] licence.

Mathematicians and scientists observe that monopoly for algorithms can deny others use mathematical formulas that are parts of these algorithms and – as a result – some parts of mathematics are removed from public domain.

ICT engineers support the idea of as wide as possible access to computers. Medical doctors support the idea of public healthcare. Both groups have to observe the intellectual property rights. MIE has to help to decide which understanding of these rights is true from the perspective of public good.

[24] The GNU Project was launched in 1984 to develop a complete UNIX style operating system which is free software: the GNU system. (GNU is a recursive acronym for "GNU's Not UNIX"; it is pronounced "guh-noo.") Variants of the GNU operating system, which use the kernel Linux, are now widely used; though these systems are often referred to as "Linux," they are more accurately called GNU/Linux systems. See http://www.gnu.org/.

2.4. Responsibility for service reliability

A few years ago in Białystok oncological hospital three patients were burned during radiological therapy. It was clear that if anyone was guilty it was one of the servicing personnel. The radiation apparatus is only a tool. But how it would be in the case of an accident when an expert system or a program was used? Is an expert system only a tool or rather is it a partner? The use of computer to support treatment implies a group of ethical questions: in what circumstances is it advisable to use an expert system? How can physicians determine if an expert system or a program is safe for human use? Who will be guilty in a case of an accident?

Many people naively believe that computers are infallible. But it is not the case. Programs are written by people. Even an often tested program may have a bug. Even lack of a comma may cause a catastrophe. Some faults are systematic as it was in the case of so called "the year 2000 problem". Computers are electronic devices and even after many tests nobody can be certain a computer used to support medical decision has no material defect. The collection of accidents caused by computer systems and related technology is enormous[25].

It is obvious that medicine supported by ICT is more efficient. The common truth that the more and better information, the better decisions are especially important in medicine. It is also confirmed by experience that intelligent systems provide better care than clinicians for patients.

> That into whatsoever house you shall enter, it shall be for the good of the sick to the utmost of your power, your holding yourselves far aloof from wrong

is promised in the Hippocratic Oath. According to the principle of beneficence the patients should be treated to the best of clinicians' abilities. Development of ICT for the purposes of medicine and its application in healthcare is a moral imperative.

Computers make errors. Who is responsible for their errors? Some authors say that it is inappropriate to hold computer responsible. Computers are not simply responsible beings. The answer seems to be acceptable for contemporary computers but it may not be the case for future ones. Computers based on learning algorithms and equipped with communication skills may be entirely able to learn interact appropriately with both its environment and its users.

[25] To find more information about this see e.g., http://www.mirrors.wiretapped.net/security/info/textfiles/risks-digest/illustrative.html.

The question of responsibility for a casualty in a treatment supported by an intelligent system – such an eventuality could never be excluded[26] – is one of the most important and complicated problems of MIE and its significance will increase as ICT employment augments medicine. AI system has two components: knowledge-base and inference engine. In medicine both these components should be based on experience of physicians. Is it possible to express this experience in computer language? For at least three reasons the answer seems to be negative: first, particularities of each individual patient; second, sociocultural and, third, axiological character of this experience.

In 1968 Marvin Minsky defined Artificial Intelligence as:

> ...the science of making machines do things that would require intelligence if done by men.

If machines behave in much the same way as humans do, the issue will arise of whether or not they are things with moral rights and responsibilities. If they behaved like us, would we be justified in treating them differently? We ask if an intelligent system is responsible or at least participates in responsibility of a man that it employs. We ask if a system with learning capabilities is a tool, a partner or a hybrid of the two? A tool could not be responsible but a partner is co-responsible. May it be co-guilty, too?

2.5. Medical ethics and Internet

The Internet is the discovery that changes life and culture, science and medicine more than any earlier discovery. Its significance for healthcare and medicine cannot be overestimated. To take advantage of it we have to understand its nature.

First of all we have to take into account anarchic nature of Internet. What does it mean in the case of medical Internet?

[26] A. Turing, one of the fathers of informatics, wrote:

> [I]f a machine is expected to be infallible, it cannot also be intelligent. There are several mathematical theorems which say almost exactly that. But these theorems say nothing about how much intelligence may be displayed if a machine makes no pretence at infallibility. See [14, p. 124]

> My contention is that machines can be constructed which will simulate the behaviour of the human mind very closely. They will make mistakes at times... and on the whole the output of them will be worth attention to the same sort of extent as the output of a human mind. The content of this statement lies in the greater frequency expected for the true statements, and it cannot, I think, be given an exact statement. See [13, p. 129]

Prophylaxis is one of *prima facie* employment of the Internet. There are many sites with such a task, e.g.,

http://www.telezdrowie.pl/,

http://diagnoza.sccs.pl/.

The first website is one of the best medical portals in the world. It offers an interactive service for diagnosis and rehabilitation of the senses responsible for communication. How can we distinguish reliable sites from unreliable ones? Anybody may have their own site. We may find sites dedicated to homeopathy, bioenergotherapy etc. Moreover, people without any license may offer medical advice. Patients have to be protected from substandard care. Any kind of censorship is excluded. Internet is a decentralized medium, thus the idea of an institution to ensure quality seems to be unrealistic.

About a third part of spam[27] that I receive deals with pharmaceutics. Some of them are available at pharmacies only with doctors prescriptions. Some of them are legally available in, e.g., USA but not admitted in Europe. Here is an example of such a spam:

> *Online pharmacy – Visit our online store and save. Save up to 80% compared to normal rates. All popular drugs are available, including Vicodin and Hydrocodone! – World wide shipping – No Doctor Visits – No Prescriptions – Next Day Priority Shipping – Discreet Packaging – Buy in Bulk and Save! We make it easier and faster than ever to get the prescriptions you need. Go here: http://ziagra.net/rx/phrm/ Simply Rx is your convenient, safe and private online source for FDA approved pharmacy prescriptions. We sell brandname and exact generic equivalents of US FDA approved prescription drugs through our fullylicensed overseas pharmacy. Upon approval of your medical information, a licensed physician will issue a free prescription which can be filled and shipped to you in one business day.*

Is it possible to control these dealings?

[27] Spam - according to standard definition – is an electronic message if:

(1) the recipient's personal identity and context are irrelevant because the message is equally applicable to many other potential recipients; AND (2) the recipient has not verifiably granted deliberate, explicit, and still-revocable permission for it to be sent; AND (3) the transmission and reception of the message appears to the recipient to give a disproportionate benefit to the sender. See http://www.nospam-pl.net/standard.php.

The name is in honour of Monty Python sketch that was first broadcast in 1970. In the sketch, two customers are trying with no result to order a breakfast without SPAM. SPAM was one of the few meats excluded from the British food rationing that began in World War II.

Medical practice without licence of a given state is a subject of criminal prosecution. We may wonder if it is permissible for a doctor from USA to consult via Internet a patient in Europe. As we know, there is no problem with such a consultation in the case of alternative medicine.

There are many ethical and legal restrictions of employment of Internet in medicine and healthcare. These restrictions do not concern the alternative medicine. MIE has to elaborate ethical standards for the Internet respecting the nature of this medium.

2.6. Cyborg

Ethical questions raised by transplantation, genetic manipulation or – generally – the questions of bioethics are subjects of many current discussions and common knowledge. Using of abiotic devices does not seem to have any ethical implications. Is anyone asked about ethical implications of wearing glasses? Are there any ethical problems with denture or an artificial limb? Professor Religa's program of construction of an artificial heart seems to escape the ethical problems raised by transplantation.

Humans dream of being immortal and perfect. For believers the accomplishment of the dream is possible in heaven. Everybody strives to accomplish this dream on the earth. ICT may help to repair imperfections and improve and lengthen human life.

The idea of the organic-artificial creature has existed in human culture. It has old roots in Indian, Chinese, Japanese, and Western culture. In the middle ages the alchemists tried to grow homunculi, "little men". Frankenstein is the commonly known name a monster animated by a man who, then abandoned his creation because its appearance horrified him. Since the 19th century the advances of science and medicine began to make the realization of such fantasies possible. In 1926 J. D. Bernal, the great British scientist, described humans involved in colonizing space. Interfaces between humans and machines allowed them to attach a new sense organ or a new mechanism to operate. The emergence of cybernetics began scientific consideration of this old idea.

In 1960 Nathan Kline from Rockland State Hospital's Research Laboratory had been asked by NASA to participate in a conference about human space exploration. Together with his colleague Manfred Clynes[28], they proposed a number of ways humans could be modified to survive in

[28] Clynes combines the artistic sensibility of a world-class pianist with a relentless technical genius powered by a restless intelligence and an exuberant enthusiasm for knowledge. It is a unique combination.

space. "Cyborg" [29] refers to the conception of an enhanced human being who could survive in extraterrestrial environments. Cyborg is a creature which is a mixture of organic and artificial parts. Generally, the aim is to add or enhance the abilities of an organism by using technology. It is a creature that combines informatics, mechanics, and organics. Clynes and Kline concluded their seminal article with the comment that cyborg developments:

> will not only make a significant step forward in man's scientific progress, but may well provide a new and larger dimension for man's spirit as well. [2, p. 33]

There are good reasons to maintain that cyborg transformations will continue and become more profound and on some day in the distant future will end with even disembodied intelligence (by Clynes labelled Cyborg V).

Advances in medical cyborg research are changing the meaning of death and life, for example. Doctors no longer speak of death plain and simple. Patients are "single-dead", "double-dead", or "triple-dead" depending on if, or how, their organs can be harvested for transplantation [6]. Our senses, memory and even such capabilities as creativity and reasoning ability can be enhanced by ITC products. Professor Kevin Warick from Reading University deems that in future people not equipped with computer parts will belong to a subspecies. The distinction between human and machine is ever more unclear. From the ethical angle it is acceptable and even desirable to help people to be "normal". The making of a superman is ethically dubious. But where is the difference? What does it mean to be "normal"?

There are prohibited experiments with atomic, chemical and biological weapons. In many countries some biotechnology experiments are prohibited. Ought it to be the same in the case of ICT? To answer this question we have to answer some other questions: Is there any pure research that should not be undertaken?, Is there any technology that should not be developed?, Are there any limits of use of a technology?

3. Conclusions

The answers to these raised questions are not simple. ICT as any other technology can be put to both beneficial and harmful uses. The beneficial consequences are not sufficient to justify any research. One thing is sure:

[29] The term "cyborg" was coined by Clynes from "cybernetic" and "organism", marrying the reality of the organic body with the idea of cybernetics.

we have to have a profound respect for life. For physicians, human life is the highest value. The human being is not only a biological entity. We have exceeded the pure biological world. Human spirit is the value that has to be protected, too. Human dignity needs to be respected. Here ME meets with any technology in particular with ICT. Medical ethics and informatics ethics have joint problems. The problems can be gathered under the name "medical informatics ethics".

References

[1] Terrell Ward Bynum. Very short history of computer ethics. *Newsletter on Philosophy and Computing*, Summer 2000. American Philosophical Association.

[2] Manfred E. Clynes and Nathan S. Kline. Cyborgs and space. *Astronautics*, pages 26–27; 74–75, September 1960. Reprinted in Gray, Mentor, and Figueroa-Sarriera, eds., The Cyborg Handbook, New York: Routledge, 1995, pp. 29–34.

[3] Kenneth Goodman, editor. *Ethics, Computing, and Medicine: "Informatics and the Transformation of Health Care"*. Cambridge University Press, 1998.

[4] Krystyna Górniak-Kocikowska. The computer revolution and the problem of global ethics. In Bynum and Rogerson, editors, *Global Information Ethics*, pages 177–190. Opragen Publications, 1996.

[5] Donald Gotterbarn. Computer ethics: Responsibility regained. *National Forum: The Phi Beta Kappa Journal*, 71:26–31, 1991.

[6] Linda F. Hogle. Tales from the cryptic: Technology meets organism in the living cadaver. In Mentor Gray and Figueroa-Sarriera, editors, *The Cyborg Handbook*, pages 203–217. Routledge, New York, 1995.

[7] Deborah G. Johnson. *Computer Ethics*. Rensselaer Polytechnic Institute, 1985. Second edition, 1994: Englewood Cliffs, New Jersey, Prentice-Hall.

[8] Deborah G. Johnson. Computer ethics in the 21st century, a keynote address at the ETHICOMP99 conference, Rome, Italy, October 1999. In Richard A. Spinello and T. Tavani Herman, editors, *Readings in CyberEthics*, pages 17–18. Jones and Bartlett, 2001.

[9] Walter Maner. *Starter Kit in Computer Ethics*. Helvetia Press, 1980. Published in cooperation with National Information and Resource Center for Teaching Philosophy. Originally self-published by Maner in 1978.

[10] Gerard J. Milburn. *Schrodinger's Machines: The Quantum Technology Reshaping Everyday Life.* Allen and Unwin, 1996. Foreword by Paul Davies.

[11] James H. Moor. What is computer ethics? In Terrell Ward Bynum, editor, *Computers and Ethics*, pages 266–275. Blackwell, Oxford, 1985. A special issue of the journal *Metaphilosophy.*

[12] Donn Parker. Rules of ethics in information processing. *Communications of the ACM*, 1968.

[13] Alan M. Turing. Intelligent machinery, a heretical theory. In *Alan M. Turing*, pages 128–134. W. Heffer, Cambridge, 1959.

[14] Alan M. Turing. Lecture to the London Mathematical Society on 20 February 1947. In B. E. Carpenter and R. W. Doran, editors, *A. M. Turing's ACE Report of 1946 and Other Papers*, pages 106–124. MIT Press, Cambridge, Mass, 1986.

[15] Joseph Weizenbaum. *Computer Power and Human Reason: From Judgment to Calculation.* W. H. Freeman, San Francisco, CA, 1976.

[16] Norbert Wiener. *The Human Use of Human Beings: Cybernetics and Society.* Houghton Mifflin, 1950. Second Edition Revised, Doubleday Anchor, 1954. This edition is better and more complete from the computer ethics angle.

Kazimierz Trzęsicki
University in Białystok
Institute of Informatics
ul. Sosnowa 64
15-887 Białystok
e-mail: kasimir@ii.uwb.edu.pl

STUDIES IN LOGIC, GRAMMAR AND RHETORIC 8 (21) 2005

Halina Święczkowska
Białystok University

IN SEARCH OF LANGUAGE ORDER
G. W. LEIBNIZ'S "UNVORGREIFLICHE GEDANKEN"

1.1. Between the objectivity and illusion

In the letter from 7th April 1699 written to Gabriel Spartvenfeld, Swedish thinker and erudite, Leibniz referred to a trend in linguistic research inspired by "not yet" the pure nationalism, but certainly by the argument on the priority of linguistic and cultural tradition, enhanced by the conscience of existence of fundamental national differences[1].

[1] H. Wieselgren (ed.), *Leibniz'bref till Spartvenfelt*, in: "Antiquarisk Tidskrift for Sverdge" 7 (3) (1884/85), s. 40: "Au reste il est plaisant de voir, comment chacun veut tout tirer de sa langue ou de celle qu'il affectionne. *Goropius Becantus* et *Rodurnus* de l'Allemand (sans distinguer les nouvelles inflextions de ce qui est de la langue ancienne), Rudbeckius du Scandinavien, un certain *Otroski* du Hongrois, cet Abbé François (qui nous promet les origins des nations) du bas Breton ou Cambien, *Praetorius* (auteur de l'orbis Gothicus) du Polonais ou Esclavon. Thomassin après plusieurs autres *Bochart* meme de l'Hebreu ou Penicien, *Ericus* Allemand établi à Venise du Grec. Et je crois, si un jour les Turcs ou Tartares deviennent sçavants à notre maniére qu'ils trouverons dans leur langue et dans leurs pais des mots ou allusions, don't ils prouveront avec autant de droit que Monsieur *Rudbeckius*, que les Argonautes, *Hercule*, *Ulysse* et les autres Heros ont été chez eux et que les Dieux sont sortis de leur pais et de leur Nation. Ils trouvent bien des passages des anciens favorables à leur hypothèse. [...] La verité est que les anciens parlent confusement et contradictoirement des choses, qu'ils ne sçvoient plus eux memes lors qu'ils ecrivoient de sorte que leur auctorités dans ces choses obscures sont à peu pres come les regles de l'Astrologie, sont on peut tirer tout ce qu'on veut, su tout après coup".

The argument over the priority of linguistic tradition had its origins in started in the end of 15th century intensive cataloguing action languages, first European, than of the Old World finally of the New World. Theoretical conclusions from at first purely mechanical arrangements of languages went in two directions: 1. determining of the original language, 2. determining of the relationship between the existing languages. In genealogical consideration nationalisms played certain role. The ambition of many researchers was proving that their language was the closest to the first language of humanity – the language of Adam. It was not however the nationalism in modern sense. Admittedly the roots of nationalism develop from the same soil as the whole western civilization and go back to the beginnings of Jewish nation and ancient Greece, but the first contemporary national country was the 17th century England. As for the scholars mentioned by Leibniz in the letter, he means the sense of a certain linguistic and cultural superiority, common also to Hebrews and Greeks. See H. Kohn, *Nationalism: Its Meaning and History*, D. Van Nostrand Company, INC., Princeton, New Yersey, New York 1955.

ISBN 83-7431-059-6 ISSN 0860-150X

A few years before in an essay entitled *Analysis linguarum* Leibniz expressed the conviction about the parity of languages. He wrote that when it comes to the contribution of a language to the process of cognizance suffice to consider whichever language since every nation is equally able to make discoveries[2]. Hence we can suppose that Leibniz, the author of genealogical classification of languages, conscious of differences between languages, especially historical differences, attributed them equal cognitive functions and did not divide them into "better" and "worse" ones. This opinion is contradictory with other Leibniz's statements. It can be without any difficulty proved that he was not much different from Goropius Becantus (criticized by Leibniz) declaring the superiority of German over other European languages from the point of view of cognizance, and from the historical and cultural point of view arguing that German is the closest to the primary language in its supreme form, and that it is particularly adapted to philosophy[3]. It seems that this discrepancy fully comes to light when we compare the general linguistic theory, the origins of which are to be found in his texts on cognitive theory and representation, and the opinions formulated by Leibniz on the grounds of comparative and historical research which he conducted. Linguistic historians claim that Leibniz in his hypotheses on the origins of German succumbed to the same illusions which led astray other scholars criticized by him[4]. There are however some extenuating circumstances which admittedly do not justify Leibniz in the aspect of scientific reliability, but they contribute to explanation of his attitude as a member of a particular linguistic and cultural community.

[2] G. W. Leibniz, *Analysis linguarum*, in: G. W. Leibniz, *Opuscules et Fragments Inédits de Leibniz*, extraits des manuscrits de la bibliothèque royale de Hanovre par Louis Couturat, Paris 1903, p. 352, quoted as C.

[3] G. W. Leibniz, Dissertatio Praeliminaris. De alienorum operum editione, de Scopo operis, de Philosophica dictione, de lapsibus Nizolii, in G. W. Leibniz, *Die Philosophishishen Schriften von G. W. Leibniz*, ed. C. I. Gerhardt, Halle 1849–1863 (repr. Hildeshein 1960) VII Vol, IV, p. 144, (quoted as GP, volumn, page). Illud tamen asserrere ausim, huic tentamento probatorio atque examine philosophematum per linguam aliquam vivam, nullam esse in Europa linguam Germanica aptiorem, quia Germanica in realibus plenissima est et perfectissima".

[4] See S. Gensini, *Leibniz Linguist and Philosopher of Language: Between 'primitive' and 'natural'*, in: M. Dascal, E. Yakira (ed.), *Leibniz and Adam*, University Publishing Projects Ltd., Tel Aviv, 1993, pp. 118–119, H. Aarsleff, *Leibniz on Locke on Language*, in: H. Aarsleff, *From Locke to Saussure*, Atlhone, London 1982, pp. 46–47.

1.2. Seventeenth century Germany

1.2.1. The history of 17th century Germany is the history of a struggle to survive which involved the political survival – maintenance of the statehood, of economy and above all it involved the cultural survival and rebirth. The statement that people living in 17th century considered war rather than peace a normal state in Europe seems not to be false. Numerous armed conflict with political, religious or social background went on in different parts of the continent. Still they all had less reach and consequences than the phenomenon called 'The Thirty Years' War'. Let the historians judge the reasons and the results of this great all-European war[5]. Suffice to say that it was conducted in the name of religious and political security. What remains important for us is the fact that the war, in which great European powers took part, was generally waged on the grounds belonging to The Holy Roman empire of The German Nation. The war causes tremendous material loss, extermination of the population, decline of crafts and trade, it left its impress on the cultural life of 17th century Germany. The war and years after it are called, with some exaggeration certainly, the times of the most deep cultural collapse of Germany. Andreas Gryphius (1616–1664), a great lyrist and playwright of the German Baroque wrote:

Wir sind doch nunmehr gantz, ja mehr den gantz verheeret.
Der frechen Völker Schaar, die rasende Posaun
Das vom Blutt fette Schwerdt, die donnernde Cartaun
Hat aller Schweiss und Fleiss und Vorrath auffgezehert.

The Holy Reich (the Holy Roman Reich) was in state of shock since the Thirty Years' War and French aggressions. Demographic catastrophe was deepened by the 1630–1640 plague. From the point of view of confession the Empire was highly differentiated; the majority of population was Lutheran, the minority Calvinist and the rest of if were Roman Catholics. In this multicultural milieu German language with its poetry and memoirs of gone power and magnitude of the Empire was the only common weal. Traditional North-South tensions based on religious criterion (Protestant Germany – Catholic Germany) subsequently adapted to the tastes of Enlightment (efficiency – education, ignorance – immorality), did not facilitate the survival of the imperial myth. In the second half of 17th century because of the threat from France, a real attempt to create imperial economic

[5] See G. Parker, *Wojna trzydziestoletnia*, in: A. Mączak (ed.), *Europa i świat w początkach epoki nowożytnej*, PWN, Warszawa 1992, part 2, pp. 98–135.

policy was undertaken. There were attempts to create common trade rules that could be followed by all Imperial countries. The Empire itself was not a country in respect of modern or national monarchy. It constituted rather a general outline of a medieval monarchy.

Scientists, lawyers, theologians struggled for the endurance of the Empire, believing that the national unity could be saved thanks to internal reforms, the tradition in such domains as the law, especially the German law more ancient and better from the Roman and canon law. Pufendorf, Reinking, Seckendorff, Arumnäus, Limänaus, Conring, Chemnitz and Leibniz should be mentioned here.

1.2.2. The Westfalian peace agreement signed in Munster in 1648 that ended the Thirty Years' War was according to historians the beginning of a gradual decomposition of the Reich, while for France it opened the way to a 40-year political dominance in Europe. The second half of 17th century could be labeled 'the French era'. The growth of its power, *'This expansion, this vigor, this glory are the signs of an intense vitality. France is an entity, a person, a moral whole. Her will to unity, her will to expand, follow one another like the steps in a logical process growing increasingly aware of itself.'*[6] France gains not only political supremacy. The most astonishing is the development of its intellectual force. Since the beginning of 17th century there is *'a miracle of perpetual profusion of masterpiece'*. processions of eminent scientists and philosophers led by Carthesius, writers and artists shaped new currents of thinking affecting with their creations the scientific and cultural life of other European countries. French became the universal language of European intellectual elites. Pierre Boyle wrote: *'The French language is the rallying point for all the countries of Europe. It is a language which we might truly call transcendental, for the same reason that philosophers bestow that epithet on natures which spread and wide, and freely manifest themselves in every clime and country'*[7].

Paul Hazard observes that intellectual hegemony was until the end of 17th century almost a family property: it remained in the Latin circle. In the age of Renaissance it belonged to Italy, then Spain had its golden era, finally France took over the legacy[8]. England did not withdraw from the fight for political influence trying to frustrate political plans of France,

[6] P. Hazard, *The European mind 1680–1717*, transl. by J. Lewis May, Penguin Books, 1964, p. 77.

[7] Ibidem, p. 80.

[8] Ibidem, p. 70.

but on the cultural battlefield England managed to get ahead France only in the beginning of 18th century. 'In 1702, no country in civilized Europe was in a more melancholy condition of intellectual emptiness than England; in 1712, not France itself could compare with us for copious and vivid production. (...) The little volume of dialogues, which Berkeley issued under the title of *Hylas and Philonous* belongs to the *annus mirabilis* 1713, when Pope, swift, Arbuthnot, Addison, Steele, were all at the brilliant apex of their genius, and when England had suddenly combined to present such a galaxy of literary talent as was to be matched, or even approached, nowhere on the continent of Europe.'[9]

If the conscience of intellectual dominance of France was the source of frustration in the motherland of Boyle's, Newton's and Locke's, the country which in the second half of 17th century created the foundations of the modern experimental science, the country where numerous scientific societies had operated with Royal Society created 15th July 1662, how should we measure the feeling of cultural slavery in those European countries which fed themselves with the memories of the lost power and were far behind from the point of view of civilization.

4.3.3. Historians of science agree that Germany was far behind France, England, Italy and also the Netherlands in terms of civilization. Experimental science was especially backward and even though Nurnberg was the center of European crafts, manual skills – crucial element of laboratory work, essential in creating research instruments – were not used as scientific base. Among the leading European researchers, engaged in experimental research the two that are mentioned are Johannes Kepler (1571–1630) and Otto Guericke (1602–1686)[10].

Although during 17th century the number of German universities increased from 17 to 39 the increase was not the result of scientific enthusiasm. It was rather caused by decentralization of the Empire. Every sovereign, even the least, aimed at establishing his own university. The majority of those were very poor and the poverty was not material at all. The utmost issue was the lack of well-formed staff. The majority of universities were owned by Protestants (23) the rest was in Jesuit's hands, except the Saltsburg university controlled by Benedictines. Religious values were dominating in Protestant and Catholic schools as well. Certain differences denoting the

[9] Ibidem, words of Edmund Gosse quoted by P. Hazard, p. 87.

[10] M. Ornstein, *The Role of the Scientific Societies in the Seventeenth Century*, The University of Chicago Press, Chicago 1928, p. 165.

dissimilarity of education courses still remained. Protestant establishments laid emphasis on the law and theology and usually had a medicine faculty. Catholic universities formed mainly philosophers. At the most progressive catholic university in Würzburg 75% of the program was devoted to philosophy and rhetoric, while the law constituted 7% and medicine 5%[11]. What was common to university education was the language – lectures were given in Latin. While Latin remained the language of universities and was the official language of German science of 17th century French became the language of enlightened intellectual, courtly and influential circles. The Germans like other European nations submitted to French replacing Latin bonding all Europeans with the language of the enemy. 'If some of our ancestors were to come back on earth – wrote Christian Thomasius in his *Discourse on imitating the French (1687)* – they simply wouldn't know us, degenerate hybrids that we are. Nowadays, everything about us has got to be French – clothes, cookery, language, all French. French are our manners, and French are our vices.'[12]

1.2.4. Martha Ornstein in her work entitled *The Role of Scientific Societies in the Seventeenth Century* observes that the civilization backwardness of 17th century Germany was mostly visible in the degree of development of the national language[13]. While eminent French and English writers and scholars were writing in their mother tongue, German was merely the language of uneducated masses. It is hard to believe, that the restoration process of Germany, 'crushed and humbled to the dust, as she was, swept and swayed'[14], had its beginnings in the process of repair and development of German language.

The realization of this program was the main goal of numerous scientific societies. It is characteristic that German societies in the beginning of their activity concentrated on the study of German and this was so to say an initial stage which was to prepare the ground for other research. Meanwhile European scientific societies proceeded in experimental research. Language societies (Sprachgesellschaften) were modeled upon Florentine Academia della Crusca, the institution brought into being in 1582 by the poet Antonio Francesco Grazzini (1503–1582) and which was engaged in purifying

[11] Ibidem, pp. 227–229.

[12] P. Hazard, op. cit., p. 81.

[13] M. Ornstein, op. cit., p. 165.

[14] P. Hazard, op. cit., p. 440.

Italian from chaff (in Italian – crusca)[15]. One of the first societies is Fruchtbringende Gesellschaft (also called Palmenorden) created in 1617, aiming at cultivating of mother tongue and obligating its members to emply the pure German. Poets and writers stood up for the language. Martin Opitz (1597–1637) recognized as the leading German poet published the same year *Aristachus sive de contemptu linguae germanicae*, pronouncing against the cult of ancient languages, and announcing the arrival of grandeur of German[16]. One of the most radical societies was the Teutschgesinnte Genossenschaft, founded in 1643 in Hamburg by Philippe von Zesen. Its members decided to purify German from all foreign loan-words even those functioning in the language for ages. Language societies attained the peak of their activity in the second half of 17th century. These highly specialized institutions were the first real centers of scientific life and movement for the safeguard and renovation of cultural legacy coming to light in Germany. The founder of Societas Eurenetica, society which motto read as follows: *'Per inductionem et experimentum omnia'*, Joachim Jungius (1587–1657) wrote: *'It is absolutely true, that all arts and sciences, as for instance, the art of governing, the knowledge of weights and measures, of medicine, architecture, fortifications, could be lectured and spread in German more easily, more properly, perfectly and clearly than in Greek, Latin or Arab'*[17].

Although German universities in 17th century were the ramparts of scholastic science with traditional curriculums – Leibniz used to call the university educational system 'minism' – some signs of change which were to lead to reform in the Enlightment era were visible in the second half of the century. Revaluation of the attitude towards mother tongue accompanied this process. Some professors had the courage to face the Latin domination, and one of the first who defended his mother tongue was Christian Thomasius.

Thomasius called the Luther of German university reform, professor of law at the conservative university of Lipsk ventured for the first time in 1679 to notify about his lecture in German. The incident was judged a sacrilege, there were even attempts to clean the blackboard with the holy water. Thomasius was however consistent, he raised German to a dignity of language of instruction and published books in German. He argued that one ought to imitate the nations that use their mother tongues: *'Greek philosophers*

[15] Académie Française engaged in language and literature research, which in 1635 cardinal Richelieu made the greatest public scientific institution in the country, also referred to the ideas spread by the same institution.

[16] See W. Czapliński, A. Galos, W. Korta, *Historia Niemiec*, Ossolineum, Wrocław, Warszawa, Kraków, Gdańsk 1981, p. 349.

[17] M. Ornstein, op. cit., p. 167.

did not write in Hebrew, but in their mother tongue'[18]. Even though the mentioned scientists had a tremendous influence on shaping the national consciousness, especially by emphasizing the linguistic unity, none of them could compete in this field and in any other scientific domain with Gottfried Wilhelm Leibniz.

1.3. *Unvorgreiffliche Gedanken...* – linguistic culture treaty

1.3.1. The above outlined social and cultural situation in Germany in 17th century justifies Leibniz daring hypothesis related to the unusual role of German in the history of European civilization. This great philosopher and untiring scholar conscious of the ravages made on the substance of national identity used arguments of psychological nature. They were to – as it seems – maintain the 'national spirit' and it is hard to assume that Leibniz believed it was possible to supply any documentary evidence to support them. After all they are contradictory with his studies in history of German. Leibniz in his voluminous correspondence with Job Ludolf exchanged his assumptions regarding the direction of migration of German tribes and their origin, relating to accessible sources, but any comment on primary role of this language appears there[19].

Renovation and improvement of a national language, and restoration of its standing became the most important duties for Leibniz. He unfolded his renovation program in dissertation *Unvorgreiffliche Gedanken, betreffend die Ausübung und Verbesserung der Teutschen Sprache*[20], which unfortunately wasn't published in his lifetime. The dissertation deals with German but it has in our opinion more universal characteristic. Leibniz's instructions and remarks might concern a whichever ethnic language and understood this way could constitute a set of elements shaping language culture of a given community.

In the very first sentence of his dissertation Leibniz observes (not for the first time after all) that a language is the finest mirror of the mind and that improvement of mind goes hand in hand with the development of the language itself. Leibniz refers here to Greeks, Roman and Arabs. The

[18] M. Ornstein, op. cit., p. 233.

[19] GP IV, 144.

[20] G. W. Leibniz, *Unforgreiffliche Gedanken, betreffend di Ausübung und Verbessrung der Teutschen Sprache*, in: G. W. Leibnitii, *Collectanea Etymologica, illustrationi linguarum, veteri celticae, germanicae, gallicae, aliarum inservientia*, cum praefatione Hohannis Georgii Eccardi, Hannoverae 1717.

same thought is present in his previous writings, among others in *Analysis Linguarum*. If a national language becomes a language of science it has to be improved and adjusted to its requirements. According to Leibniz the case of French and English speaks volumes about it. They achieved considerable progress in philosophical science since having abandoned Latin they began to philosophize in their own languages. This made possible for simple people and 'even women' to form their own opinion on the matter[21].

Leibniz a skilful politician and diplomat while joining the struggle for the linguistic rebirth refers to the achievements of the German nation and points to the domains in which Germans scored some success. He is aware of the fact that he cannot mention any scientific achievements, German scientists having recourse solely to Latin and letting their mother tongue take its own course. German could not develop properly being a language of uneducated majority[22]. 'The majority' reached according to Leibniz high level of knowledge in such areas as: mining, hunting, forestry, mechanics and navigation. Specialized vocabulary adopted by other national languages is the evidence[23]. Leibniz reminds also his nation's military victories and writes that the nation which distinguished itself by courage and prowess is capable of intellectual effort. It is possible solely thanks to development of one's own language[24].

1.3.2. Leibniz reasoning is to a certain extent emotional, it is not however deprived of an important dose of common sense. Leibniz acts like a 'real psychologist' trying to cure German spirit severely experienced by war-time adversities. The Thirty Years' War among other things caused – he wrote – that *'our language was in a chaos as well as our homes'*[25] and after the war we were dominated by French power and language. *'France set [...] a model of elegance'*[26]. He judged foreign influence differently than his contemporary linguistic purists. Being aware of hazards Leibniz highlighted certain benefits resulting from interpenetration of different cultures. Germans learned about prevention from contagious diseases from Italians, form the French they got the knowledge on improving their military structures[27]. Further-

[21] GP IV, 144.

[22] UG, § 9.

[23] GP IV, 144.

[24] UG, § 4.

[25] UG, § 25.

[26] UG, § 26.

[27] UG, § 27.

more relations with French culture which added some delicacy to a serious 'German nature' permitted some change in aesthetical tastes and lifestyle. The language itself was enriched by expressions introduced to German in a natural way, like plants, that grow in a breeding ground.

Leibniz in general was against the usage of any foreign language in common every day life as well as in any social, scientific or institutional activity. '*It would be shameful and scandalous – he writes if our basic language, the one of our heroes vanished due to our negligence*'[28]. Employing foreign languages, which could never be mastered by all, causes confusion in the way of thinking. One who is not aware of various meanings of foreign vocabulary and expressions cannot write or think properly. Nothing good can result from adopting a foreign language, and there is danger of loosing freedom[29]. Leibniz relating to documents from Imperial archives writes that after analyzing them one can make a thorough study of progressive decline of German which in the times of Reformation still kept its integrity. Undertaking of a repair program with the aim of giving German back its due standing depends on the goodwill of scholars, ecclesiastics and educated social elites[30].

1.3.3. Leibniz presents his program by enumerating lacks and defaults which should be coped with. He highlights above all the lack of specialized terminology in such domains as logic, mathematics or theology. It is obviously due to secular addiction to Latin, the official language of science. Leibniz thinks however that it is not due to German lack of skill which restrains them from improving their language but absence of goodwill. If '*everything done by a simple man could be expressed well in German, undoubtedly matters proper to eminent and educated people, could be, if they were willing to do so, expressed well or even better in pure German*'[31]. It seems Leibniz returns to his own thoughts presented in a radical way in the 'Introduction to Nizolius' '*if something cannot be expressed in common parlance it should be removed from philosophy*'[32]. Similarly to 'Introduction' certain incoherence of Leibniz's views can be observed. Leibniz maintains that in principle every colloquial language is to the same extent suited to the needs of everyday life and adjusts the requirements of science. Development of science depends

[28] UG, § 21.
[29] UG, § 20, 21.
[30] UG, § 24.
[31] UG, § 10.
[32] GP IV, 143–144.

on formulating thoughts in a clear and precise way especially in the mother tongue. It is therefore unimportant if we philosophize in English, German or French the only important matter is that each of these languages follows the traces of our inventions and reflects in the best manner the inner order of thoughts. Thus, if Germans overcome some psychological barriers they will be able to succeed in every domain by perfectioning their language. In the struggle for enhancement of 'national spirit' this democratic sounding argument appeared too weak for Leibniz. He probably believed in philosophical mission of German language when he reached for more substantial measures of persuasion. Leibniz explained to his compatriots in a rather publicist than a scientific style that none of European languages is as well adapted to verification of various philosophical doctrines as German[33]. It is so because '*We Germans, we have got a special measure of thoughts, unknown to others* [...] *Empty words, are void and foam and are not accepted in German.*'[34]. If we put aside arguments of propagandist nature as the readers of *Unvorgreiffliche Gedanken* we will appreciate the objective analysis of mother tongue especially in areas where – according to Leibniz – its specialization should take place.

Improvement of onomathology and enrichment of a language, apart from the above mentioned, relates to such areas as morality, psychology (Leidenschafften des Gemüths), customs, management, national and internal services and politics as well as the law[35]. Leibniz notices the need of protection and restoration of German language not only as a means of everyday communication but above all as the national official language of power and administration. National documents, legal deeds, the Government representatives should procure examples of appropriate use of the mother tongue to the people. It is also useful to read theological writings since even though theologians gave themselves up to fanatical fantasies, expressions and terms which testify for wealth of German are present in their works[36].

Leibniz considers the language a living system consumed by a disease. He enumerates the reasons for that, diagnoses and prescribes the treatment. An important element of this treatment are institutional activities engaging influential and educated representatives of 'higher' classes of society. Leibniz values the contribution of linguistic societies to the safeguard and restoration of German. He reminds that the basic mottos of their acti-

[33] GP IV, 144.
[34] UG, § 11.
[35] UG, § 15, 67, 90.
[36] UG, § 14.

vity were wealth, purity and transparence of the language. Even though Leibniz agrees on the general idea of their activity, he criticizes their procedures claiming that the members of the societies went too far clearing the German of any foreign influence[37]. There exist, according to him, fundamental difficulties in selection of criteria allowing to decide on the right origin of expressions. Job Ludolf, a friend of Leibniz who refused cooperation with Palmenorden shared his doubts. In a letter to Leibniz in January, 22nd 1692 he wrote '*the first thing that must be decided is which words are to be considered foreign and which ones of native stock, for not everything associated with foreigners is foreign. Who would be so senseless as to assert that our ancestors lacked noses, ears, mouths, or eyes before learning these words from the Romans*'[38]. Ludolf, the same as Leibniz doubted the actual influence of linguistic societies on the change of linguistic behavior of common users even though he observed some benefits resulting from their activity. The behavior in his opinion cannot be shaped by means of any law.

1.3.4. Leibniz postulates methodic review of all German expressions. The foundations of a language – as he writes – are words, on which idioms grow. Vocabulary analysis should include both common parlance in all its environmental varieties and every possible dialect. Leibniz points out that though the written language is ruled by High-German still Low-German, the Margraviate of Brandenburg dialect, Swabian dialect, Bavarian and others should be treated equally. Languages related to German ought to be studied as well since there are some words and expressions having their source in German. Documents and texts written in Old German, Old Saxon, or Old Frankon provide valuable information. It should be added that Leibniz did not limit himself to postulates only. Studying German dialects was one of his passions. In *Collectanea Etymologica...* one can find Low-German lexicon prepared by Leibniz on the grounds of materials provided by Johan Justus Kelp, a clergyman from Brema[39]. Leibniz collaborated in this field with numerous scholars, he encouraged Gerhard Meier to initiate work on his lexicon of Saxon, Dutch and Danish[40].

[37] UG, § 19.

[38] J. T. Waterman (ed.), Leibniz and Ludolf on Things Linguistic, Excerpts from Their Correspondence (1688–1703), University of California Press, Berkeley, Los Angeles, London 1977, p. 23.

[39] See Leibniz's letter to Ludolf form April 17th 1692, in J. T. Waterman (ed.), *Leibniz and Ludolf on Things Linguistic...*, p. 25 and Waterman's commentary, p. 66.

[40] J. T. Waterman (ed.), *Leibniz and Ludolf on Things Linguistic...*, p. 31.

Linguistic material demands a detailed analysis and classification on the basis of which it is possible to isolate vocabulary belonging to common parlance, technical vocabulary, expressions typical of rural areas, and also words of foreign origin and archaisms. It is the starting point for works aiming at creation of dictionaries and books, which would determine what we call today a linguistic standard. Leibniz observes the necessity to elaborate a German dictionary of colloquial speech, a specialist dictionary covering technical terms and an etymological dictionary containing information on the meaning and origin of expressions. He relates here to the experience of Académie Française and Florentine Crusca, but also underlines the fact that the main objective of these institutions was merely to create colloquial speech dictionaries, however the specialized vocabulary is not less important since thanks to it science can be understood, developed and spread[41]. One ought to ask representatives of all domains of science for help and cooperation in elaboration of a dictionary of technical terms. Linguistic fluency which is the essential element of the ability to express clearly one's thoughts is to be achieved through knowledge of the meanings, origin and relations between expressions. This ability is developed through etymological dictionaries which enable to make a thorough study of a given nation. Since languages – as it is highlighted by Leibniz – are the oldest documents of humanity, Glossarium Etymologicum of German would permit to re-create the origins of the nation and would indicate its due role in shaping the European civilization[42]. It is characteristic of Leibniz to concentrate mainly on vocabulary, he does not devote much attention to other aspects of linguistic correctness. In his dissertation comprising 114 paragraphs only a few deal with grammar and rules of standardization of orthography. Leibniz observes that even though Germans have no room for shame when it comes to grammar, it is still nowhere near perfection. He postulates indeed simplification of its rules, he observes however that real experts of the matter are missing among scholars.

Leibniz advising to avoid barbarisms and provincialisms in formal language tolerates using them in informal language. If they are appropriately used they do not shock – in Leibniz's opinion – in the parlance of ordinary people. He is also quite liberal when it comes to the use of foreign expressions and phrases, but advises avoiding them when one is not sure the interlocutor knows their meaning. Leibniz proves to have a lot of common sense when expressing his opinion about the language of national and legal documents

[41] UG, § 36, 37.

[42] UG, § 46, 48.

for unlike purists under the banner of Palemnorden he admits foreign technical terms, especially Latin ones which have been functioning for ages with the same meaning in legislation of many countries[43].

When considering the process of naturalization of expressions of foreign origin in German Leibniz notices that it is one of the ways of enriching a language, especially when the loan-words come from the same linguistic family. A thorough historical study of these expressions would permit the reconstruction of the source of Germanic languages[44]. Another way of adjusting the language to new needs is creation of new words or conferring new meanings to existing ones. Acceptance of newborn expressions by a wide circle of users depends on – according to Leibniz – 'blind luck'[45]. It appears that the author is conscious that the language is governed by its own laws and it is possible to form it by propagating certain patterns of linguistic behavior and not by administrative actions.

1.3.5. Literature in the first place provides models of such behavior. Leibniz spots exceptional role of poetry and sets certain norms of poetic language. And so poetic language should steer clear of foreign expressions and words of unsound origin should be recognized German. Their orthography should be standardized and adjusted to German orthography[46]. Leibniz quotes here the example of Martin Opitz, who outlined some theoretical instructions for German poetry and who as one of the first undertook actions in favor of rebirth of national literature. Leibniz highlights an urgent need to spread such models, encouraging all who write in German to a productive attitude towards their own language[47].

The program of repair should comprise all applications of a language. Leibniz as the fervent adherent of public education, addressed scholars to present the results of their studies in their mother tongue and to undertake translations of writings of eminent authors. The nation – in his opinion – was too long kept away from knowledge. Real scientists should not fear the language of their nation, mostly because the more accessible their knowledge is the more witnesses of their grandeur. A well developed language, like a well polished glass, improves the sharpness of thought and gives mind a lucid clarity[48].

43 UG, § 82, 83, 85, 89, 92.
44 UG, § 67–72.
45 UG, § 76.
46 UG, § 95, 96.
47 UG, § 111.
48 See M. Ornstein, op. cit., p. 182.

Realization of this program ought to be assisted by organized institutional actions. Leibniz the enthusiast of the idea of scientific societies, 'academic movement' that spread over 17th century Europe, led to creation in 1700 of the Science Academy in Berlin, the institution modeled upon the Royal Society and French Académie Française. One of its statutory goals was consolidation of 'fame, prosperity and significance of German nation, erudition and language'. Even though creation of the Academy was Leibniz's personal success, still his ambitions were greater. The public access to knowledge would be guaranteed – in his view – by scientific societies only – modern centers of research and popularizing work, that should be created in various cities belonging to the Empire and coordinated by the Academy[49].

1.3.6. It is difficult to underestimate Leibniz's contribution to the rebirth of the cultural integrity of German nation, the source of which was the language reflecting both its power and its decline. Paul Hazard wrote '*the theory of Racial superiority had not yet come to the fore. The profound significance of the expressions 'native land' had not been fully gauged. No notion had been formed as yet of the dynamic potentialities of the idea of nationality*'[50], but the value of the most important binder which for people forming a society is language became visible. If we put aside Leibniz's arguments concerning the uncommon philosophical mission and paradisiacal origins of German the *Unvorgreiffliche Gedanken* appear to be a universal treatise of history and culture of a nation in a perspective of the history of its language. Writing about the intellectual potential of his nation Leibniz informs the lecturers that it is initiated through language. The improvement of minds demands continuous improvement of the language since language is the mind's reflection and renders best creative possibilities of its users. Moreover it allows to recreate their aesthetic tastes, customs and character.

Leibniz was not privileged to rejoice at the reach of the influence of his thoughts, *Unvorgreiffliche Gedanken...* were published only posthumously, but it did not take one century for Kant, Goethe or Schiller to prove the admirable power hidden in their language. Johann Gottfried Herder, one of the most eminent representatives of the German Enlightment, referring to Leibnizian legacy wrote that the most beautiful attempt to explore the history and to characterize the diversity of human intellect and heart would

49 Ibidem, p. 195.
50 P. Hazard, op. cit., p. 443.

be *philosophical comparison of languages*, for in each of them intellect and character of a given nation is reflected. Finally the most valuable *architectonics of human notions, incomparable logic, and common sense metaphysics* would emerge. Laurel of victory is not yet awarded and another Leibniz will be honored when the right moment comes[51].

Translated by Marta Głowacka

[51] http://www.textlog.de/herder_menschheit.html, http://www.odysseetheater.com/gothe/herder/ideen.html

STUDIES IN LOGIC, GRAMMAR AND RHETORIC 8 (21) 2005

Renata Botvina
Politechnika Białostocka

FRANCIS BACON'S NATURAL PHILOSOPHY AS A UNIVERSAL LANGUAGE

Sola perpetuo manent
Subiecta nulli, mentis atque animi bona
Florem decoris singuli carpunt dies...

Seneca

The liberty of manconsists lies solely in this:
that he obeys natural laws because
he has "himself" recognised them as such,
and not because they have been externally imposed
upon him by any extrinsic will whatever,
divine or human, collective or individual...

Michail Bakunin

Disputes over the power of word were a commonplace in the seventeenth century. Numerous political affairs, economic stagnation, and religious disagreements taking place in Europe interwove in what has come to be regarded as "the crisis of the seventeenth century"[1]. Naturally, prominent thinkers of the century devoted their minds to the search for a universal remedy for the crisis. Consequently, to a large and surprising extent, an intense course of history in the seventeenth-century Europe led towards a detailed preoccupation with the power of word bringing to memory the myth of the Adamic language with all its blessings.

Bruno Latour calls contemporary scientists "the tribe of readers and writers"[2]. If he is right, then that tribe certainly has its origins in the persona of Francis Bacon who perfected and highlighted both arts of reading and writing all his life. In his philosophical program there remains the issue of language which Bacon sees as of high importance on the way to the

[1] See T. Munck, *Seventeenth-Century Europe: State, Conflict, and the Social Order in Europe, 1598–1700*, The Macmillan Press Ltd, New York and London, 1990, pp. 83–84.

[2] B. Latour and S. Woolgar, *Laboratory Life: The Social Construction of Scientific Facts*, Beverly Hilles, California, Sage, 1979, p. 69.

ISBN 83-7431-059-6 ISSN 0860-150X

clear perception of things. For him, a language, as every product of the human mind, succumbs to the fallacy of human thinking and therefore needs a thorough cleansing. He would certainly have agreed with Huxley's statement: "A man who habitually writes and speaks correctly is one who has cured himself, not merely of conscious and deliberate lying, but also (and the task is much more difficult and at least as important) of unconscious mendacity"[3].

In the light of the seventeenth-century belief in the magic power of word, a case of Francis Bacon's natural philosophy deserves a special consideration. Facing a political crisis in the monarchy, he eagerly grasped at the opportunity to exploit language in terms of a universal remedy for all evils in the state. Therefore, "[...] the new philosophy of Bacon [...] emerges against specific circumstances of the country's pursuit of a universal language"[4]. His natural philosophy and critique of language derived from his mistrust of the scholastic philosophy after indicating its failure to see the dawn of the new science with its new aims and needs.

Not only did Bacon see a need to instaurate the science, but he also saw that instauration as a means to prop the model of a perfect state on. In the course of his philosophical program, Francis Bacon clearly realized that language was its integral part because of its capability of being either an aid or an obstacle in the process of human perception of the real world. Being corrupted mostly by the idol of the market place, language as seen by Bacon was a kind of sponge constantly absorbing ambiguity and vagueness of the thinking processes governed mostly by motions of desire.

In other words, language was a mirror reflecting the imperfect reality with its errors. Just as scholastic philosophy, erroneous in its aspects, language needed a catharsis on which both political harmony and further scientific progress depended. Therefore, Bacon's program of natural philosophy based on the inductive method presupposed a thorough cleansing of the human fallacies present in language, which would consequently lead towards achieving power over nature, which, in turn, would give a key to the natural harmony in the state and science.

Bacon's desire to achieve power over nature remained in accordance with the general goal of the Renaissance movement, which among other

[3] A. Huxley, *Words and Their Meanings*, in "The Importance of Language," (ed.) M. Black, Prentice-Hall Inc., Englewood Cliffs, N. J., 1962, p. 1.

[4] R. E. Stillman, *The New Philosophy and Universal Languages in Seventeenth-Century England Bacon, Hobbes, and Wilkins*, London: Associated University Press, London, 1995, p. 263.

aims also presupposed a victory over nature[5]. In his book entitled "Francis Bacon: From Magic to Science" Paolo Rossi discusses the Renaissance tradition of alchemy, astrology, and hermeticism as a prologue to the new philosophy, mentioning Bacon's desire for experimental works, mechanical arts, and the goal of mastering nature by mastering the language of its signatures[6]. In accordance with Paolo Rossi remains Charles Whitney, who in the book entitled "Francis Bacon and Modernism" argues that Bacon's natural philosophy marked the beginning of "a revolution in thinking that will lead to radical changes in culture", for in many ways Bacon's discovery of natural philosophy was "itself a kind of reform or fulfillment of rhetorical ideals and practices"[7]. Having mentioned the importance of language in Bacon's natural philosophy, there arises a need to analyze the links between Bacon's natural philosophy and his views on the issue of language.

Bacon's *New Atlantis* is a good example integrating all his theoretical and practical views and desires, presenting a perfect state built on the laws of nature which, in turn, constitute the foundation of the Atlantic society. The laws the society is built on are natural and, therefore, successful. Although they are not revealed like natural knowledge, they can be examined when some effort is put and a certain doze of observation is employed. In *The Advancement of Learning* Bacon writes that "the just and lawful sovereignty over men's understanding, by force of truth rightly interpreted, is that which approacheth nearest to similitude of divine rule"[8]. Truth, which is the product of logical method in act, regulates thought according to the fixed laws providing a proper vehicle for domination not only on the intellectual level but also in the world[9]. In this way, the concept of law remains in the centre of Bacon's program of natural philosophy. Indeed, his permanent reference to the concept of law is not surprising bearing in

[5] Compare P. Rossi, *Francis Bacon: from Magic to Science*, (trans.) Sacha Rabinovitch, University of Chicago Press, Chicago, 1968, p. 16; C. Whitney, *Francis Bacon and Modernism*, Yale University Press, New Haven, 1986, p. 12; and M. Wiszniewski, *Bacona metoda tłumaczenia natury*, Państwowe Wydawnictwo Naukowe, 1976, pp. 34–50.

[6] P. Rossi, *Francis Bacon: from Magic to Science*, op. cit., p. 16.

[7] C. Whitney, *Francis Bacon and Modernism*, op. cit., p. 12.

[8] F. Bacon, *The Advancement of Learning*, BI, VIII:3 – HTML edition, 1998 (based on G. W. Kitchin's 1861 edition; paragraph sections according to J. Spedding's 1854 edition; available to be reproduced freely in unaltered form provided that this editorial comment is included; copyright 1998 by Dr. Hartmut Krech, Bremen, Germany (kr538@zfn.uni-bremen.de), displayed at: http://darkwing.uoregon.edu/%7Erbear/adv1.htm; as retrieved on 21.III.2005.

[9] See R. E. Stillman, *The New Philosophy and Universal Languages in Seventeenth-Century England Bacon, Hobbes, and Wilkins*, op. cit., p. 87.

mind the fact that he was a lawyer himself, which explains his need to regulate everything by the means of the written and legalized word, that is, a law.

Regarding Bacon's concept of law, it appears to have its origins in the basic assumptions about the fallacy of human nature. Adam fell from paradise, which brought the destruction of the principal order on earth, bringing the consequences visible in every field of life. The direct heritage of that dramatic fall was man's predisposition towards idleness and pleasure, the root of all evil. Therefore, man was doomed to fight with his inclination towards evil even in learning. In the first book of the *Advancement of Learning* Bacon concludes that it is necessary "to keep and defend the possession of the mind against idleness and pleasure, which otherwise at unawares may enter to the prejudice of both"[10]. In the *Advancement of Learning* Bacon also refers to "the lawfulness of the phrase or word"[11] in the context of superfluous decorum of the written text which consequently darkens a clear understanding of the writer's purpose, which is also a heritage after the fall of the first people.

While referring to the concept of law, Bacon also alludes to the role of the king as the one after God to rightly keep it. He gives the examples of Moses and David who are the true "pastors of their people"[12]. The conclusion he draws on that premise is simple: "That Kings ruled by their laws as God did by the laws of Nature"[13]. Therefore, the king is the head who governs the state according to the laws which are based on truth: "the fundamental laws of nature, with the branches and passages of them, [are] an original and first model, whence to take and describe a copy and imitation for government"[14]. In this way, "law is a vehicle for achieving sovereignty over men's understanding in politics, natural philosophy, and language, and it achieves theological sanction from God"[15].

If law is central in the order of nature, it must be central in the order of natural philosophy as well as in the structure of language. In the passage taken from *Cogitationes de Natura Rerum* (*Thoughts on the Nature of Things*) written in 1624 Bacon returns to the issue of language in order to

10 F. Bacon, *The Advancement of Learning*, B1, II:7.

11 Ibid., BI, IV:2.

12 Ibid., BII, XXI:8.

13 Ibid., BII, XXI:8.

14 Ibid., BII, XXI:8.

15 R. E. Stillman, *The New Philosophy and Universal Languages in Seventeenth-Century England Bacon, Hobbes, and Wilkins*, op. cit., p. 87.

find a model for his desired "course of application"[16]. He writes: "Surely as the words or terms of all languages, in an immense variety, are composed of a few simple letters, so all the actions and powers of things are formed by a few natures and original elements of simple motions"[17]. Therefore, natural philosophy "is conceived as a logically constructed alphabet of nature enabling the user to decipher its laws of motion"[18]. The search for unifying laws in nature consequently leads him to the idea of natural philosophy and universal language's laws as being similar to those present in nature.

Bacon's desire to find a universal key in nature acquires a metaphysical context when he refers to the law created by God and hidden in nature's primary order. Not surprisingly, allusions to Adam and the Adamic language are frequent in Bacon's writings. Naturally, Adam is presented as "possessing the pure knowledge of nature and universality, a knowledge by the light whereof man did give names unto other creatures in Paradise, according unto their properties"[19]. Having created Adam after his image, God let him govern the nature. The language was given to Adam as a precious gift so that he could communicate with Eve and manage the earth's treasures. Unfortunately, the gift was lost as a result of Adam's fall and from that moment on "the freeing of the minds depends upon men's revision of the world"[20]. Moreover, such a revision is "a part of the reform of the knowledge"[21]. Hopefully, the discovery of Creator's primary vision is possible:

> God forbid that we should give out dream of our own imagination for a pattern of the world; rather may he graciously grant to us to write an apocalypse or true vision of the footsteps of the Creator imprinted on his creatures...[22]

Therefore, the parallels between the natural philosophy and the Adamic language are obvious: Bacon's sees natural philosophy as "a kind of second scripture" promising the return to the primary Edenic perfection whereas natural philosophy, itself a perfect language, promises a quick return to

16 F, Bacon, *Thoughts on the Nature of Things*, in "The Philosophical Works of Francis Bacon", op. cit., p. 467.

17 Ibid., 467.

18 R. E. Stillman, *The New Philosophy and Universal Languages in Seventeenth-Century England Bacon, Hobbes, and Wilkins*, op. cit., p. 90.

19 F. Bacon, *The Advancement of Learning*, BI, I:3.

20 P. Rossi, *Francis Bacon From Magic To Science*, op. cit., p. 163.

21 Ibid.

22 F. Bacon, *The Great Instauration*, displayed at: http://www.whale-hunter.net/dongli/ShowArticle.asp?ArticleID=1206 as retrieved on 14.V.2005.

the Edenic (perfect) language[23]. In other words, Bacon held that Adam possessed metaphysical knowledge to a very high degree. Moreover, for him the whole nature was a book which he could read with ease. Paolo Rossi reminds on Bacon's behalf that "the pages of nature's great book should be read with patience and reverence, pausing and meditating over each one and discarding all easy interpretations"[24]. Bacon stubbornly believed that metaphysical knowledge could be retrieved and again and again he proclaimed his longings to possess the secret of the inner things which Deborah Taylor Bazeley rightly calls "a Baconian desire to appropriate the power of naming"[25].

While searching for the universal key to the laws of nature, Bacon comes up with the idea of scientific communication. Since language is a mirror reflecting the real picture of the world, scientific communication must be characterized above all by brevity, precision, and plainness: these are the general standards by which he proposes to guide a philosophical discourse. The appeal to precision as regards to words once again appears in Bacon's *Preparative Toward Natural and Experimental History* written in 1620:

> ...never cite an author except in a matter of doubtful credit: never introduce a controversy unless in a matter of great moment. And for all that concerns ornaments of speech, similitudes, treasury of eloquence, and such like emptinesses, let it be utterly dismissed. And let all these things which are admitted be themselves set down briefly and concisely, so that they may be nothing less than words...[26]

Again, according to Bacon, a perfect model of scientific communication is hidden in man's understanding of nature and its laws which seem to be definitely underrated or even neglected:

> It seems to me that men look down and study nature as from remote and lofty tower. Nature presents to their gaze a certain picture of herself, or a cloudy semblance of a picture, in which all the minute differences of things on which the practice and prosperity of men rest, are blurred by distance. So men toil and strive, straining the eye of the mind, fixing their gaze in prolonged meditation, or shifting it about to get things into better focus. Finally they construct the arts of disputations, like ingenious perspective glasses, in order to seize and master the subtle differences of nature. A ridiculous kind of ingenuity, is it not, and misdirected energy for a man to climb his tower, arrange his lenses, and screw up his eyes to get a closer view, when he might avoid all that laborious contrivance and tedious industry and achieve his end by a way not only easy but far superior in its benefits and utility, namely by getting down from his tower and coming close to things?[27]

In the article entitled "The Seventeenth-Century Context: The Discourse of The new Science As The Ultimate Masculine Register" Deborah Taylor Bazeley discusses the language theories of the new science movement in terms of gender, giving an interesting account of Francis Bacon's vision of the language of the new science based on Bacon's statement that "the true philosophy which echoes most faithfully the voice of the world itself"[28].

Therefore, when Bacon approaches the creation of a philosophical language, he clearly attempts at what R. E. Stillman calls "closing the gap between a natural philosophy conceived on the model of a perfected language"[29]. The model to refer to is exclusively nature; the data to analyze is well hidden in its laws. Everything man has to do is to get down from his tower and come close to things.

While searching for a perfect language, Bacon tackles the very nature of language. Bearing in mind Bacon's admiration for the Chinese system of writing, it is clear that for Bacon, language is an "instrument of transmission", capable of benefiting from other means besides words and letters:

> We are handling here the currency (so to speak) of things intellectual, and it is not amiss to know that as moneys may be made of other material besides gold and silver, so other Notes of Things may be coined besides words and letters...[30]

Bacon's *De Augmentis Scientiarum* contains perhaps the most complete discussions of the communication sciences, or what he calls the Art of Trans-

[23] R. E. Stillman, *The New Philosophy and Universal Languages in Seventeenth-Century England Bacon, Hobbes, and Wilkins*, op. cit., p. 99.

[24] P. Rossi, *Francis Bacon From Magic To Science*, op. cit., p. 32.

[25] D. Taylor Bazeley, *The Seventeenth-Century Context: The Discourse of the New Science As The Ultimate Masculine Register*, displayed at http://www.she-philosopher.com/library/diss-appB.html as retrieved on 17.VI.2005.

[26] F. Bacon, *Preparative Toward Natural and Experimental History*, aphorism III, displayed at: http://www.constitution.org/bacon/preparative.htm as retrieved on 25.VI.2005.

[27] F. Bacon, *Redargutio Philosophiarum*, in "The Philosophy of Francis Bacon", (ed.) B. Farrington, Liverpool University Press, Liverpool, 1964, p. 129.

[28] Claiming that Bacon was the first Englishman to wrestle with the language issues relating to the new science, she suggests that Bacon's discourse was similar to his society's "feminine speech ideal" where nature is referred to in the masculine gender, as Pan, whereas language is feminine, being referred to as either the good wife Echo or the bad daughter Iambe. See D. Taylor Bazeley, *The Seventeenth-Century Context: The Discourse of the New Science As The Ultimate Masculine Register*, displayed at: http://www.she-philosopher.com/library/diss-appB.html as retrieved on 17.VI.2005.

[29] R. E. Sillman, *The New Philosophy and Universal Languages in Seventeenth-Century England Bacon, Hobbes, and Wilkins*, op. cit., p. 86.

[30] F. Bacon, in "The Philosophical Works of Francis Bacon", op. cit., p. 343.

mission[31]. In a letter of advice to Fulke Greville he outlines two principal methods of note taking: "He that shall out of his own reading gather notes for use of another, must (as I think) do it by Epitome, or Abridgment, or under Heads or Common Places. Epitomes may also be of two sorts: of any one Art or part of Knowledge out of many Books, or of one Book by itself"[32].

Accordingly, the first method, that is, by epitome or abridgment, is to summarize or paraphrase the original texts; these notes, generally presented in the order of the text from which they were produced, are often called *adversaria*. The second method is to select passages of interest for their content or style, which are copied and sorted under a thematic heading to facilitate retrieval[33]. These categories and the notes that correspond to them are usually called commonplaces. Bacon favoured the latter as of "far more profit and use"[34]. According to R. E. Stillman, Bacon's arts of transmission represents "his most sophisticated efforts to wrestle with the language", at the same time being "the culmination and the end limit to Bacon's philosophical thought about language"[35]. The project of the language cleansed of the idol of the market place and presented in the *Novum Organum* echoed the perfected language based on the new philosophy and elaborated according to the Art of Transmission. The new language meant to free knowledge from the organum.

Bacon's concerns with the issue of language also find their way in *The Advancement of Learning*, where he presents the science of grammar which, being a product of man's considerations about speech and words, in many cases is to blame for the "confusion of tongues":

> Concerning speech and words, the consideration of them hath produced the science of grammar: for man still striveth to reintegrate himself in those benedictions, from which by his fault he hath been deprived; and he hath striven against the first general curse by the invention of all other arts, so hath he sought to come forth of the second general curse which was the confusion of tongues, by the art of grammar...[36]

[31] The art of transmission in its broad sense refers to the different forms of writing and printing through which texts were preserved through times.

[32] F. Bacon, quoted in V. F. Snow, *Francis Bacon's Advice to Fulke Greville on Research Technique*, in "Huntington Library Quaterly" 23, 1960, p. 370.

[33] Ibid., p. 370.

[34] Ibid.

[35] R. E. Stillman, *The New Philosophy and Universal Languages in Seventeenth-Century England Bacon, Hobbes, and Wilkins*, op. cit., p. 107.

[36] F. Bacon, *The Advancement of Learning*, BII, XVI:4.

While further elaborating on the use of grammar, Bacon mentions its two natures, that is, practical and philosophical, out of which it is definitely philosophical that is more perfect than the natural one:

> The duty of it is of two natures; the one popular, which is for the speedy and perfect attaining languages as well for intercourse of speech as for understanding of authors; the other philosophical, examining the power and nature of words, as they are the footsteps and prints of reason [...] and therefore I cannot report it deficient, though I think it very worthy to be reduced into a science by itself...[37]

The importance of the nature of words is revealed in the definition which Bacon's attached to them; words are seen as "the footsteps and prints of reason"[38]. The words, therefore, were to be deprived of their ambiguity and vagueness, which were the main features of the idol of the market place. To do that, Bacon set out to analyze the nature of the ciphers, the smaller units of the words, which "are commonly in letters or alphabets but may be in words"[39] and whose three virtues Bacon highly praises: "they be not laborious to write and read; that they be impossible to decipher, and, in some cases, that they be without suspicion"[40]. The allusions to the language of nature deciphered by God is visible here: "this art of ciphering hath for relative an art of deciphering, by supposition unprofitable, but, as things are, of great use; but suppose that ciphers were well managed, there be multitudes of them which exclude the decipher"[41].

In *The Advancement of Learning* Bacon gives a high value to the importance of letters, the smallest units of the words, concluding: "Nay, there is a ground of discourse [...] which is a discourse upon letters, such as are wise and weighty [...]. For letters have a great and more particular representation of business and either chronicles of life"[42]. However, words, being "chronicles of life" are subjected to absorbing ambiguity and vagueness remaining part of human thinking. A universal language may therefore be constructed on the basis of words as well as on "signs":

> This then may be laid as rule; that whatever can be divided into differences sufficiently numerous to explain the variety of notions (provided those differen-

[37] Ibid., BII, XVI:4.

[38] Ibid., BII, XVI:4.

[39] Ibid., BII, XVI:6.

[40] Ibid., BII, XVI:6.

[41] Ibid., BII, XVI:6.

[42] F. Bacon, *The Advancement of Learning*, BII, XXIII:9.

ces be perceptible to the sense) may be made a vehicle to convey the thoughts of one man to another...[43]

Consequently, he recognizes two types of notions: ex congruo (iconic or motivated) and ad placitum (arbitrary or conventional). Ad placitum can be regarded as "a real notion" if it refers not to the defined word but directly to the thing[44]. In other words, signs may act as symbols; to do that, they must be perceptible to the human senses and represent different elements to refer to a particular idea or thing. Gestures, hieroglyphics, and ideograms are the direct symbols of notions and things.

Bacon's conviction as to the power hidden in letters is visible in his numerous analogies between natural philosophy and the alphabet. As R. E. Stillman observes, Adam's naming of the creatures is a significant trope for Bacon since Adam penetrated in the very nature of things and was therefore able to grasp the link between the signifier and the signified[45]. Bacon's frequent references to nature as a book inscribed with divine signatures is apparent in the *New Atlantis*, where the state is organized according to the laws of nature and nature is read and interpreted as a divine scripture.

The conventional approach to the issue of language, which was a result of human inclination towards ambiguity and abstraction, finds its way in Bacon's statement that "the false appearance imposed upon us on words, which are framed and applied according to the conceit and capacities of the vulgar sort"[46]. The key is a search for the primary means hidden in the scripture of nature, where everything has its definite meaning: "So as it is almost necessary in all controversies and disputations to imitate the wisdom of the mathematicians, in setting down in the very beginning the definitions of our words and terms that others may know how we accept and understand them, and whether they occur with us or no"[47].

Finally, the parallels between the natural philosophy and language reveal Bacon's appeal to translate the laws of nature: "Human beings are the executives and interpreters of nature"[48]. If God is a code-maker, nature is decipherable and man is challenged to become a code-breaker. As

[43] Ibid., BII, XXIII:9.

[44] See F. Bacon, *The Advancement of Learning*, BII, XVI:3.

[45] R. E. Stillman, *The New Philosophy and Universal Languages in Seventeenth-Century England Bacon, Hobbes, and Wilkins*, op. cit., p. 98.

[46] F. Bacon, *The Advancement of Learning*, BII, XIV:11.

[47] Ibid., BII, XIV:11.

[48] F. Bacon, *Novum Organum*, B1:I.

R. E. Stillman notes, the perception of knowledge as understood by Bacon involves always one and the same operation: the translation of the voice of the world's order from one language, that is, the order of signatures, into another language, the order of natural philosophy; thus, natural philosophy becomes an objective transcription, "a neutral and passive dictation from the language of things"[49].

Therefore, Bacon's natural philosophy becomes a universal language whereas a universal language becomes an integral part of such philosophy. The establishment of each promises the return to the Edenic perfection and universality, that is, the key to the perfect state for, as Bacon notes, "it is impossible to capture nature in other way than by listening to it"[50]. The message is clear – the language of nature must be learnt anew for, as Paolo Rossi has rightly grasped Bacon's message, "it had suffered the confusion of the Tower of Babel and man must come to it again, not searching for marvels and surprises but handling, like a little child, each letter of its alphabet"[51].

[49] R. E. Stillman, *The New Philosophy and Universal Languages in Seventeenth-Century England Bacon, Hobbes, and Wilkins*, op. cit., p. 98.

[50] F. Bacon, *Novum Organum*, B1:III.

[51] P. Rossi, *Francis Bacon From Magic To Science*, op. cit., p. 32.

STUDIES IN LOGIC, GRAMMAR AND RHETORIC 8 (21) 2005

Katarzyna Doliwa
Białystok University

WILLIAM OF OCKHAM AND THOMAS HOBBES – ON THE NATURE OF GENERAL CONCEPTS

William of Ockham and Thomas Hobbes are three centuries away, each of them created in different social and political situation, and based on different paradigms and scientific standards. Ockham times is the end of the Middle Ages, the period when religion ruled every domain of life, the ages of the scholastic rule. The period when Hobbes lived, the great crisis of 17^{th} century, when the fundaments of the old order collapsed, new science was in creation and the struggle with scholastic methods was the anthem of many thinkers.

It seems however that the two philosophers have a lot in common – nationality and studies at Oxford – great polemic temperament and engagement in politics. They were both contestants of contemporary social order, they did not hesitate to speak their disagreement and both had a lot of trouble because of unpopular views they propagated (Ockham was judged at the pontifical Court at Avignon from where he resorted to flight, Hobbes almost perished at the stake).

Common to both philosophers is their attitude in the argument over universals which engaged the greatest thinkers of all ages, starting with Plato and lasting until today[1] especially on the grounds of philosophy of mathematics. From the classical point of view the argument divides philosophers into partisans of realism recognizing the existence of generalities[2],

---

1 See *Spór o uniwersalia a nauka współczesna*, (ed.) M. Heller, W. Skoczny, J. Życiński, Cracow, 1991

2 Plato, architect of theory of ideas – forms existing beyond time and space, saw in generalities the elements of the real world, the only object of knowledge. Some modern partisans of the existence of generalities consider them the ideas forming the 'field of potentiality' which delimits the range of possible beings and which reality is disclosed in formulation of the laws of nature. The role of this 'field of potentiality' is especially underlined, by discoveries related to heredity, gravitation effect, physical fields or artificial intelligence. See J. Życiński, *Poza granicami konkretu. Spór o powszechniki w kontekście rozwoju nauki nowożytnej*, in: *Spór o uniwersalia a nauka współczesna*, p. 56–57.

ISBN 83-7431-059-6 ISSN 0860-150X

conceptualists accepting the generality of concepts (and their equivalents – general appellations) as the abstraction of features vested in various objects and existing in the mind[3], and also nominalists, who prove that general appellations relate only to substantial objects that they indicate. In the argument from a dichotomic point of view the conceptualism is treated as a reasonable form of nominalism[4].

Both Ockham and Hobbes were declared antirealists and this is what determined their respective philosophies. The aim of this report is to present the nominalist doctrine in versions proposed by Ockham and Hobbes as well as to make a thorough study of direct implications of nominalist assumptions in the sphere of socio-political phenomena and to point out the similarity in philosophical decisions accepted as consequences of 'thinking in accordance to nominalism'.

Logic occupies a specific place in Ockham's hierarchy of sciences and it is not understood the way Aristotle did, as a tool helpful in the process of gaining knowledge. Complying with scholastic method of complex and scrupulous description of a given domain, Ockham included his views on logic in *Summa logicae*. In the introduction to *Summa...*, relating to Aristotle and Boecius (what he does repeatedly), he underlines the role of terms – signs that designate objects (for this reason his logic is called 'terminative logic'[5]), he also draws an important differentiation between term and concept. *'Now certain differences are found among these (kinds of) terms. One is that a concept or passion of the soul signifies naturally whatever it signifies. But a spoken or written term signifies nothing except according to arbitrary institution. From this there follows another difference, namely that a spoken or written term can change its significate at (the user's) will, but a conceived term does not change its significate for anyone's will.'*[6]

Hence concepts are natural signs, whereas terms (in a more narrow meaning) are arbitrarily established conventional signs. Concepts are in Ockham's opinion common to all people, the same thing produces the same concept in the mind of every human; so it is a sense, a logical meaning of a conventional sign. Concepts are created as a result of a natural influence of objects on the mind and the only 'place of their existence' are minds[7].

[3] See *Mały słownik terminów i pojęć filozoficznych*, Warsaw 1983, p. 183.

[4] See T. Kotarbiński, *Wykłady z dziejów logiki*, Warsaw 1985, pp. 52, 55.

[5] A. Heinz, *Dzieje językoznawstwa w zarysie*, Warsaw 1978, p. 81.

[6] W. Ockham, *Summa logicae*, displayed at: http://pvspade.com/Logic/docs/ockham.pdf

[7] Ibid.

The concepts by nature cannot be equivocal, only conventional sings can be, and they are if they relate to several concepts[8].

A term can occur in various suppositions – this feature is observable when a term is entangled in a sentence[9]. Ockham distinguishes personal, ordinary and material suppositions. An important achievement of Ockham's logic is interrelated with the theory of suppositions – indication of the levels of language. He observes that among signs there are 'terms of first intension', which relate to objects and which are elements of objective language and terms that relate to signs, namely, to the 'terms of second intention'[10]. These terms are employed in logic, definitions in logic are metalinguistic definitions. Scientific knowledge, unlike logic refers to objects[11].

Ockham underlines that relations between the denotations of names do not have to be connected with relations characterizing the structure of a given object[12], there is no parallelism between the linguistic form and reality – such a statement is a consequence of his antirealism.

Observation is the necessary condition of knowledge of the surrounding reality, without it even the most advanced logical research is solely a vain speculation. Observation of the world is the one of singular objects, common objects according to Ockham do not exist. (Names are singular too – a name becomes a sign for various objects through convention). Specific objects are identified in an intuitional experience, in other words, experimental, or perceptive[13]. Only terms and concepts, that denominate other terms and concepts have a universal character[14]. Generalities – are terms only, they are signs common to several objects, brought into being by creative intellectual effort and nominalism (in general) is conceptualism in Ockham's version.

Disciplines of knowledge such as mathematics or physics, basing on abstract cognition need general concepts generated by minds. The knowledge develops thanks to general terms[15] and – according to Stanisław Kamiński, researcher of Ockham's philosophy – absolutely definite knowledge is the analytical one, and one achieved through experiment is deprived of the

[8] Ibid.

[9] Ibid.

[10] Ibid.

[11] W. Ockham, *Wstęp do wykładu VIII ksiąg Fizyki Arystotelesa*, translated by R. Palacz, in: Idem, *Ockham*, Warsaw 1982, p. 263.

[12] See R. Palacz, *Ockham*, op. cit., p. 113.

[13] S. Kamiński, *Ockhama koncepcja wiedzy przyrodniczej*, (in:) *Metoda i język. Studia z semiotyki i metodologii nauk*, Lublin 1994, pp. 259–261.

[14] W. Ockham, *Summa logice*, op. cit.

[15] W. Ockham, *Wstęp do wykładu VIII ksiąg Fizyki Arystotelesa*, op. cit., pp. 261–263.

attribute of objective certainty – it is a probable knowledge, even though subjectively sure[16].

To sum up, Ockham's logic by employing general concepts combined into sequences of correct syllogistic proofs shows how to achieve knowledge which occurs on the conceptual path and is the synthesis of syllogistic inference and the experience of the senses[17]. Ockham especially concentrates on semantic functions performed by a term and on the ways of rational presentation of objects, he is interested in relation combining objects and minds.

Ockham's logical views, it seems, became the foundation of his philosophy. He postulated care in formulating new concepts, in accordance with the professed methodological principle, which went down to posterity known as 'Ockham's razor'. The ground for creation of concepts is delimited by the reality – and the reality only. Ockham argued, for instance with the differentiation of being and existence done by Thomas of Aquinas – in his opinion being of every object is identical with its existence. If Thomas's point of view was true, creation of existence without the being – for example creation of an angel deprived of its angelic nature or of a being without existence – would lie in God's hands[18].

Accepted methodological assumptions and consistent antirealism constituted, apart from logical discipline of not frequent occurrence, Ockham's antlers in the struggle with numerous hipostases in scholastic metaphysics. It should be highlighted that the philosopher recognized the contest with the hipostases the aim of his actions on all polemical fronts. Treating abstract as real, existing objects is in his opinion, the cause of many errors in science.

Ockham reproaches unaquaintance with logic to those who took the wrong path on their way to knowledge. He is strikingly consistent when defending the primacy of laws of reason, identified with the laws of logic, over the whole of humanity. This consistency often leads to the situation unparalleled in the middle ages – the negation of authority[19]. Ockham writes: *I believe, that it is a very dangerous and daring to put in irons any mind and to force anybody to accept what his own mind recognizes as false...*[20] Of course, the above declaration is secured by a condition – what the Bible

[16] S. Kamiński, *Ockhama koncepcja wiedzy przyrodniczej*, op. cit., p. 260.

[17] Ibid., p. 261.

[18] See R. Palacz, *Ockham*, op. cit., p. 128.

[19] See Ibid., p. 74.

[20] W. Ockham, *De Corpore Christi*, I, 6, quoted by R. Palacz, *Ockham*, op. cit., p. 74.

says and what results from the statements of the Church or from the declarations of 'eminent doctors' should be accepted. Limitations concerning the Bible result certainly from Ockham's orthodoxy, quoting 'eminent doctors' is as it seems the effect of his prudence – yet he did not hesitate, away from Avignon, and under the protection of the Emperor, to make interpellations regarding recognizing heretical of certain theses of the Pope John XXII. He did not spare other authorities, responsible, in his opinion, for distortion of the doctrine of the Church.

The consequence of accepting the experience as a necessary element of the process leading to knowledge is the exclusion of theology from the domains accessible to cognizance. Revealed truths could be objects of faith solely. Observation of accidental facts permits the formulation of certain regularities occurring in the world, they are however not absolute – this thesis harmonizes with Ockham's antirealism. The God is not limited by already created ideas, He is totally free and unrestricted by any rules, He is therefore omnipotent and what seems to be unchangeable law of nature, might be transformed freely by God and changed or cancelled at any time. The fact that morality – established by God's unlimited will – is compulsory, is the result of this establishment and not of a whatever necessity. God's command is compulsory for it is His command, and not because of what it proclaims, it is good autonomously – such a statement is the correlate of Ockham's nominalistic assumptions[21].

The conception of the law – the command the essence of which is the fact that it is compulsory, that is, it has legal force, will be the foundation of legal positivism hundreds of years later. It seems there is some anticipation of this doctrine in the work of Ockham (remembering that his considerations concern unlike positivist research the God's law).

Ockham's antirealism is also noticeable in his attitude towards the matter of infallibility of the pope – if generalities do not exist, the pope cannot be the symbol of the Church as a whole, and his interpretation of the Bible cannot be recognized as the only one possible. The philosopher propagating the priority of a particular being, an entity, over what is by nature general, pronounced univocally for the superiority of conciliar decisions, or of a aggregation of individuals over the decisions of the pope – being the embodiment of the Church[22].

The postulate of the separation of theology from the scientific knowledge was in keeping with another one proclaiming the separation of secular autho-

[21] G. L. Seidler, *Myśl polityczna średniowiecza*, Cracow 1961, pp. 334–335.

[22] See Z. Kuksewicz, *Zarys filozofii średniowiecznej*, Warsaw 1973, pp. 470–473.

rity from the one of the church. Ockham in conflict both with John XXII and his successors Benedict XII and Clemens VI and hiding from the judgment at the court of Louis the Bavarian[23], was a declared advocate of the autonomy of imperial authority. He presents his attitude in several works of political character, the most important one, written between 1338–1342 is entitled *'Dialogus inter magistrum et discipulum de imperatorum et pontificum potestate'*. One of the main reasons for his political conceptions is conviction about the human free will, the conviction having its source in several observations – man, as an intelligent being, is able to make decisions freely[24]. The emperor's authority is not derivative of papal authority, it is the result of the will of the nation understood as the sum of individuals (at the same time the will of the nation can be expressed by the will of the electors). The theory advocated by Ockham can be explained according to the spirit of nominalism: individuals agree on the authority of the sovereign, who thereafter acts as their representative. The fact of agreeing is essential here, for it is the realization of natural right to choose a ruler. The election itself and succession of authority is a question of convention, that is to say of national law[25].

Ockham a fourteenth century theologian, formed on scholastic writings had philosophical views which seem to be surprisingly modern. Many of the trends of his thoughts, having their origins in his philosophy of language, are to revive in 17th century. That is why, the thesis that 14th century was the first decisive phase of scientific revolution, which rise occurs three centuries later, is probably not unfounded. Researchers, who are the partisans of the above thesis, highlight a startlingly sudden and stormy character of seventeenth century changes in science and their firm direction[26]. It is possible that this first revolutionary wave did not gain sufficient durability to produce a new paradigm, only because of the external limitations – the relations

[23] See G. L. Seidler, *Myśl polityczna średniowiecza*, op. cit., pp. 336–337.

[24] Ibid., p. 339.

[25] The sovereign's authority, to whom state laws are subordinated is according to Ockham one of the necessary elements of a state, and this subordination of the laws to an arbitrary will is secured by one condition; it cannot be contradictory to natural right or to the good of the citizens. In case of illegitimate violation of freedom of subordinates, they have the right to overthrow the tyrant. Among the functions of the state the philosopher enumerates legislation and preservation of justice. Execution of these is the duty of the sovereign. In the light of Ockham's ontological assumptions crucial is the fact that even though authority and property belong to the laws of nature, God's laws – the realization of these laws is the result of a substantial, positive law formed by a sovereign. See G. L. Seidler, *Myśl polityczna średniowiecza*, op. cit., pp. 336–337.

[26] See S. Kamiński, *Ockhama koncepcja wiedzy przyrodniczej*, op. cit., p. 225.

between science, philosophy and theology and the lack of print[27]. When external limitations binding the development of science came to an end, the second phase of scientific revolution exploded, transforming the thinking about the world. The opinion about its two phases is supported by an unusual convergence of certain trends of Ockham's thoughts and the philosophy of Thomas Hobbes.

It appears that the views of the two philosophers were shaped to a significant extent by antirealist attitude. They were both convinced that paying special attention to language, its structure, and semantics could be helpful in revision of hitherto philosophical methods (scientific cognizance).

Hobbes much the same as Ockham defines constituent parts of a language – language is composed of artificial signs – names, it is a creation of convention, and the diversity of national languages is the best proof[28]. The birth of a national language is preceded by a state in which everybody creates their own, private language, free from ambiguity, but inaccessible to others. This inaccessibility, and impossibility of transmission of knowledge, drives people – probably through convention – to accept signs common to several of them namely 'indications' – in Hobbes terminology[29]. The feature of the first rank of names is the fact that they awaken in minds a thought similar to bygone thought and allow to reason. A derivative function and equally important is that names arranged in a sentence become the indications, that is signs legible to all members of a given society[30].

The names are in Hobbes opinion signs (indications) of concepts – thoughts about a given object are not the signs of objects[31] (sense, the meaning of a name is according to Hobbes a subjectively understood concept). The philosopher draws the following conclusion: '*...notorious dispute as to whether names signify matter, form, or a compound of both, and other such disputes of the metaphysicians, are disputes of muddled thinkers who do not even understand the words they are arguing about*'[32].

The goals of Hobbes and Ockham are convergent when it comes to the struggle with 'with scholastics' jargon', Hobbes fights with unauthorized – in his opinion – misuse or abuse of words with a particular passion; in philosophy which leads to knowledge there is no room for metaphorical expressions

[27] Ibid., p. 265.

[28] T. Hobbes, *De corpore*, displayed at: http://www.philosophy.leeds.ac.uk/GMR/hmp/texts/modern/hobbes/decorpore/decorp1.html

[29] Ibid.

[30] Ibid.

[31] Ibid.

[32] Ibid.

and considerations on the nature of imaginary beings. Such words as *'hypostatical'*, *'transsubstantiate'*, *'consubstantiate'*, *'eternal-now'*[33] or *'immaterial substance'*[34] were absurd for him – *'...words whereby we conceive nothing but the sound are those we call absurd, insignificant, and nonsense'*[35]. Hobbes postulated a clear and lucid language believing that it is genetically interconnected with an order of thoughts. Liberation of language from the ballast of ambiguous metaphors will bring profits not only to science but it will contribute to establishment of harmony in socio-political sphere.

Not all names are the names of objects[36] – even such words as 'man', 'tree', 'stone' relating to objects could indicate fiction of the objects in dreams, in language there are also names evidently related to fiction – e.g. 'nothing', 'less than nothing' or 'what is improbable'. In connection with the fact that *'name is related to something named'* Hobbes proposes to recognize what is named as an object for cohesion of theoretical considerations[37]. Being close to indicating the differentiation of subjective language and metalanguage (this differentiation was introduced and justified by Ockham three centuries before) he is ready to recognize that words are objects[38] too. And even though he writes about the names of the primary and secondary intentions, the reasons for the introduced division are not clear to him – he supposes only, that the first ones are connected to everyday life while the other ones relate to knowledge[39].

The names of the names, that is to say 'the names of secondary intention' are generalities, general names, common to several things – they are distinguished by the fact that they do not indicate objects existing in nature, ideas or images of the imagination. The philosopher maintains that there are no general objects, this feature is vested only in names[40]. Concepts corresponding to these names are images of substantial, singular objects in the mind[41] – this is how Hobbes nominalistic credo sounds.

33 T. Hobbes, Leviathan, displayed at: http://socserv.mcmaster.ca/econ/ugcm/3ll3/hobbes/Leviathan.pdf

34 Ibid.

35 Ibid.

36 It concerns the denotation of a name, its designates. Hobbes not consistently enough differentiated name and its meaning.

37 T. Hobbes, *De corpore*, displayed at: http://www.philosophy.leeds.ac.uk/GMR/hmp/texts/modern/hobbes/decorpore/decorp1.html

38 Ibid.

39 See S. Kamiński, *Hobbesa teoria definicji*, op. cit., p. 35–36.

40 T. Hobbes, *De corpore*, displayed at: http://www.philosophy.leeds.ac.uk/GMR/hmp/texts/modern/hobbes/decorpore/decorp1.html

41 Ibid.

Explorers of Hobbes thoughts do not agree on the estimation of his nominalism – some write about radical and firm nominalism[42], quoting the significant Hobbes' statement: *'...there being nothing in world universal but names; for the things named are every one of them individual and singular.'*[43] Others demonstrate that in spite of the above declaration he was a temperate nominalist[44]; he notices that common names are employed in relation to many objects with regard to similarity bringing them together[45]. In the opinion of some, similarity understood that way – as a relation permitting to determine the denotation of a given name – allows to maintain the thesis on his limited or temperate nominalism.

Although any object in nature corresponds to general names – they remain important, since they allow to think and reason without the need to identify the object every time[46]. Reasoning is particularly understood by Hobbes – it is the execution of arithmetic rules on names[47] (his manual of logic had a symptomatic title *'Computatio sive logica'*). General names express human knowledge of universal character.

As Ockham, Hobbes makes the experience of the senses the necessary though not sufficient condition of cognizance. In his view: *'there is no conception in a man's mind which hath not at first, totally or by parts, been begotten upon the organs of sense'*[48]. Fundamental knowledge on the external world in accordance with mechanistic view of the philosopher is based on images provoked by external objects in people's minds. And even though a real philosophical knowledge is the result of reasoning, that is operation on general names, the sensorial data remains a starting point.

According to Hobbes the appreciation of deductive method coexists with the above empiristic trend. He devotes a lot of space to the deductive method in his considerations, convinced that it presents a certain method of collecting and processing the knowledge. Euclid's method of explaining the terms should apply in all domains of science, in the study of state as well. Rejecting the Aristotelian conception of definition understood as discovering the essence of defined matter, Hobbes admitted definition to

42 See e.g. B. Russell, *A History of Western Philosophy*, Warsaw 2000, p. 632, R. Tokarczyk, *Hobbes*, Warsaw 1987, pp. 72–73.

43 T. Hobbes, *Leviathan*, op. cit.

44 See K. Lee, *The Legal – Rational State*, Avebury 1993, pp. 15–20, J. W. N. Watkins, *Hobbes' system of ideas*, London 1965, pp. 147–149.

45 T. Hobbes, *Leviathan*, op. cit.

46 R. Tokarczyk, *Hobbes*, op. cit., p. 69.

47 See T. Hobbes, *De corpore*, op. cit. and T. Hobbes, *Leviathan*, op. cit.

48 T. Hobbes, *Leviathan*, op. cit.

be a manipulation executed on the language, and concerning names[49]. The term 'definition' is for Hobbes a designation of the sense of words. Hobbes has a particular attitude towards the problem of the existence of the defined object; as mentioned above, he was inclined to treat every designate of a name as an object, as if the creation of a definition was connected with existence of a defined object[50]. In connection with such formulation of the problem a question about the relation of scientific statements and the real world arises, the question to which Hobbes does not answer univocally.

Defining is also present in Hobbes' state science; moral categories such as right and wrong depend on the will of legislator who ought to define them in the prime of life of the state – Hobbes repeats Ockham's thought here as well, yet in his system the place of the God – legislator is taken by sovereign – legislator.

The state is according to Hobbes a construct of a man, law is also a convention understood as the ruler's order. Though the philosopher assumes that a sovereign acting in accordance to the instructions of the reason will incorporate the laws of nature within the legal order still sovereign's will has a decisive meaning in the process of constitution of laws. Hobbes cognitive nominalistic universalism, inherent in his state science, assumes the existence of a great number of 'correct' legal orders.

Ockham's attitude in the argument over the generalities described above and (only outlined here) the implications of his nominalistic ontology allow to claim, that he anticipated essential trends of Hobbes' thought. And even though versions of Ockham's and Hobbes' antirealism differ (Ockham recognizes the existence of general concepts, Hobbes recognizes only the universality of names) its consequences are similar when it comes to the opinion on the nature of scientific knowledge, the essence of law and morality.

Translated by Marta Głowacka

---

49 See T. Hobbes, *De corpore*, op. cit.

50 S. Kamiński, *Hobbesa teoria definicji*, op. cit., p. 36.

STUDIES IN LOGIC, GRAMMAR AND RHETORIC 8 (21) 2005

Dariusz Surowik
Białystok University

ON THE COMPUTATIONAL POWER OF SOME MODELS OF COMPUTATION

Introduction

In the 1930's mathematicians began to think about: What it means to be able to compute a function? It was the time, when a computability theory started. We may ask the next question: What is a computation? A simple answer would be as follows: a computation involves the mapping of a set of numbers to another set of numbers. But computation involves more. Namely, computation involves the use of finite procedures or algorithms to generate number mappings.

A computation model is an entity that is capable of carrying out computations. One famous example of a computation model is the Turing Machine.

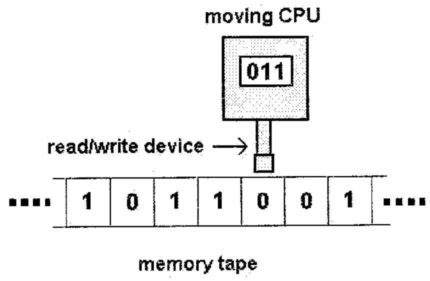
Turing machines

A Turing Machine (TM) is a mathematical model for a computing device. A TM has a potentially infinite tape to hold the input data and to store the results. The tape is divided into cells. Any cell may contain a symbol from a finite alphabet. The TM has also a read/write head which moves along the tape and reads one symbol and replaces it by another. There is some restriction, namely: there can be only finitely many non-blank cells on the tape.

This behavior of the machine is completely determined by three parameters:

1. the state the machine is in,
2. the number on the cell it is scanning, and
3. a table of instructions.

ISBN 83-7431-059-6 ISSN 0860-150X



The table of instructions specifies, for each state and each binary input, what the machine should write, which direction it should move in, and which state it should go into. The table can list only finitely many states, each of which becomes implicitly defined by the role it plays in the table of instructions.

Although a Turing Machine is only a mental construction, but any given Turing Machine can be realized or implemented on a different physical computing devices.

Formal definition of deterministic Turing Machine is as follows:

Definition

A *Turing Machine* $M = (Q, \delta, q_0, F)$, where:

- Q is a finite set of *states*,
- q_0 is a *start state*
- F is a set of *accepting states* and
- δ is a function such that: $\delta : Q \times X \to Q \times X \times \{L, R\}$, where the *alphabet* X contains a symbol B ("blank").

Definition

Let M be a class of machines. The class of functions computed by M is the set $\{\phi^n_{\mathcal{M}} \in M \text{ and } n \in \mathbb{N}\}$.

The class of functions computed by Turing Machines is the class of partial recursive functions of Kleene[1].

Variants of Turing machine

Nondeterministic Turing Machines

A Nondeterministic Turing Machine (NTM) has a finite number of choices of next move (state, new symbol, and head move) for each state and symbol scanned, i.e., its transition function

$$\delta(q, X) = \{(q_1, Y_1, D_1), (q_2, Y_2, D_2), \ldots, (q_k, Y_k, D_k)\}.$$

[1] Full discussion on this class of functions you may see in Marvin L. Minsky. *Computation: Finite and Infinite Machines.* Prentice-Hall, 1967, chapter 10.

The following theorem holds:

Theorem

Every non-deterministic TM has an equivalent deterministic TM.

The above theorem says, that deterministic Turing Machines are just as powerful as non-deterministic ones, and so they accept the same languages.

Multi-tape Turing Machine

We can to consider a multi-tape Turing Machine. The transition function of this machine is as follows:

$\delta : Q \times \Gamma^k \to Q \times \Gamma^k \times \{L, R\}^k$, k: *the number of types*

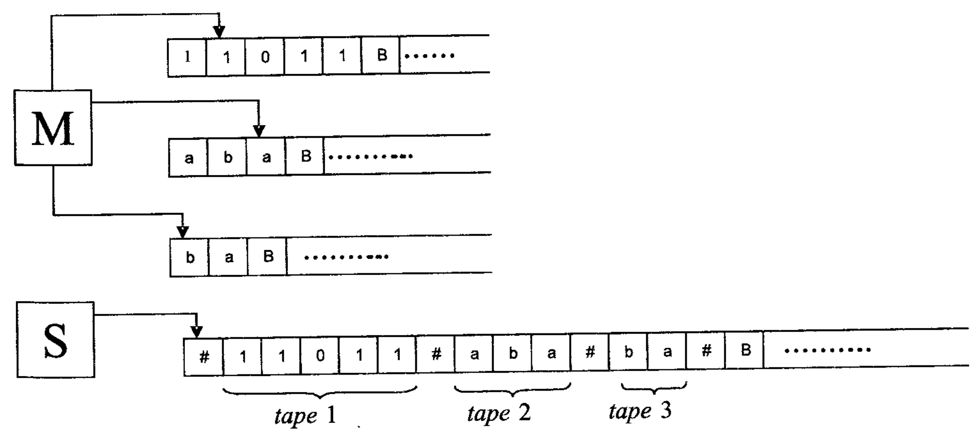
$\delta(q_1, a_1, \ldots, a_k) = (q_j, b_1, \ldots, b_k, L, R, L, \ldots, L)$.

More tapes in the machine not increase of computational power of the machine.

Theorem

Every multi-tape Turing Machine has an equivalent single tape Turing Machine[2].

We can emulate a multi-tape Turing Machine with single-tape Turing Machine.



Universal Turing Machine

For the computation of each function, it is necessary to construct a separate Turing Machine. We can build a Turing Machine that can do the

[2] Equivalent only in the sense that the classes of computable functions are the same. From the point of view of complexity classes, these models are not equivalent.

job of any other Turing Machine. This is known as a *Universal Turing Machine*[3] (UTM). The UTM interprets the input symbols on the tape as the program. Every conventional computer is logically (not physically) equivalent to a UTM[4].

Parallel Turing Machines

Parallel Turing machines (PTM) can be viewed as a generalization of cellular automata (CA) where an additional measure called processor complexity can be defined which indicates the "amount of parallelism" used.

The definition of PTM is as follows:

Definition[5]

A *Parallel Turing Machine* consists of a usual one-dimensional Turing tape, on which a number of finite automata are working. It is characterized by an 8-tuple $P = (Q, q_0, F_+, F_-, B, A, \Box, \delta)$. Q is the set of states and contains an initial state q_0. The disjoint subsets F_+ and F_- of Q contain the accepting respectively rejecting final states. It is required that $q_0 \notin F_+ \cup F_-$. B is the tape alphabet containing at least the blank symbol $\Box$ and the symbols of the input alphabet A.

A configuration of a PTM $P = (Q, q_0, F_+, F_-, B, A, \Box, \delta)$ is a pair $c = (p, b)$ of mappings $p : \mathbb{Z} \to 2^{\mathbb{Q}}$ and $b : \mathbb{Z} \to B$ where $p(i)$ is the set of states of the finite automata currently visiting square i and $b(i)$ is the symbol written on it.

Each step of a PTM, i.e. the transition from a configuration c to its successor configuration $c' = (p', b')$ is determined by the transition function $\delta : 2^Q \times B \to 2^{Q \times D} \times B$ where D is the set $\{-1, 0, 1\}$ of possible movements of a finite automaton. In order to compute c', δ is simultaneously applied at all tape positions $i \in \mathbb{Z}$. The arguments used are the set of states of the finite automata currently visiting square i and its tape symbol. Let $(M'_i, b'_i) := \delta(p(i), b(i))$. Then the new symbol on square i in configuration c' is $b'(i) := b'_i$. The set of finite automata on square i is replaced by a new set of finite automata (defined by $M'_i \subseteq Q \times D$) each of which has to change the tape square according to the indicated direction of movement, i.e., $p'(i) := \{q | (q, 1) \in M'_{i-1} \vee (q, 0) \in M'_i \vee (q, -1) \in M'_{i+1}\}$.

[3] Wolfram describes a Turing Machine with 2 states and 5 symbols per cell, currently the *smallest known Universal Turing Machine*.

[4] All computer instruction sets, high level languages and computer architectures, including multi-processor parallel computers, can be shown to be UTM-equivalent.

[5] Thomas Worsch, *Parallel Turing Machines With One-Head Control Units And Cellular Automata*.

Thomas Worsch defines for total functions s, t and h from $\mathbb{N}_+$ into $\mathbb{N}_-$, the complexity class $PTM - STP(s, t, h)$ as the family of all languages L for which there is a PTM recognizing L and satisfying, for all $n \in \mathbb{N}_+$, Space$p(n) \leq s(n)$, Time$p(n) \leq t(n)$, and Proc$p(n) \leq h(n)$. He also uses $PTM - ST(s, t)$, and so on. Furthermore he writes $PTM - T(O(t))$ instead of $\bigcup_{t' \in O(t)} PTM - T(t')$ and so on.

There is a close relation between Parallel Turing Machines and Cellular Automata. The following theorem holds:

Theorem[6]

For all functions $s(n) \geq$, $t(n) \geq n$ and $h(n) \geq 1$ where h is fully PTM processor constructible in space s, time t, and with h processors, holds:

- $PTM - STP(O(s), O(t), O(h)) \subseteq CA - ST(O(s), O(t))$
- $CA - ST(O(s), O(t)) \subseteq PTM - STP(O(s), O(st/h), O(h))$
- $PTM - STP(O(s), O(t), O(s)) = CA - ST(O(s), O(t))$

The relations between computational power of Turing Machines and Parallel Turing Machines are explained by the following theorem:

Theorem

For all functions $s(n) \geq n$, $t(n) \geq n$ holds

- $TM - ST(O(s), O(t)) \subseteq PTM - ST(O(s), O(t), O(s))$
- $PTM - ST(O(s), O(t), O(h)) \subseteq TM - ST(O(s), O(t\sqrt{h}))$

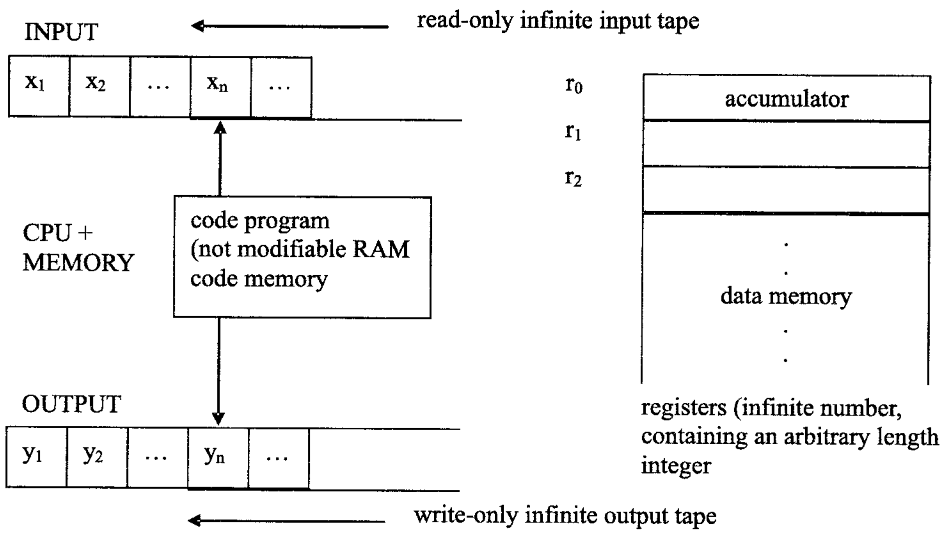
It means that computational power of Turing Machines and Parallel Turing Machines is the same, but Parallel Turing Machines are faster than Turing Machines.

Random access machines[7] (RAMs)

A *Random Access Machine* is an idealized computer with a random access memory consisting of a finite number of idealized registers (i.e., they can hold any sized number) $R_1, R_2, \ldots$ whose contents are strings over some alphabet $\sum_k$ and which has a finite set of machine instructions. The scheme of such machine is as follows:

[6] Thomas Worsch, *Parallel Turing Machines...*

[7] AV Aho, JE Hopcropft, JD Ullman, *The Desing and Analysis of Computer Algorithms*, addison-Wesley Publishing Copmpany, London, 1974.



One of a possible (equivalent) set of instructions (optimally labeled):

1. LOAD operand	; loads to accumulator contents of a register
2. STORE operand	; store accumulator to one of the registers
3. ADD operand	; integer addition: AC + op, result $\rightarrow$ AC
4. SUB operand	; subtraction: AC – op, result in AC
5. MULT operand	; multiplication (note, no overflow)
6. DIV operand	; integer division: AC/op
7. READ operand	; integer from input tape to a register
8. WRITE operand	; contents of a register to an output tape
9. JUMP label	; program counter set to label value
10. JGTZ label	; jump if accumulator greater than zero
11. JZERO (or JLTZ) label	; jump if zero
12. HALT	; stop execution

Other possible computer instructions can be simulated by 12 RAM instructions.

Theorem

Computational power of a RAM machine is the same as a Turing Machine.

This means that a sequential algorithm can be computed by RAM machine. Turing Machines have the same computational power as RAM machines but they have different computational time and different memory complexity.

Theorem (Aho, Hopcropft, Ullman)[8]

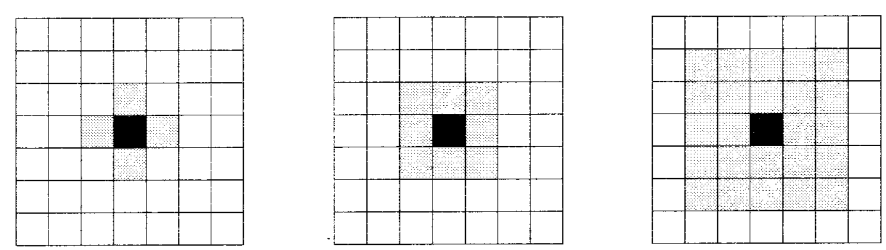
If algorithm is accepted by a RAM machine in time $T(n)$ using logarithmic cost criterion and RAM does not perform multiplications or divisions, then there exists a multi-tape Turing Machine accepting the same algorithm in time $O(T^2(n))$.

This means that although RAM machine has the same computational power as a Turing Machine, a *RAM machine is faster than a Turing Machine.*

Cellular automata

Cellular Automata were introduced by John von Neumann[9] and Stanislaw Ulam in the late 1940's. From the more practical point of view Cellular Automata was introduced in the late 1960's when John Horton Conway developed the *Game of Life*.

The basic element of a Cellular Automata is the cell. A cell is a kind of a memory element and stores states. In the simplest case, each cell can have the binary states 0 or 1 but in more complex case the cells can have more different states. These cells are arranged in a spatial web – a lattice. The simplest one is the *one dimensional* "lattice", meaning that all cells are arranged in the form of a tape. The most common CA's are built in one or two dimensions. To introduce *dynamic* into the system, there are some rules. These rules define the state of the cells for *the next time step.* In cellular automata a rule defines the state of a cell in dependence of the neighborhood of the cell. Most famous neighborhoods:



von Neumann Neighborhood — Moore Neighborhood — Extended Moore Neighborhood

[8] AV Aho, JE Hopcropft, and JD Ullman, "The Desing and Analysis of Computer Algorithms", addison-Wesley Publishing Copmpany, London, 1974.

[9] Von Neumann proved that the typical feature of living systems and their tendency to reproduce themselves, can be simulated by an automaton with 200,000 cells, if each cell has 29 possible states and the four orthogonal neighboring cells as environment. Although this idea is justified by a mathematically precise proof, it is hard to realize in technical computers.

Definition

Cellular Automata are quadruples (d, Q, N, δ),
where:

d is the *dimension* of the space the cellular automaton works on.

Q is a finite set of *states* of cells.

$N = (x_1, \ldots, x_k)$ is *neighborhood.* It is a k-tuple of distinct vectors of $\mathbb{Z}^d$. The x_i's are the relative positions of the neighbor cells with respect to the cell, whose new state is being computed. The states of these neighbors are used to compute the new state of the center cell by the *local function* of the cellular automaton $\delta : Q^k \rightarrow Q$.

Definition

A Cellular Automata simulates a Turing Machine if there is a bijection from the possible instantaneous descriptions of the Turing Machine to the possible instantaneous descriptions of the Cellular Automata, so that if the Cellular Automata is run with initial state M, for a fixed number i, the i^{th} instantaneous description of the Cellular Automata is equal to the i^{th} instantaneous description of the Turing Machine if run with the initial state M.

A Turing Machine can be simulated by Cellular Automata[10].

Theorem[11]

Any $TM[n;m]$[12] can be simulated by a k-states CA where:

1. $k = (n+1) \cdot m$
2. $k = m + 2n$
3. $k = m + n + 4$
4. $k = m + n + 2$
5. $k = \max[m;n] + 4$

Wolfram shows how Turing Machines can be built to emulate cellular automata and vice versa. Therefore, these two different machines are effectively equivalent in their computational power and he argues, many systems in nature are equivalent as well. *Because Turing machines have*

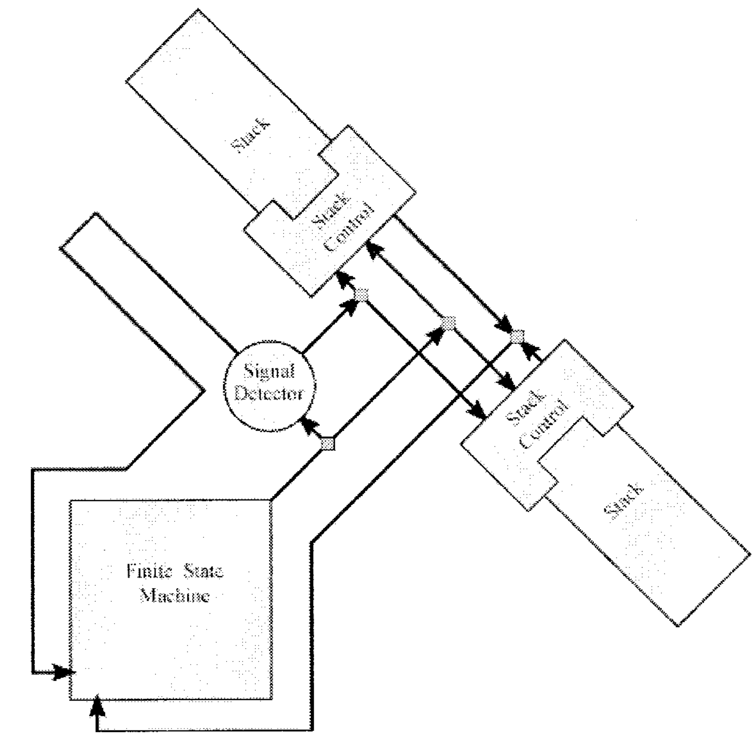
[10] Any 1-tape Turing Machine can be simulated without loss of time by an invertible partitionned cellular automaton. Dubacq Jean-Christophe, *How to simulate Turing machines by invertible one-dimensional cellular automata*, Departement de Mathematiques et d'Informatique, Ecole Normale Sup'erieure de Lyon, 1997.

[11] Claudio Baiocchi, *Some results on cellular automata.* Rend. Mat. Acc. Linceis. 9, v. 9: 307–316 (1998).

[12] A $TM[n;m]$ is a Turing Machine with n internal-states and m tape-symbols.

been shown to be universal computers capable of arbitrarily complex computations, so are universal cellular automata which emulate Turing machines[13].

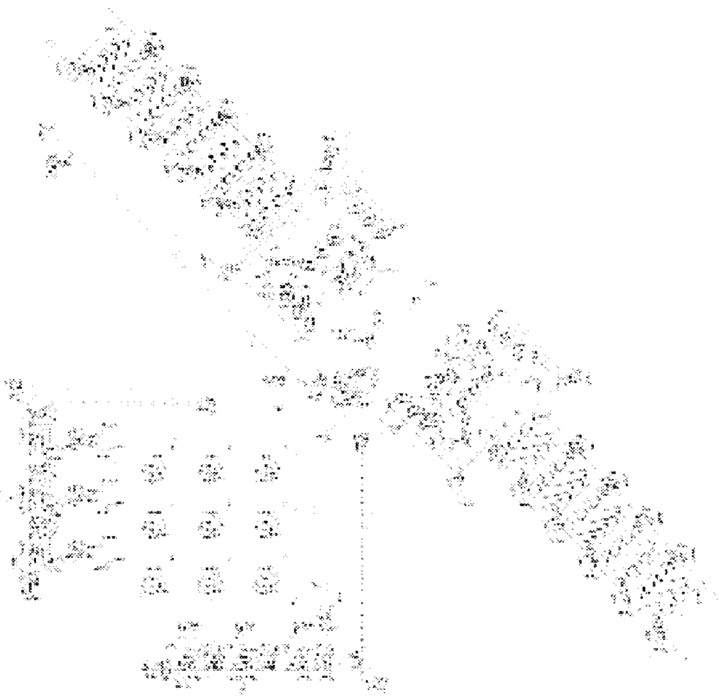
It was proved a long time ago that a Turing Machine could be simulated in The Game of Life. This proof is based on the fact, that simple logic can be performed and therefore simulation can be built.



The above figure shows a diagram of the Turing Machine. The finite state machine contains the memory unit built up of the memory cells. In each cycle of the Turing Machine the finite state machine sends its output to the signal detector and the stacks[14]. The simulation of this machine in the Game of Life is as follows:

[13] Sam Reid Virtual Machines: *Turing Machines, Cellular Automata and Java*, 2003.

[14] AA Book P. Rendell Chapter Draft 3.



Many alternative schemes for simulating Turing Machines in cellular automata have been formulated over the years. From the other hand, the following theorem has been proved:

Theorem
A cellular automaton can be simulated by a 2-tape Turing Machine.

Neural network

Artificial neural networks were proposed as a tool for machine learning. Many results have been obtained by their application to practical problems. Usually, a neural network is trained during a *supervised* training session to recognize associations between inputs and outputs. These associations are incorporated into the weights of the network, which encode a representation of the information contained in the input. Once trained, the network will compute an input/output mapping which, if the training data was representative enough.

Definition[15]
A (discrete) neural network is defined as a 6-tuple $N = (V, I, O, \Lambda, w, h)$, where:
V is a finite set of units, which we assume are indexed as $V = \{1, \ldots, p\}$,
$I \subseteq V$ and $O \subseteq V$ are the sets of input and output units, respectively,
$\Lambda \subseteq V$ is a set of initially active units, of which we require that $\Lambda \cap I = \emptyset$,
$w : V \times V \rightarrow Z$ is the edge weight matrix, and
$h : V \rightarrow Z$ is the threshold vector.
The size of a network is its number of units, $|V| = p$, and the weight of a network is defined as its sum total of edge weights, $\sum_{i,j \in V} |w_{ij}|$[16].

Given a neural network N, let us denote $|I| = n$, $|O| = m$. Moreover, let us assume that the units are indexed so that the input units appear at indices 1 to n. The network computes a partial mapping $f_N : \{0,1\}^n \rightarrow \{0,1\}^m$ as follows. Given an input x, $|x| = n$ the states s_i of the input units are initialized as $s_i = x_i$. The states of the units in set Λ are initialized to 1, and the states of the remaining units are initialized to 0. Then new states s'_i, $i = 1, \ldots, p$ are computed simultaneously for all the units according to the rule $s'_i = \text{sgn}(\sum_{i=1}^{p} w_{ij}s_j - h_i)$ where $\text{sgn}(t) = 1$ for $t \geq 0$, and $\text{sgn}(t) = 0$ for $t < 0$.

Simulation of Turing Machines by neural networks was done first by McCulloch and Pitts in 1943[17]. The result that neural networks can simulate Turing Machines[18] is well-known. The computational power increases considerably for rational weights[19] and thresholds. For instance, a "rational" recurrent net is, up to a polynomial time computation, equivalent to a Turing Machine. In particular, a network that simulates a universal Turing Machine does exist and one could refer to such a network as "universal" in the Turing sense. It is important to note that the number of nodes

---

[15] Pekka Orponen, *The Computational Power of Discrete Hopfield Nets with Hidden Units.*

[16] A Hopfield net, for example, (with hidden units) is a neural network N whose weight matrix is *symmetric.*

[17] McCulloch, W. and Pitts, W., *A logical calculus of the ideas immanent in nervous activity. Bulletin of Mathematical Biophysics*, 7: 115–133, 1943.

[18] Any computatable function can be computed on a neural network.

[19] *Irrational weights provide a further boost in computation power. If the net is allowed exponential computation time, then arbitrary Boolean functions (including non-computable functions) are recognizable. However, if only polynomial computation time isallowed, then nets have less power and recognize exactly the languagescomputable by polynomial-size Boolean circuits.* See: Bhaskar DasGupta, Georg Schnitger, *On the Computational Power of Analog NeuralNetworks*, p. 11.

in the simulating recurrent net is fixed (i.e., *does not grow* with increasing input length)[20].

The computational power of recurrent neural networks was investigated by Siegelmann and Sontag. They proved the following theorem:

Theorem (Siegelmann and Sontag 1991, 1995)[21]
A finite recurrent neural network with rational weights can compute, in real time, any function computable by a Turing Machine[22].

However, in the presence of noise, the behavior of recurrent neural network with analytic function of activation of neuron is not good. The computational power of this kind of network falls to a level below the computational power of finite automata[23].

There are some neural networks, which have very interesting properties, for example probabilistic recurrernt networks. The following theorem holds:

Theorem
Probabilistic Recurrent Networks (PRN) and Probabilistic Turing Machines (PTM) are polynomially equivalent. More specifically a PRN can be simulated by a PTM with a polynomial increase in running time and conversely a PTM of time complexity $T(n)$ can be simulated by a PRN of size at most polynomial in $T(n)$.

This is an important result. The significance of the above result lies in the fact that a PRN with polynomial number of processors can therefore learn NP language problems. whereas for example a Hopfield network with polynomial number of processors is already proved not to have such capabilities even if allowed to run for exponentially many steps and hence PRNs are more powerful.

And finally we can to write the following theorem:

20 Bhaskar DasGupta, Georg Schnitger, *On the Computational Power of Analog Neural Networks.*

21 Siegelmann and Sontag shows that if one moves from binary state to analog-state neurons, then arbitrary machines may be simulated by single, finite recurrent networks. The original construction required 1058 saturated-linear eurons to simulate a universal Turing Machine, but this has later been improved to even 25 neurons.

22 The universal network possesses at most 884 nodes. The computation time is essentially enchanged. The potential infinity of rational values plays the role of the infinite tape in the Turning Machine.

23 Wolfgang Maass, Eduardo D. Sontag. *Analog neural nets with gaussian or other common noise distribution cannot recognize arbitrary regular languages.* Neural Computation, 1998 or Siegelmann H. T., Roitershtein A. *Noisy analog neural networks and definite languages: stochastic kernels approach.* Technical Report, Technion, Haifa, Israel, 1998.

Theorem
Every self-map $T : \mathbb{Z} \to \mathbb{Z}$ realizable on a cellular automaton can be implemented by some neural network, and every neural network can be implemented by some random neural network, i.e.

$$TM \subset CA \subseteq NN \subseteq RN$$

Sketch of proof
TM can be simulated by one-dimensional cellular automaton thus $TM \subseteq CA$. The inclusion is proper, because there exist problems solvable by Celullar Automata, but not by Turing Machine. For example, one-dimensional Celullar Automata taking as an input a real number (as an infinite binary expansion) and stabilizing iff it is an integer. A Turing Machine cannot solve the problem since the integer 1 could be given as 0.999... and hence it will not even finish reading its input in finite time, i.e. $TM \subset CA$.

Every Celullar Automata can be simulated by Neural Network, i.e. $CA \subseteq NN$ because if $\delta : Q \times Q^d \to Q$, $|Q| = m$, $\delta(q_0, q_1, \ldots, q_d) = q'_0$ then we construct Neural Network using the same digraph as for CA.

Every NN can be simulated by Random Network, i.e. $NN \subseteq RN$ because NN cell (neuron) is a FSM with local transition: $\delta(x_i, x_{i_1}, \ldots, x_{i_d}) := f_i(\sum w_{i_j} x_{i_j})$.

We have to agree with a version of the Church-Turing thesis:

No one has ever invented a more powerful computing model than a Turing Machine[24].

Bibliography

[1] AV Aho, JE Hopcropft, and JD Ullman, *The Desing and Analysis of Computer Algorithms*, addison-Wesley Publishing Copmpany, London, 1974

[2] Baiocchi Claudio, *Some results on cellular automata.* Rend. Mat. Acc. Linceis. 9, v. :307–316, 1998

24 It is now known that quantum computing gives us unprecedented possibilities in solving problems beyond the abilities of classical computers. For example Shor's algorithm gives a polynomial solution (on a quantum computer) for the problem of prime factorization, which is believed to be classically intractable. There are some proponents for the idea that *Quantum Neural Networks* may be developed that have abilities beyond the restrictions imposed by the Church-Turing thesis.

[3] Bhaskar DasGupta, Georg Schnitger, *On the Computational Power of Analog NeuralNetworks*

[4] Dubacq Jean-Christophe, *How to simulate Turing machines by invertible one-dimensional cellular automata*, Departement de Mathematiques et d'Informatique, Ecole Normale Sup'erieure de Lyon, 1997

[5] Kroll Jason, *Introduction to Neural Networks and Computational Complexity*

[6] Minsky Marvin L., *Computation: Finite and Infinite Machines.* Prentice-Hall, 1967

[7] Mikel L. Forcada, *Neural Networks: Automata and Formal Models of Computation*, 2002

[8] McCulloch, W. and Pitts, W., *A logical calculus of the ideas immanent in nervous activity. Bulletin of Mathematical Biophysics*, 7:115–133, 1943

[9] Melanie Mitchell, *Computation in Cellular Automata: A Selected Review*

[10] Heikki Hyötyniemi, *Turing Machines are Recurrent Neural Networks*

[11] Orponen Pekka, *The Computational Power of Discrete Hopfield Nets with Hidden Units*

[12] Reid Sam, *Virtual Machines: Turing Machines, Cellular Automata and Java*, 2003

[13] Ricardo Joel Marques dos Santos Silva, *On the Computational Power of Sigmoidal Neural Networks*, Diploma Thesis, Universidade Tecnica de Lisboa, 2002

[14] Saharon Shelah, John T. Baldwin, *On the classifiability of Cellular Automata*

[15] Siegelmann H. T., *Neural Networks and Analog Computation: Beyond the Turing Limit*, Birkhauser publishers, 1998

[16] Siegelmann H. T. and Sontag E. D., *Analog computation, neural networks, and circuits*, Theoretical Computer Science, 131, 331–360, 1994

[17] Siegelmann H. T. and Sontag E. D., *On the Computational Power of Neural Nets*, Journal of Computer and System Sciences, 50, 132–150, 1995

[18] Siegelmann H. T., Roitershtein A., *Noisy analog neural networks and definite languages: stochastic kernels approach.* Technical Report, Technion, Haifa, Israel, 1998

[19] Wolfram, S., *Computation theory of cellular automata.* Communications in Mathematical Physics, 96, 1984

[20] Wolfram, S., *Universality and complexity in cellular automata.* Physica D, 10, 1984

[21] Wolfgang Maass, Eduardo D. Sontag. *Analog neural nets with gaussian or other common noise distribution cannot recognize arbitrary regular languages.* Neural Computation, 1998

[22] Worsch T., *Parallel Turing Machines With One-Head Control Units And Cellular Automata*

Dariusz Surowik
Department of Logic, Informatics and Philosophy of Science
Białystok University
e-mail: surowik@hum.uwb.edu.pl

Anna Zalewska
University of Białystok

INTRODUCTORY REMARKS ON INFERENCE RULES FOR ALGORITHMIC LOGIC WITH PROCEDURES

1. Algorithmic logic (AL) ([2]) supplies a set of logical axioms and inference rules appropriate for reasoning about properties of programs. The problem of AL axiomatization was completely solved for programs without procedures. A certain proof system for AL with program variables has been presented in [3]. It bases on a suitable Gentzen-type axiomatization and uses the notion of a tree of sequents as a basis tool. In this paper we consider a certain extension of the proof system in which algorithmic properties of programs with simple procedures (without procedure parameters) can be proved. We add new constructs to our algorithmic language (blocks and procedures) and appropriate new Gentzen-like rules of inference. According to [1] we treat procedure text as a text constant and declaration of procedure as an assignment of this text constant to the name of the procedure.

2. Procedures allow us to express in evident way a program structure by logical closed elements. Procedures are named sequences of instructions. The connection between the name and the sequence of instructions is expressed by the following procedure declaration:

procedure name;
declaration of local variables
begin
procedure_body
end {name}.

The declaration of procedure (without parameters) consists of two parts: the procedure header (i.e. the keyword **procedure** and the procedure identifier: name) and the procedure text (i.e. instructions between keywords **begin** and **end**). In order to indicate that the procedure instructions

ISBN 83-7431-059-6 ISSN 0860-150X

should be done in a given point of the program it is enough to write procedure identifier. Some examples of procedure declarations are presented below.

EXAMPLES.

The example of simple procedure declaration without local variables.

```
procedure exchange;
begin
 t:=x;
 x:=y;
 y:=t;
end {exchange}
```

The example of simple procedure declaration with a local variable.

```
procedure exchange;
 var t:integer;
begin
 t:=x;
 x:=y;
 y:=t;
end {exchange}
```

The example of procedure declaration with nested procedure declaration.

```
procedure exchange;
 var t:integer;
 procedure add;
  var t:integer;
 begin
  t:=x;
  x:=x+y;
  y:=x+t;
 end {add};
begin
 t:=x;
 x:=y;
 y:=t;
 add;
end {exchange}.
```


3. The algorithmic logic is an extension of the first-order logic by the expressions of the following form

$$M\alpha$$

where M is:

- *a program variable*;
- *assignment statement*: (x:=τ) or (q:=γ) where x is an individual variable, τ is a classical term, γ is an open formula and q is a propositional variable;
- *composed program*: **begin** M; M' **end**;
- *branching program*: **it** γ **then** M **else** M' **fi**;
- *iteration program*: **while** γ **do** M **od** where γ is an open formula and M and M' are programs;
- *block*: **beginblock** D; I_1; ... I_n **endblock** where D is a declaration of local variables and procedures without parameters and $I_1; \ldots I_n$ are instructions.

Some examples of blocks are presented below.

EXAMPLES.

The example of simple block with a procedure declaration without local variables.

```
beginblock
 procedure exchange; begin t:=x; x:=y; y:=t; end {exchange};
 exchange;
endblock;
```

The example of block with a procedure declaration (with a local variable).

```
beginblock
 procedure exchange; var t:integer;
 begin t:=x; x:=y; y:=t; end {exchange}
 exchange;
endblock;
```

The example of block with procedure declaration (with nested procedure declaration).

```
beginblock
 procedure exchange; var t:integer;
  procedure add; var t:integer; begin t:=x; x:=x+y; y:=x+t; end {add};
 begin t:=x; x:=y; y:=t; add; end {exchange};
 exchange;
endblock;
```

In general we are basing on the notion of realization of language and valuation of variables given in [1, 3]. Programs are interpreted as partial functions. The informal meaning of the formula M is "*after execution of the program M the formula α holds*".

4. We are basing on the Gentzen-like system given in [3]. In the system each of the decomposition rules describes relation between its conclusion (written over a line) and its premise or premises (written under the line):

$$\frac{\text{conclusion}}{\text{premise}_1;\ \text{premise}_2;\ \ldots\ \text{premise}_n}$$

The set of inference rules is extended by the following schemes;

I) The scheme of the inference rule for simple block with a procedure declaration without local variables.

$$\frac{\begin{array}{l}\{\Gamma, s\ \textbf{beginblock}\\ \quad \textbf{procedure}\ \text{name\_p}_1;\ \textbf{begin}\ \text{body\_p}_1\ \textbf{end}\ \{\text{name\_p}_1\};\\ \quad \ldots\ldots\ldots\\ \quad \textbf{procedure}\ \text{name\_p}_n;\ \textbf{begin}\ \text{body\_p}_n\ \textbf{end}\ \{\text{name\_p}_n\};\\ \quad I_1;\ \text{name\_p}_1;\ \ldots\ I_n;\ \text{name\_p}_n;\ I_{n+1};\\ \quad \textbf{endblock}\ \alpha, \Delta\}\end{array}}{\{\Gamma, s\ \textbf{begin}\ I_1;\ \text{body\_p}_1;\ \ldots\ I_n;\ \text{body\_p}_n;\ I_{n+1};\ \textbf{end};\ \alpha, \Delta\}}$$

II) The scheme of the inference rule for block with a procedure declaration (with a local variable).

$$\frac{\begin{array}{l}\{\Gamma, s\ \textbf{beginblock}\\ \quad \textit{declaration of local variables}\ x_1, \ldots, x_m;\\ \quad \textbf{procedure}\ \text{name\_p}_1;\ \textbf{begin}\ \text{body\_p}_1\ \textbf{end}\ \{\text{name\_p}_1\};\\ \quad \ldots\ldots\ldots\\ \quad \textbf{procedure}\ \text{name\_p}_n;\ \textbf{begin}\ \text{body\_p}_n\ \textbf{end}\ \{\text{name\_p}_n\};\\ \quad I_1;\ \text{name\_p}_1;\ \ldots\ I_n;\ \text{name\_p}_n;\ I_{n+1};\\ \quad \textbf{endblock}\ \alpha, \Delta\}\end{array}}{\{\Gamma, s\ \textbf{begin}\ I_1';\ \text{body\_p}_1';\ \ldots\ I_n';\ \text{body\_p}_n';\ I_{n+1};\ \textbf{end}\ \alpha, \Delta\}}$$

where x_j' $(j = 1, \ldots, m)$ does not occur on the lefthand side of the sequent
$I_i' = I_i(x_j/x_j')$
$\text{body\_p}_i' = \text{body\_p}_i(x_j/x_j')$

III) The scheme of the inference rule for block with procedure declaration (with nested procedure declaration).

$$\frac{\begin{array}{l}\{\Gamma, s\ \textbf{beginblock}\\ \quad \textbf{procedure}\ \text{name\_p};\\ \quad\ \textit{declaration of local variables}\ x_1, \ldots, x_m;\\ \quad\ \textbf{procedure}\ \text{name\_np};\\ \quad\quad \textit{declaration of local variables}\ y_1, \ldots, y_k;\\ \quad\quad \textbf{begin}\ \text{body\_np}\ \textbf{end}\ \{\text{name\_np}\};\\ \quad\ \textbf{begin}\ IP_1;\ \text{body\_np};\ IP_2;\ \textbf{end}\ \{\text{name\_p}\};\\ \quad I_1;\ \text{name\_p};\ I_2;\ \textbf{endblock}\ \alpha, \Delta\}\end{array}}{\{\Gamma, s\ \textbf{begin}\ I_1';\ IP_1';\ \text{body\_np'};\ IP_2';\ I_2'\ \textbf{end}\ \alpha, \Delta\}}$$

where x_j' $(j = 1, \ldots, m)$ does not occur on the lefthand side of the sequent
y_j' $(j = 1, , k)$ does not occur on the lefthand side of the sequent
$\text{body\_np'} = \text{body\_np}(x_j/x_j', y_j/y_j')$
$\text{body\_p'} = \text{body\_p}(x_j/x_j')$
$IP_i' = I_i(x_j/x_j',\ , y_j/y_j')$ $(i = 1, 2)$
$I_i' = I_i(x_j/x_j')$ $(i = 1, 2)$

5. Some examples of applications of the above rules are given below.

I)

$\{(t = 0) \wedge (x = 1) \wedge (y = 2) \rightarrow$
beginblock
procedure exchange; **begin** t:=x; x:=y; y:=t; **end** {exchange};
exchange; **endblock** $((t = 1) \wedge (x = 2) \wedge (y = 1))\}$

$\{(t \neq 0), (x \neq 1), (y \neq 2),$
beginblock
procedure exchange; **begin** t:=x; x:=y; y:=t; **end** {exchange};
exchange; **endblock** $((t = 1) \wedge (x = 2) \wedge (y = 1))\}$

$\{(t \neq 0), (x \neq 1), (y \neq 2),$
begin t:=x; x:=y; y:=t **end** $((t = 1) \wedge (x = 2) \wedge (y = 1))\}$

$\{(t \neq 0), (x \neq 1), (y \neq 2),$ (t:=x)(x:=y)(y:=t) $((t = 1) \wedge (x = 2) \wedge (y = 1))\}$

$\{(t \neq 0), (x \neq 1), (y \neq 2),$ (t:=x)(x:=y) $((t = 1) \wedge (x = 2) \wedge (t = 1))\}$

$\{(t \neq 0), (x \neq 1), (y \neq 2),$ (t:=x) $((t = 1) \wedge (y = 2) \wedge (t = 1))\}$

$\{(t \neq 0), (x \neq 1), (y \neq 2), (x = 1) \wedge (y = 2) \wedge (x = 1)\}$

$\{(t \neq 0), (x \neq 1), (y \neq 2), (x = 1)\} \mid \{(t \neq 0), (x \neq 1), (y \neq 2), (y = 2)\}$
$\mid \{(t \neq 0), (x \neq 1), (y \neq 2), (x = 1)\}$

II)

$\{(t = 0) \wedge (x = 1) \wedge (y = 2) \rightarrow$
beginblock
var t: integer;
procedure exchange; **begin** t:=x; x:=y; y:=t; **end** {exchange};
exchange; **endblock** $((t = 0) \wedge (x = 2) \wedge (y = 1))\}$

$\{(t \neq 0), (x \neq 1), (y \neq 2),$
beginblock
var t: integer;
procedure exchange; **begin** t:=x; x:=y; y:=t; **end** {exchange};
exchange; **endblock** $((t = 0) \wedge (x = 2) \wedge (y = 1))\}$

$\{(t \neq 0), (x \neq 1), (y \neq 2),$
begin t':=x; x:=y; y:=t' **end** $((t = 0) \wedge (x = 2) \wedge (y = 1))\}$

$\{(t \neq 0), (x \neq 1), (y \neq 2),$ (t':=x)(x:=y)(y:=t') $((t = 0) \wedge (x = 2) \wedge (y = 1))\}$

$\{(t \neq 0), (x \neq 1), (y \neq 2),$ (t':=x)(x:=y) $((t = 0) \wedge (x = 2) \wedge (t' = 1))\}$

$\{(t \neq 0), (x \neq 1), (y \neq 2),$ ((t':=x) $((t = 0) \wedge (y = 2) \wedge (t' = 1))\}$

$\{(t \neq 0), (x \neq 1), (y \neq 2), ((t = 0) \wedge (y = 2) \wedge (x = 1)\}$

$\{(t \neq 0), (x \neq 1), (y \neq 2), (t = 0)\} \mid \{(t \neq 0), (x \neq 1), (y \neq 2), (y = 2)\}$
$\mid \{(t \neq 0), (x \neq 1), (y \neq 2), (x = 1)\}$

III)

$\{(t = 0) \wedge (x = 1) \wedge (y = 2) \rightarrow$
beginblock
procedure exchange; **var** t:integer;
procedure add; **var** t:integer;
begin t:=x; x:=x+y; y:=x+t; **end** {add};
begin t:=x; x:=y; y:=t; add; **end** {exchange};
exchange;
endblock; $((t = 0) \wedge (x = 3) \wedge (y = 5))\}$

$\{(t \neq 0), (x \neq 1), (y \neq 2),$
beginblock
procedure exchange; **var** t:integer;
procedure add; **var** t:integer;
begin t:=x; x:=x+y; y:=x+t; **end** {add};
begin t:=x; x:=y; y:=t; add; **end** {exchange};
exchange;
endblock; $((t = 0) \wedge (x = 3) \wedge (y = 5))\}$

$\{(t \neq 0), (x \neq 1), (y \neq 2),$
begin t':=x; x:=y; y:=t'; t":=x; x:=x+y; y :=x+t"
end $((t = 0) \wedge (x = 3) \wedge (y = 5))\}$

$\{(t \neq 0), (x \neq 1), (y \neq 2),$
(t':=x)(x:=y)(y:=t')(t":=x)(x:=x+y)(y:=x+t")
$((t = 0) \wedge (x = 3) \wedge (y = 5))\}$

$\{(t \neq 0), (x \neq 1), (y \neq 2),$
(t':=x)(x:=y)(y:=t')(t":=x)(x:=x+y) $((t = 0) \wedge (x = 3) \wedge (x + t'' = 5))\}$

$\{(t \neq 0), (x \neq 1), (y \neq 2),$
(t':=x)(x:=y)(y:=t')(t":=x) $((t = 0) \wedge (x + y = 3) \wedge (x + y + t'' = 5))\}$

$\{(t \neq 0), (x \neq 1), (y \neq 2),$
(t':=x)(x:=y)(y:=t') $((t = 0) \wedge (x + y = 3) \wedge (x + y + x = 5))\}$

$\{(t \neq 0), (x \neq 1), (y \neq 2),$ (t':=x)(x:=y) $((t = 0) \wedge (x + t' = 3) \wedge (x + t' + x = 5))\}$

$\{(t \neq 0), (x \neq 1), (y \neq 2),$ (t':=x) $((t = 0) \wedge (y + t' = 3) \wedge (y + t' + y = 5))\}$

$\{(t \neq 0), (x \neq 1), (y \neq 2), (t = 0) \wedge (y + x = 3) \wedge (y + x + y = 5)\}$

$\{(t \neq 0), (x \neq 1), (y \neq 2), (t = 0)\} \mid \{(t \neq 0), (x \neq 1), (y \neq 2), (y + x = 3)\}$
$\mid \{(t \neq 0), (x \neq 1), (y \neq 2), y + x + y = 5)\}$

6. In the paper we give some introductory remarks on inference rules for algorithmic logic with simple procedures. The extended in this way Gentzen-like system is sound and for propositional part of algorithmic logic is complete. The system ([3]) for the first-order algorithmic logic can be extended by inference rules for procedures with parameters called by value or variable. The declaration of such procedures c be expressed in one of the following ways:

declaration of procedures with parameters called	
by variables	**by value**
procedure name(**var** x, **var** y); *declaration of local variables* **begin** *procedure_body* **end** {name}.	**procedure** name(x,y); *declaration of local variables* **begin** *procedure_body* **end** {name}.

References

[1] Mirkowska G., *A Complete Axiomatic Characterization of Algorithmic Properties of Block-Structured Programs with Procedures*, in: it Proc. MFCS'76 (A. Mazurkiewicz ed.), LNCS 45, Springer Verlag, 602–606

[2] Mirkowska G., Salwicki A., *Algorithmic Logic*, D. Reidel Publishing Company, Dordrecht, 1987

[3] Zalewska A., *Program Verification with Algorithmic Logic*, Philomath, Warsaw 2001

Anna Zalewska
University of Białystok
Department of Computer Science
e-mail: zalewska@uwb.edu.pl

STUDIES IN LOGIC, GRAMMAR AND RHETORIC 8 (21) 2005

Robert Kublikowski
Catholic University of Lublin

CIRCULAR DEFINITION

The main aim of this paper is to construct a circular definition, which illustrates *The Revision Theory of Definition* (henceforth *RTD*). *RTD* was extensively presented in *The Revision Theory of Truth* (henceforth *RTT*) written by Anil Gupta and Nuel Belnap[1]. The concept of circular definition, given and logically justified in *RTD*, is useful for showing that some other concepts are also circular (e.g. truth, belief, rational choice). According to Gupta and Belnap, every kind of change of meaning of the predicate "true" (both ordinary and pathological, i.e. non-categorical) can be displayed by the use of circular definitions.

Let us analyse a constructed exemplification to get to know the process of revision and to understand how different parts (aspects) of the apparatus of revision work[2].

Definition 1.1.

Let $\mathfrak{L}$ $(= \langle L, M, \tau \rangle;\ M = \langle D, I \rangle)$ be an interpreted classical language with an ordered triple $\langle L, M, \tau \rangle$.

(i) L is a language with the whole syntactic information (characteristic) of the language $\mathfrak{L}$.

(ii) $\mathfrak{L}$ has a model (structure) M, that gives the interpretation of non-logical constants, i.e. M describes how denotation is assigned to predicates.

(iii) τ is a semantic scheme by which the interpretation of the logical constants is delivered.

[1] These authors teach in the Department of Philosophy at The University of Pittsburgh (USA). Professor Gupta was invited as a distinguished guest to a conference *Applications of Logic in Philosophy and Foundations of Mathematics* (Poland, Karpacz 2000), where he presented three lectures: *Definition, Truth and Rational Choice.*

[2] In my exemplification I use a typical, standard notation of mathematical logic and set theory. A certain notation (e.g. the notation of the rule of revision) is taken from *RTT* (see also Gupta 2001, pp. 102–103).

ISBN 83-7431-059-6 ISSN 0860-150X

(iv) M is an ordered pair $\langle D, I\rangle$, which consists of a non-empty domain of discourse D, to which certain subsets h belong.
(v) Subsets h of a domain D will be called *hypotheses* (e.g. objects $\mathbf{a}, \mathbf{b}, \mathbf{c}, \ldots$).
(vi) X is the subset of D (symbolically $X \subseteq D$) and X is the set being a hypothetical initial extension of *definiendum* G.
(vii) Symbols $F, G, \ldots$ represent predicates; $x, y, \ldots$ are symbols of variables, representing names; R represents a function (e.g., the function of being a parent).
(viii) The function (interpretation) I assigns an element $D^n \rightarrow D$ to each n-ary function symbol, an element $D^n \rightarrow \{\mathbf{t}, \mathbf{f}\}$ to each n-ary predicate (where $\mathbf{t}$ is a symbol of a predicate "true", and $\mathbf{f}$ is a symbol of "false").
(ix) The interpretation (extension) I of $F, G, \ldots$ is respectively represented by $I(F), I(G), \ldots$; $I(a), I(b), \ldots$ represent interpretations of names $a, b, \ldots$ (The interpretations of $a, b, \ldots$ are respectively objects $\mathbf{a}, \mathbf{b}, \ldots$).

Definition 1.2.
Let L^+ be the extended language (*syntactically* constructed) that is achieved by adding the *definienda* to L. Let $\mathfrak{L}^+$ be the extended language *semantically* constructed. Let $M + h$ be the model of a language L^+, which is the same as the model M, with an exception, that M assigns an interpretation h to a predicate G.

Definition 1.3.
Let $\mathfrak{D}$ be a set of definitions d which introduces new predicates to $\mathfrak{L}$. $\mathfrak{D}$ contains definitions d having the following scheme:

$$G(x_1, \ldots, x_n) =_{\mathrm{Df}} A(x_1, \ldots, x_n, G),$$

where $x_1, \ldots, x_n$ are variables, A is a formula of L^+ having free variables $x_1, \ldots, x_n$.

Definition 1.4.
Let $\delta^n_{D,M}$ be a rule (i.e. scheme, function) of revision for $\mathfrak{D}$ in M on the set $D \rightarrow \{\mathbf{t}, \mathbf{f}\}$ constructed by D, which meets the following condition:

$$\delta^n_{D,M}(h)(d) = \mathbf{t} \longleftrightarrow d \textit{ satisfies } A(x, G) \text{ in } M + h,$$

$$\delta^n_{D,M}(h)(d) = \mathbf{f} \longleftrightarrow \neg(d \textit{ satisfies } A(x, G) \text{ in } M + h)$$

Definition 1.5.
Let $\delta^n_{D,M}$ fix a stable (categorical) extension of *definiendum* by taking initially a hypothetical extension X and assigning X to $\delta^n_{D,M}(X)$.
This process can be displayed as follows:

$$\delta^n_{D,M}(X) = X,$$
$$\delta^{n+1}_{D,M}(X) = \delta_{D,M}(\delta^n_{D,M}(X)),$$

where n stands for the number of the stage of revision:

$$X^0 = \delta^0_{D,M}(X^0),$$
$$X^1 = \delta^1_{D,M}(X^0) = \delta^1_{D,M}(\delta^0_{D,M}(X^0)),$$
$$X^2 = \delta^2_{D,M}(X^1) = \delta^1_{D,M}(\delta^1_{D,M}(X^0)) \text{ etc.}$$

Example
The definition (a) is a definition given in a natural language as follows:
(a) Someone (let us say) x is *hereditarily intelligent*, means that x is intelligent and if x is a parent of someone else (let us say) y, then y is *hereditarily intelligent.*

Symbolising (a) we obtain:
(a*) x is $G =_{\mathrm{Df}} x$ is F and for every y (if xRy, then y is G),
where G represents the name "hereditarily intelligent", F represents the name "intelligent", R stands for "... is a parent of ...".
Finally we get a symbolic form of (a*)
(a**) $G(x) =_{\mathrm{Df}} F(x) \wedge \forall y(xRy \rightarrow G(y))$.
It is obvious that (a**) is circular.

Definition 1.6.
Let $D = A \cup \{\mathbf{b}, \mathbf{c}, \mathbf{d}\}$ be the arbitrary domain of (a**). Let the following parameters be also arbitrary:

$$A = \{\mathbf{a}_0, \mathbf{a}_1, \mathbf{a}_2, \ldots, \mathbf{a}_i, \ldots\};$$
$$I(a_i) = \{\mathbf{a}_0\},\ I(b) = \{\mathbf{b}\},\ I(c) = \{\mathbf{c}\},$$
$$I(F) = \{\mathbf{a}_0, \mathbf{b}, \mathbf{c}\},$$
$$I(R) = \{\langle x, y\rangle : (x = y = \mathbf{b}) \vee (x = \mathbf{a}_i \wedge y = \mathbf{a}_j \wedge i < j\}, \text{ so}$$
$$I(R) = \{\langle \mathbf{b}, \mathbf{b}\rangle, \langle \mathbf{a}_0, \mathbf{a}_1\rangle, \langle \mathbf{a}_1, \mathbf{a}_2\rangle, \ldots, \langle \mathbf{a}_0, \mathbf{a}_2\rangle, \ldots, \langle \mathbf{a}_1, \mathbf{a}_2\rangle, \langle \mathbf{a}_1, \mathbf{a}_3\rangle, \ldots\}.$$

We initially take a certain hypothetical value of X, e.g. $X = \emptyset$, it means $I(G) = \emptyset$ (i.e. no element has got a property G). Later we take another hypothetical value of X, e.g. $X = A$, which means that $I(G) = A$ (only elements of A belong to the set, which is the hypothetical, initial extension of *definiendum* G). Later we take $X = \{\mathbf{b}\}$, etc. We check (a**) for a certain hypothetical value of X, for all supposed names – $a_i, b, c\ldots$ (and objects – $\mathbf{a}_i, \mathbf{b}, \mathbf{c}\ldots$), and for all stages of the revision.

Theorem 1.1.

If a hypothetical, initial extension of the *definiendum* G (input) $I(G) = \emptyset$, then $X^1 = \delta^1_{D,M}(X^0) = \{\mathbf{c}\}$.

Proof

Let $X^0 = \emptyset$.

1) Initially, we check (a**) for a_0 and we get $F(a_0) \wedge \forall y(a_0 Ry \rightarrow G(y))$. Since $I(F) = \{\mathbf{a}_0, \mathbf{b}, \mathbf{c}\}$, $F(a_0) = \mathbf{t}$.

Since $I(R) = \{\langle x, y\rangle : (x = y = \mathbf{b}) \vee (x = \mathbf{a}_i \wedge y = \mathbf{a}_j \wedge i < j\}$, for $j > 0$ $a_0 Ra_j = \mathbf{t}$. If $I(X) = \emptyset$, then $G(a_j) = \mathbf{f}$. So $\forall y(a_0 Ra_j \rightarrow G(a_j)) = \mathbf{f}$. Since $F(a_0) = \mathbf{t}$, $F(a_0) \wedge \forall y(a_0 Ra_j \rightarrow G(a_j)) = \mathbf{f}$. Thus $X^0 = \emptyset \rightarrow \mathbf{a}_0 \notin \delta^1_{D,M}(X^0)$.

2) Now, we check (a**) for b. So $F(b) \wedge \forall y(bRy \rightarrow G(y))$. Since $I(F) = \{\mathbf{a}_0, \mathbf{b}, \mathbf{c}\}$, then $F(b) = \mathbf{t}$. Since $I(R) = \{\langle x, y\rangle : (x = y = \mathbf{b}) \vee (x = \mathbf{a}_i \wedge y = \mathbf{a}_j \wedge i < j\}$, $bRb = \mathbf{t}$. If $I(X) = \emptyset$, then $G(b) = \mathbf{f}$. So $\forall y(bRb \rightarrow G(b)) = \mathbf{f}$. Since $F(b) = \mathbf{t}$, $F(b) \wedge \forall y(bRb \rightarrow G(b)) = \mathbf{f}$. Thus $X^0 = \emptyset \rightarrow \mathbf{b} \notin \delta^1_{D,M}(X^0)$.

3) Now we check (a**) for c. We get $F(c) \wedge \forall y(cRy \rightarrow G(y))$. Since $I(F) = \{\mathbf{a}_0, \mathbf{b}, \mathbf{c}\}$, then $F(c) = \mathbf{t}$. Since $I(R) = \{\langle x, y\rangle : (x = y = \mathbf{b}) \vee (x = \mathbf{a}_i \wedge y = \mathbf{a}_j \wedge i < j\}$, $cRc = \mathbf{f}$. If $I(X) = \emptyset$, then $G(c) = \mathbf{f}$. So $\forall y(cRc \rightarrow G(c)) = \mathbf{t}$. Since $F(c) = \mathbf{t}$, $F(c) \wedge \forall y(cRc \rightarrow G(c)) = \mathbf{t}$. Hence $X^0 = \emptyset \rightarrow \mathbf{c} \in \delta^1_{D,M}(X^0)$. Thus $X^0 = \emptyset \rightarrow \delta^1_{D,M}(X^0) = \{\mathbf{c}\}$. ■

Theorem 1.2.

If $X^1 = \delta^1_{D,M}(X^0) = \{\mathbf{c}\}$, then $X^2 = \delta^2_{D,M}(X^1) = \{\mathbf{c}\}$.

Proof

Let $X^1 = \delta^1_{D,M}(X^0) = \{\mathbf{c}\}$.

1) We check (a**) for a_0. So, $F(a_0) \wedge \forall y(a_0 Ry \rightarrow G(y))$. Since $I(F) = \{\mathbf{a}_0, \mathbf{b}, \mathbf{c}\}$, $F(a_0) = \mathbf{t}$. Since $I(R) = \{\langle x, y\rangle : (x = y = \mathbf{b}) \vee (x = \mathbf{a}_i \wedge y = \mathbf{a}_j \wedge i < j\}$, for $j > 0$ $a_1 Ra_j = \mathbf{t}$. If $I(X) = \{\mathbf{c}\}$, then $G(a_j) = \mathbf{f}$. So $\forall y(a_0 Ra_j \rightarrow G(a_j)) = \mathbf{f}$. Since $F(a_0) = \mathbf{t}$, $F(a_0) \wedge \forall y(a_0 Ra_j \rightarrow G(a_j)) = \mathbf{f}$. Thus $X^1 = \{\mathbf{c}\} \rightarrow \mathbf{a}_0 \notin \delta^2_{D,M}(X^1)$.

2) We check (a**) for b. $F(b) \wedge \forall y(bRy \rightarrow G(y))$. Since $I(F) = \{\mathbf{a}_0, \mathbf{b}, \mathbf{c}\}$, $F(b) = \mathbf{t}$. Since $I(R) = \{\langle x, y\rangle : (x = y = \mathbf{b}) \vee (x = \mathbf{a}_i \wedge y = \mathbf{a}_j \wedge i < j\}$, $bRb = \mathbf{t}$. If $I(X) = \{\mathbf{c}\}$, then $G(b) = \mathbf{f}$. So $\forall y(bRb \rightarrow G(b)) = \mathbf{f}$. Since $F(b) = \mathbf{t}$, $F(b) \wedge \forall y(bRb \rightarrow G(b)) = \mathbf{f}$. Thus $X^1 = \{\mathbf{c}\} \rightarrow \mathbf{b} \notin \delta^2_{D,M}(X^1)$.

3) We check (a**) for c. So, $F(c) \wedge \forall y(cRy \rightarrow G(y))$. Since $I(F) = \{\mathbf{a}_0, \mathbf{b}, \mathbf{c}\}$, $F(c) = \mathbf{t}$. Since $I(R) = \{\langle x, y\rangle : (x = y = \mathbf{b}) \vee (x = \mathbf{a}_i \wedge y = a_j \wedge i < j\}$, $cRc = \mathbf{f}$. If $X^1 = \{\mathbf{c}\}$, then $G(c) = \mathbf{t}$. So $\forall y(cRc \rightarrow G(c)) = \mathbf{t}$.

Since $F(c) = \mathbf{t}$, $F(c) \wedge \forall y(cRc \rightarrow G(c)) = \mathbf{t}$. So $X^0 = \{\mathbf{c}\} \rightarrow \mathbf{c} \in \delta^2_{D,M}(X^1)$. Thus $X^1 = \{\mathbf{c}\} \rightarrow \delta^2_{D,M}(X^1) = \{\mathbf{c}\}$. ■

Theorem 1.3.

If $X^2 = \delta^2_{D,M}(X^1) = \{\mathbf{c}\}$, then $X^3 = \delta^3_{D,M}(X^2) = \{\mathbf{c}\}$.

The proof as above, etc., $\{\mathbf{c}\}$ is the fixed point.

Theorem 2.1.

If $I(G) = A = \{\mathbf{a}_0, \mathbf{a}_1, \mathbf{a}_2, \ldots, \mathbf{a}_i, \ldots\}$, then $X^1 = \delta^1_{D,M}(X^0) = \{\mathbf{a}_0, \mathbf{c}\}$.

Proof

Let $X^0 = A = \{\mathbf{a}_0, \mathbf{a}_1, \mathbf{a}_2, \ldots, \mathbf{a}_i, \ldots\}$.

1) We check (a**) for a_0. So, $F(a_0) \wedge \forall y(a_0 Ry \rightarrow G(y))$. Since $I(F) = \{\mathbf{a}_0, \mathbf{b}, \mathbf{c}\}$, $F(a_0) = \mathbf{t}$. Since $I(R) = \{\langle x, y\rangle : (x = y = \mathbf{b}) \vee (x = \mathbf{a}_i \wedge y = \mathbf{a}_j \wedge i < j\}$, for $j > 0$ $a_0 Ra_j = \mathbf{t}$. If $I(X) = A$, then $G(a_j) = \mathbf{t}$. So $\forall y(a_0 Ra_j \rightarrow G(a_j)) = \mathbf{t}$. Since $F(b) = \mathbf{t}$, $F(a_0) \wedge \forall y(a_0 Ra_j \rightarrow G(a_j)) = \mathbf{t}$. Thus $X = A \rightarrow \mathbf{a}_0 \in \delta^1_{D,M}(X^0)$.

2) We check (a**) for b. So $F(b) \wedge \forall y(bRy \rightarrow G(y))$. Since $I(F) = \{\mathbf{a}_0, \mathbf{b}, \mathbf{c}\}$, $F(b) = \mathbf{t}$. Since $I(R) = \{\langle x, y\rangle : (x = y = \mathbf{b}) \vee (x = \mathbf{a}_i \wedge y = \mathbf{a}_j \wedge i < j\}$, $bRb = \mathbf{t}$. If $I(X) = A$, then $G(b) = \mathbf{f}$. So $\forall y(bRb \rightarrow G(b)) = \mathbf{f}$. Since $F(b) = \mathbf{t}$, $F(b) \wedge \forall y(bRb \rightarrow G(b)) = \mathbf{f}$. Thus $X = A \rightarrow \mathbf{b} \notin \delta^1_{D,M}(X^0)$.

3) We check (a**) for c. So $F(c) \wedge \forall y(cRy \rightarrow G(c))$. Since $I(F) = \{\mathbf{a}_0, \mathbf{b}, \mathbf{c}\}$, $F(c) = \mathbf{t}$. Since If $I(R) = \{\langle x, y\rangle : (x = y = \mathbf{b}) \vee (x = \mathbf{a}_i \wedge y = \mathbf{a}_j \wedge i < j\}$, $cRy = \mathbf{f}$. If $I(X) = A$, then $G(c) = \mathbf{f}$. So $\forall y(cRy \rightarrow G(c)) = \mathbf{t}$. Since $F(c) = \mathbf{t}$, $F(c) \wedge \forall y(cRy \rightarrow G(c)) = \mathbf{t}$. So $X = A \rightarrow \mathbf{c} \in \delta^1_{D,M}(X^0)$. Thus $X^0 = A \rightarrow \delta^1_{D,M}(X^0) = \{\mathbf{a}_0, \mathbf{c}\}$. ■

Theorem 2.2.

If $X^1 = \delta^1_{D,M}(X^0) = \{\mathbf{a}_0, \mathbf{c}\}$, then $X^2 = \delta^2_{D,M}(X^1) = \{\mathbf{c}\}$.

The proof as above, etc., $\{\mathbf{c}\}$ is the fixed point.

Theorem 2.3.

If $X^2 = \delta^2_{D,M}(X^1) = \{\mathbf{c}\}$, then $X^3 = \delta^3_{D,M}(X^2) = \{\mathbf{c}\}$.

The proof as above, etc., $\{\mathbf{c}\}$ is the fixed point.

Theorem 3.1.

If $I(G) = X^0 = \{\mathbf{b}\}$, then $X^1 = \delta^1_{D,M}(X^0) = \{\mathbf{b}, \mathbf{c}\}$.

Proof

Let $X^0 = \{\mathbf{b}\}$.

1) We check (a**) for a_0. So $F(a_0) \wedge \forall y(a_0 R y \rightarrow G(y))$. Since $I(F) = \{\mathbf{a}_0, \mathbf{b}, \mathbf{c}\}$, $F(a_0) = \mathbf{t}$. Since $I(R) = \{\langle x, y\rangle : (x = y = \mathbf{b}) \vee (x = \mathbf{a}_i \wedge y = \mathbf{a}_j \wedge i < j\}$, for $j > 0$ $a_0 R a_j = \mathbf{t}$. If $I(X) = \{\mathbf{b}\}$, then $G(a_j) = \mathbf{f}$. So $\forall y(a_0 R a_j \rightarrow G(a_j)) = \mathbf{f}$. Since $F(a_0) = \mathbf{t}$, $F(a_0) \wedge \forall y(a_0 R a_j \rightarrow G(a_j)) = \mathbf{f}$. Thus $X = \{\mathbf{b}\} \rightarrow \mathbf{a}_0 \notin \delta^1_{D,M}(X^0)$.

2) We check (a**) for b. So $F(b) \wedge \forall y(bRy \rightarrow G(y))$. Since $I(F) = \{\mathbf{a}_0, \mathbf{b}, \mathbf{c}\}$, $F(b) = \mathbf{t}$. Since $I(R) = \{\langle x, y\rangle : (x = y = \mathbf{b}) \vee (x = \mathbf{a}_i \wedge y = \mathbf{a}_j \wedge i < j\}$, $bRb = \mathbf{t}$. If $I(X) = \{\mathbf{b}\}$, then $G(b) = \mathbf{t}$. So $\forall y(bRb \rightarrow G(b)) = \mathbf{t}$. Since $F(b) = \mathbf{t}$, $F(b) \wedge \forall y(bRy \rightarrow G(y)) = \mathbf{t}$. Thus $X = \{\mathbf{b}\} \rightarrow \mathbf{b} \in \delta^1_{D,M}(X^0)$.

3) We check (a**) for c. So $F(c) \wedge \forall y(cRy \rightarrow G(y))$. Since $I(F) = \{\mathbf{a}_0, \mathbf{b}, \mathbf{c}\}$, $F(c) = \mathbf{t}$. Since $I(R) = \{\langle x, y\rangle : (x = y = \mathbf{b}) \vee (x = \mathbf{a}_i \wedge y = \mathbf{a}_j \wedge i < j\}$, $cRc = \mathbf{f}$. If $X = \{\mathbf{b}\}$, then $G(c) = \mathbf{f}$. So, $\forall y(cRc \rightarrow G(c)) = \mathbf{t}$. Since $F(c) = \mathbf{t}$, $F(c) \wedge \forall y(cRc \rightarrow G(c)) = \mathbf{t}$. So $X = \{\mathbf{b}\} \rightarrow \mathbf{c} \in \delta^1_{D,M}(X^0)$. Thus $X^0 = \{\mathbf{b}\} \rightarrow \delta^1_{D,M}(X^0) = \{\mathbf{b}, \mathbf{c}\}$. ■

Theorem 3.2.
If $X^1 = \delta^1_{D,M}(X^0) = \{\mathbf{b}, \mathbf{c}\}$, then $X^2 = \delta^2_{D,M}(X^1) = \{\mathbf{b}, \mathbf{c}\}$
The proof as above, etc., $\{\mathbf{b}, \mathbf{c}\}$ is the fixed point.

Theorem 3.3.
If $X^2 = \delta^2_{D,M}(X^1) = \{\mathbf{b}, \mathbf{c}\}$, then $X^3 = \delta^3_{D,M}(X^2) = \{\mathbf{b}, \mathbf{c}\}$
The proof as above, etc., $\{\mathbf{b}, \mathbf{c}\}$ is the fixed point.

So, finally there are two different fixed points $\{\mathbf{c}\}$, $\{\mathbf{b}, \mathbf{c}\}$.

A circular definition is understood as a scheme (rule, function) of revision $\delta_{D,M}$. The function $\delta_{D,M}$ takes an arbitrary hypothetical extension X of the predicate G as an initial argument (an input value), and assigns to this argument X a unique value $\delta_{D,M}(X)$, i.e. a set, which is a new revised hypothetical extension of a predicate G, calculated on the basis of the given circular definition. The revision begins from e.g. $\emptyset$-hypothesis, taken as an initial hypothesis and can be repeated many times. The new revision at a still higher and higher level, is caused by following applications of a hypothetical rule of revision $\delta_{D,M}$. The process of revision consists in obtaining new succeeding hypotheses (i.e. candidates or versions) for the extension of the predicate G appearing in a *definiendum* of a given circular definition. These new hypothetical extensions of G are expected to be improved, or at least as good as an initial hypothesis (i.e. hypotheses, which repeat regularly, are counted as better ones). In following revisions some objects always belong to the set, which is an extension of G. These objects are positively stable for an initial hypothesis. But some objects finally do not belong to

the extension G in following revisions. These objects are negatively stable for an initial hypothesis. The result is unstable for other objects; in some cases they belong to the extension G, and in some cases they do not. Such objects are unstable. In the case of objects, which are positively or negatively stable, a circular definition gives a definitive result on the basis of the initial hypothesis, but it does not do this in the case of unstable objects. So, a circular definition is not able to fix an exact set as an extension of the predicate G. Nevertheless such a definition remains a scheme (rule), which is capable to calculate which set will be an extension of the *definiendum G* if another fixed set is taken as an initial hypothesis. This is why the meaning, assigned by a circular definition to the *definiendum G*, is hypothetical (*RTT*, pp. 117–125; see Gupta 1981, pp. 735–736; Gupta 1982, pp. 1–60; Gupta 1988–89, pp. 234–237; Gupta, Belnap 1994, pp. 632–636; Gupta 1997, pp. 419–443; Gupta 2001, pp. 102–103; Belnap 1982, pp. 103–116; Koons 1994, pp. 614–615; Kublikowski 2005, pp. 143–156).

It is intriguing how an initial, hypothetical and an unstable extension of the *definiendum G* changes into a stable, categorical extension in the process of revision. This transition is possible on the basis of the fact that *all possible* initial hypotheses of the extension of G, are taken into account in the revision process. If in the revision process for all possible hypotheses, a certain object always belongs to the extension of the *definiendum G* then it is sure that this object is categorically G. The scheme of revision gives intuitionally correct categorical statements about ordinary, non-problematic (non-pathological) sentences, which always stabilise at the same value, independently of an initial hypothesis, which was taken in the revision process. Some other sentences stabilise for all hypotheses, but sometimes they stabilise as true and sometimes as false. The remaining sentences never stabilise in the revision process (i.e. they always change). We can say that the behaviour of different kinds of pathological sentences can be displayed in the revision process, in which analysed objects behave in a typical way, independently of an initial hypothesis (Gupta 1988–89, pp. 236, 242).

The application of the mathematical machinery of revision theory not only shows us how a circularity of definitions works, but this mathematical apparatus also allows us to obtain the same value (or values) for all possible, initial and arbitrary hypotheses of the extension of the *definiendum G*[3].

---

[3] For very helpful remarks and corrections I would like to thank Anil Gupta (The University of Pittsburgh), Michael Kremer (The University of Chicago) and Grzegorz Malinowski (The University of Łódź). The first draft of this paper was written at The University of Notre Dame (USA), where I was a visiting researcher in the summer semester of 2002.

References

Belnap, Nuel (1982), *Gupta's Rule of Revision Theory of Truth*, "Journal of Philosophical Logic" (11), pp. 103–116.

Gupta, Anil (1981), *Truth and Paradox* (abstract), "Journal of Philosophy" (78), pp. 735–736.

Gupta, Anil (1982), *Truth and Paradox*, "Journal of Philosophical Logic" (11), pp. 1–60.

Gupta, Anil (1988–89), *Remarks on definitions and the concept of truth*, Proceedings of the Aristotelian Society (89), pp. 227–246.

Gupta, Anil and Nuel Belnap (1993), *The Revision Theory of Truth*, Cambridge, MA: MIT.

Gupta Anil and Nuel Belnap (1994), *Reply to Robert Koons*, "Notre Dame Journal of Formal Logic" (35) 4, pp. 632–636.

Gupta, Anil (1997), *Definition and Revision: A Response to McGee and Martin*, "Philosophical Issues" (8), pp. 419–443.

Gupta, Anil (2000), *On circular concepts*, in: Andre Chapuis, Anil Gupta, *Circularity, Definition, and Truth*, New Delhi: Indian Council of Philosophical Research, pp. 123–154.

Gupta, Anil (2001), *Truth*, in: Lou Goble (ed.), *The Blackwell Guide to Philosophical Logic*, Oxford: Blackwell Publishers, pp. 90–114.

Koons, Robert (1994), *Book Review: The Revision Theory of Truth*, "Notre Dame Journal of Formal Logic" (35) 4, pp. 606–631.

Kublikowski, Robert (2005), *Alfreda Tarskiego Schemat T jako równość definicyjna*, „Roczniki Filozoficzne" (53) 1, pp. 143–156.

Robert Kublikowski
Department of Logic and Theory of Knowledge
Catholic University of Lublin
Al. Raclawickie 14
20–950 Lublin POLAND
e-mail: robertk@kul.lublin.pl

STUDIES IN LOGIC, GRAMMAR AND RHETORIC 8 (21) 2005

Jacek Waldmajer
University of Opole

ON STRUCTURES AND THEIR ADEQUACY*

1. Introduction

While asking about the adequacy of our knowledge we most often ask about the adequacy of these sentences which represent this knowledge. We normally understand by this that the manner in which sentences describe objects to which they refer must be in accordance with what the objects really are. A precise reply to the question of adequacy requires thus the acceptance of a possibly broad notion of ontology. By ontology, following on from J. Perzanowski ([2004], p. 93), we shall understand the general theory of all essential possibilities. Within this framework we shall be able to speak about an ontology of the world (metaphysics), an ontology of language, an ontology of meanings, formal ontology, etc.

How to formulate questions about adequacy? Let us begin with quoting certain comments offered by R. Wójcicki, who – pointing to the work by H. Putnam [1989], as an introduction to the problem of adequacy, writes as follows: "*... problems of adequate representation of knowledge constitute, at present, one of the very intensively discussed issues of the so-called cognitive science*" ([1991], p. 85), and "*A lot of confusion around the definition of truth results from either mixing up truth with adequacy, or the mistaken view that either of these notions is redundant*" ([1996], p. 69).

So far the theory of adequacy, which would refer to the above-indicated problem area, has not yet been built. The most significant philosophical categories and the relations between them to determine a formulation of the problem of adequacy in the sense proposed by Wójcicki [1991] are indicated in Fig. 1. According to the diagram presented in it, posing the problem of adequacy of knowledge in science is a result of the two-way

* This paper is an expanded version of the paper, which was presented at the 2nd Logical Workshops in Bielsko-Biała, December 16th–17th, 2004.

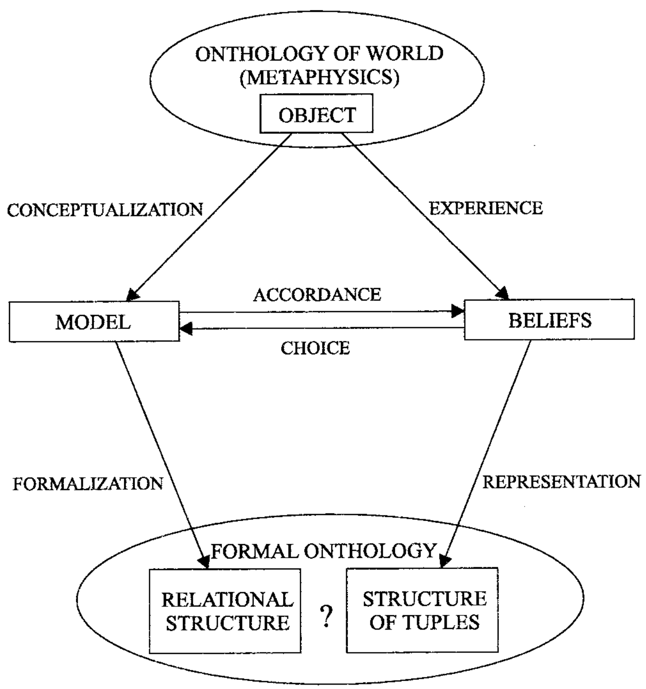
ISBN 83-7431-059-6 ISSN 0860-150X

character of cognition: theoretical conceptualization and experimental verification.

Conceptualization in research into a concrete object leads to an appearance of its descriptive *models* understood as a certain kind of systems of representation of knowledge. These systems are imposed by linguistic communication or technical means (generally, through certain creations of culture) in such a way that the formalization of the processing of information on the object examined becomes possible on the grounds of a formal ontology, being most often the set theory or a mathematical theory built over the set theory (e.g. the theory of differential equations). Formal ontology is then treated here as a certain idealization of the ontology of the world. A result of the formalization of the knowledge of the object is the determination of the *relational structure* that reflects the features, properties and behaviour of the object (e.g. determination of the system of differential equations with appropriate boundary conditions).

Figure 1

A question about adequacy: Is the relational structure describing the object examined in accordance with the structure of tuples referring to the states of things which characterize the object by means of belief?



By means of experiments the knowledge about the states of things reflecting the features, properties and behavior of the object examined is collected. In this way *beliefs* are formed, which – in the language – are expressed by statements referring to the states of things which were discovered. Beliefs are represented on the grounds of formal ontology by means of expressions corresponding to *structures of tuples*, that is sets of ordered systems (*tuples*) of objects reflecting features, properties and behaviour of the object examined.

The model of the object, which is formed as a result of conceptualization, can be intended, i.e. chosen in such a way that the beliefs are in accordance with it (see Fig. 1). Then we can say that the model is adequate to the beliefs. All the models of a given object that are adequate to the same set of beliefs referring to this object are called *adequate models*. Accepting that E is any object, we can repeat after Wójcicki [1991], that it is an *adequate model of object p* iff the conditions E1–E7 as formulated below are satisfied:

E1. There has been determined *potential scope of applicability* E to p, i.e. a set of statements Ξ referring to E, which (see E2) can be translated into statements concerning p.

E2. There has been determined an effective procedure – *interpretation code* I – which allows translating any sentence of set Ξ into a sentence concerning p.

E3. There has been determined a set of procedures Δ, which allow effective decibility of any sentence $\alpha \in \Xi$.

E4. There has been determined the *real scope of applicability*, for short – *scope of applicability of E to p*, i.e. a set of statements $\Xi^* \subsetneq \Xi$ such that for any sentence $\alpha \in \Xi^*$ the equivalence called condition of adequacy of model E is satisfied:

$$(*) \qquad \alpha \Leftrightarrow \mathrm{I}(\alpha).$$

E5. All the sentences belonging to the set of interpretation $\mathrm{I}(\Xi^*)$ are empirically decidable.

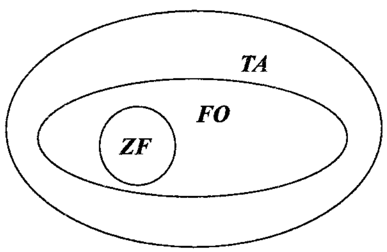
E6. There has been determined together with E, in practice, an infinitely numerous set of models Σ, satisfying the conditions E1–E3,

E7. For each model belonging to Σ there is determined the probability of this model being adequate.

Precise checking whether the condition of adequacy for the model of the object examined is satisfied, is possible only on the ground of formal ontology. Then, the formula which states being a *relational structure* – corresponding to being a descriptive model – is well defined. So is the formula which states being a *structure of tuples* – corresponding to being belief. Moreover, the formula settling the unambiguous relation between the object satisfying the first formula and the object satisfying the second one is defined in such a way when that knowing the first object, the second one could be determined.

Let us pay attention to intuitions which are a motivation to introduce formal ontology on the grounds of the set theory. These intuitions refer to the following cognitive schemata: the first stage of cognition, on the way of abstraction, of any object is the distinguishing of elements of this object and all the tuples which bind these elements. In this way, we get to know the *structure of tuples* of the object (the structure of the object) separately from the relations that allow determining tuples, which bind the elements of the object. We recognize these relations at the second stage of cognition of the object. The set of all the tuples belonging to these relations is a kind of set of generators (a *base structure*) forming all the tuples determining the structure of the object. According to intuition, the tuples binding elements of the object are allowed connections of elements of the object examined when they are determined by accessible cognitive means. Elements that are directly available to cognition are represented by a one-element tuple.

Figure 2



As regards the above-mentioned intuitions, the following question may be asked: How, having given relations, that is having imposed relational structure, can all the tuples be generated out of the structure of tuples characterizing the object (see Fig. 1.)? An answer to this question, on the ground of formal ontology, allows us to solve the problem of the adequacy (accordance) of the descriptive model with beliefs. The present work is an attempt at formulating basis of a formal theory of adequacy ***TA***. The theory of adequacy refers to the concept of adequacy in the sense proposed by Wójcicki, yet it takes into account the above intuitions. The formulated theory also makes references to certain notions of adequacy applied in logic and philosophy (ontology). Theory ***TA*** will be built over formal ontology ***FO*** developed over the set theory ***ZF*** (see Fig. 2).

2. A structure of tuples determined by the relational structure

Let U be any established, nonempty set of objects called the ***universe***.

Definition 1.
Each finished sequence $\langle \alpha_1, \alpha_2, \ldots, \alpha_n \rangle \in U^n$ of elements $\alpha_1, \alpha_2, \ldots, \alpha_n$ of the universe U, where $n \geq 1$ and U^n is the Cartesian product of n sets U, is called a ***tuple****. If the tuple has n-elements, then the number n is called its* ***length****.*

Definition 2.
Let t, t_1 be tuples.

$$t_1 \varepsilon t \Leftrightarrow t_1 \text{ is a subsequence of } t.$$

The expression "$t_1 \varepsilon t$" is read: the tuple t_1 is a ***subtuple*** of the tuple t.

Let us accept a certain convention concerning one-element tuples and introduce certain notation:
1) For any $\alpha \in U$, a tuple $\langle \alpha \rangle$ will be identified with α.
2) For any set Z, the family of all subsets of Z is denoted by $\boldsymbol{P}(Z)$.
3) For any momentary set Z the set of all tuples of elements of Z is denoted by $S(Z)$, i.e.

$S(Z) = \bigcup_{i \in N} Z^i$, where Z^i is the Cartesian product of i sets U.

Definition 3.
Any subset of the set $S(U)$ is called a ***structure of tuples of elements of the universe*** *U.*

In subsequent parts of this paper, instead of writing: *a structure of tuples of elements of the universe U*, we shall write short from: *a structure of tuples.*

The connections between the objects of the universe U are usually determined by means of relations defining the relational structure with the universe U. These relations are subsets of U^n for any established n; the one-argument relations are identified with certain distinguished subsets of the universe U. The family of these relations (certain sets of tuples of elements of the universe U) – called a *base* here – is characterized by the following definition:

Definition 4.
a) *The nonempty family of sets $\boldsymbol{B} \subseteq \boldsymbol{P}(S(U))$ is a* ***base*** *iff*

$$\forall X \in \boldsymbol{B} \; \exists n \in N \; (X \subseteq U^n).$$

b) *The nonempty family of sets* $\boldsymbol{B}_1$ *is a* **subbase** *of the base* $\boldsymbol{B}$ *iff*

$$\forall X \in \boldsymbol{B}_1 \ \exists Y \in \boldsymbol{B} \ (X \subseteq Y).$$

c) *The set* $BS^{\boldsymbol{B}} = \bigcup \boldsymbol{B}$ *is called a* **base structure** *for the base* $\boldsymbol{B}$.

Thus, the base structure is a certain structure of tuples to which belong all the tuples determined by relations from the base $\boldsymbol{B}$.

Fact 1.

If the base $\boldsymbol{B}_1$ *is a subbase of the base* $\boldsymbol{B}_2$*, then* $BS^{\boldsymbol{B}_1} \subseteq BS^{\boldsymbol{B}_2}$.

Further along in the work, we will understand by a relational structure an ordered system $\boldsymbol{Re} = \langle U^{\boldsymbol{Re}}, \{R_k\}_{k\in N}, I\rangle$, where $U^{\boldsymbol{Re}}$ is a nonempty subset of the universe U, $\{R_k\}_{k\in N}$ is a nonempty family of distinguished relations determined in $U^{\boldsymbol{Re}}$, and I is a distinguished subset of $U^{\boldsymbol{Re}}$.

Fact 2.

For any relational structure $\boldsymbol{Re} = \langle U^{\boldsymbol{Re}}, \{R_k\}_{k\in N}, I\rangle$*, the family of sets* $\boldsymbol{B}(\boldsymbol{Re}) = \{R_k\}_{k\in N} \cup \{I\}$ *is a base.*

Definition 5.

Let $\boldsymbol{Re} = \langle U^{\boldsymbol{Re}}, \{R_k\}_{k\in N}, I\rangle$ *be a relational structure. Then the base* $\boldsymbol{B}(\boldsymbol{Re}) = \{R_k\}_{k\in N} \cup \{I\}$ *is called a* **base of relational structure Re**.

We can determine a certain relational structure for any structure of tuples. This relational structure will be defined in the following way:

Definition 6.

Let $S \subseteq S(U)$*. We denote*

a) $U(S) = \{\alpha \in U : \exists t \in S \ \exists i \in N \ (t = \langle \alpha_1, \ldots, \alpha_i, \ldots, \alpha_n\rangle \wedge \alpha_i = \alpha)\}$.
$U(S)$ is a set of certain elements of the universe U, being – at the same time – elements of tuples of the structure S.

b) $\boldsymbol{R}(S) = \{R : \exists t \in S \ (R = \{t\})\}$.
$\boldsymbol{R}(S)$ is a set of one-element relations composed from particular tuples of the structure S.

c) *The relational structure* $\boldsymbol{Re}(S) = \langle U(S), \boldsymbol{R}(S), \emptyset\rangle$ *is called the* **relational structure for structure of tuples** *S.*

Fact 3.

Let $S \in S(U)$*. Then*

a) $\boldsymbol{B}(\boldsymbol{Re}(S)) = \boldsymbol{R}(S)$,

b) $BS^{\boldsymbol{B}(\boldsymbol{Re}(S))} = S$.

Now, we shall give some intuitions connected with the next definition referring to the **structure $S^{\boldsymbol{Re}}$ of tuples determined by any relational**

structure Re. The set (structure) of all connections (tuples) of the object examined is a set of connections available to cognition and determined by the relational structure $\boldsymbol{Re}$. The available connections are obtained in the following way: (C1) from connections of a set of generators, (C2) by joining, to produce chains, the connections obtained earlier, (C3) if a certain tuple is an allowed connection of distinguished elements with a given element by means of other elements, forming an allowed connection with this element, then also the distinguished elements form an allowed connection with it, and (C4) if a certain tuple represents an available to cognition (allowed) connection of elements, which are directly available to cognition, with a certain element, then this element is also directly available to cognition.

Let us first accept the definition of the *composition of two tuples.*

Definition 7.

Let $r = \langle \alpha_1, \alpha_2, \ldots, \alpha_j, \beta_1, \beta_2, \ldots, \beta_k\rangle \in S(U)$*,* $s = \langle \beta_1, \beta_2, \ldots, \beta_k, \gamma_1, \gamma_2, \ldots, \gamma_l\rangle \in S(U)$ *and* $\langle \beta_1, \beta_2, \ldots, \beta_k\rangle \in S(U)$ *for* $j, k, l \geq 1$*, where k is the greatest number of common elements of r and s. Then the* **composition of tuples** *r and s (symbolically: $r \bullet s$) is the tuple* $\langle \alpha_1, \alpha_2, \ldots, \alpha_j, \beta_1, \beta_2, \ldots, \beta_k, \gamma_1, \gamma_2, \ldots, \gamma_l\rangle$.

Definition 8.

Let $BS^{\boldsymbol{B}(\boldsymbol{Re})}$ *be the base structure for the base* $\boldsymbol{B}(\boldsymbol{Re})$*. The set* $S^{\boldsymbol{Re}}$ *is a* **structure of tuples determined by the relational structure Re** *iff*

$$S^{\boldsymbol{Re}} = \bigcap\{S \subseteq S(U) : S \text{ satisfies the conditions C1-C4}\},$$

i.e. $S^{\boldsymbol{Re}}$ *is the smallest set among the sets* $S \subseteq S(U)$ *satisfying the following conditions:*

(C1) ***(generators)**:*

$$BS^{\boldsymbol{B}(\boldsymbol{Re})} \subseteq S,$$

(C2) ***(composition of tuples)**:*

$$\forall t, s \in S \ (\exists r \in S(U) \ (t \bullet s = r) \Rightarrow t \bullet s \in S),$$

(C3) ***(reduction of tuples)**:*

for any $\beta_1, \ldots, \beta_k, \alpha, \alpha_1, \ldots, \alpha_j, \alpha_{j+1}, \alpha_{j+2}, \ldots, \alpha_l \in U$*, where* $j, k, l \geq 1$*,*
if $\langle \beta_1, \ldots, \beta_k, \alpha\rangle \in S$ *and* $\langle \alpha_1, \ldots, \alpha_j, \beta_1, \ldots, \beta_k, \alpha, \alpha_{j+1}, \alpha_{j+2}, \ldots, \alpha_l\rangle \in S$
then $\langle \alpha_1, \ldots, \alpha_j, \alpha, \alpha_{j+1}, \alpha_{j+2}, \ldots, \alpha_l\rangle \in S$,

(C4) ***(detachment)**:*

for any $\beta_1, \ldots, \beta_k, \alpha \in U$*, where* $k \geq 1$*,*
if $\langle \beta_1, \ldots, \beta_k, \alpha\rangle \in S$ *and* $\forall i = 1, \ldots, k \ (\beta_i \in S)$*, then* $\alpha \in S$.

A structure of tuples $S^{\boldsymbol{Re}}$ is a nonempty set, because the base structure $BS^{\boldsymbol{B}(\boldsymbol{Re})} \neq \emptyset$.

From the above definition and Fact 1 there follows:

Fact 4.

Let $\boldsymbol{Re1}$ *and* $\boldsymbol{Re2}$ *be two relational structures of the same universe. If the base* $\boldsymbol{B}(\boldsymbol{Re1})$ *is a subbase of the base* $\boldsymbol{B}(\boldsymbol{Re2})$, *then* $S^{\boldsymbol{Re1}} \subseteq S^{\boldsymbol{Re2}}$.

3. Adequacy of structures

One of the most important conditions of the adequacy of structures is their homomorphism. The adequacy (agreement) of known conceptual constructions in logic and mathematics is often determined by means of this notion. Let us pay more attention to this notion.

Let two relational structures $\boldsymbol{Re1} = \langle U_1, \boldsymbol{R}_1, I_1\rangle$ and $\boldsymbol{Re2} = \langle U_2, \boldsymbol{R}_2, I_2\rangle$ be given, where $U_1, U_2 \subseteq U$. Let $\boldsymbol{B}(\boldsymbol{Re1})$ be the base of $\boldsymbol{Re1}$ and $\boldsymbol{B}(\boldsymbol{Re2})$ be the base of $\boldsymbol{Re2}$. Let $S^{\boldsymbol{Re1}}$ and $S^{\boldsymbol{Re2}}$ are structures of tuples determined by the relational structures $\boldsymbol{Re1}$ and $\boldsymbol{Re2}$, respectively.

We say that the relational structures $\boldsymbol{Re1}$ and $\boldsymbol{Re2}$ are *similar* (have the same signature), if there exists a function of *interpretation*

$$int : \boldsymbol{B}(\boldsymbol{Re1}) \longrightarrow \boldsymbol{B}(\boldsymbol{Re2})$$

such that:

1. $\forall A \in \boldsymbol{R}_1$ [$(int(A) \in \boldsymbol{R}_2)$ and tuples belonging to the relation A and $int(A)$ have the same length]
2. $int(I_1) = I_2$.

The relational structures $\boldsymbol{Re1}$ and $\boldsymbol{Re2}$ are *homomorphic* (cf. Marciszewski [1987], p. 164), if they are similar and there exists such a function

$$h : U_1 \longrightarrow U_2,$$

mapping the set U_1 in the set U_2, such that:

1. If $A \in \boldsymbol{R}_1$, then for any tuple $\langle \alpha_1, \ldots, \alpha_n\rangle \in A$, $\langle h(\alpha_1), \ldots, h(\alpha_n)\rangle \in int(A)$,
2. If $i \in I_1$, to $h(i) \in I_2$.

The criterion of homomorphism of the relational structures $\boldsymbol{Re}(S_1)$ and $\boldsymbol{Re}(S_2)$, for the structures of the tuples $S_1 \subseteq S(U)$ and $S_2 \subseteq S(U)$ (see Definition 6), is as follows:

Theorem 1.

Let two structures of tuples $S_1 \subseteq S(U)$ *and* $S_2 \subseteq S(U)$ *be given and also let the function* $f : S(U) \longrightarrow S(U)$ *be given. If the function* f *satisfies the conditions:*

1) $f(S_1) \subseteq S_2$ *(The image of the structure of tuples* S_1 *with respect to the function* f *is included in the structure of tuples* S_2*),*

and for any tuple $\langle \alpha_1, \alpha_2, \ldots, \alpha_n\rangle \in S_1$

2) $f(\langle \alpha_1, \alpha_2, \ldots, \alpha_n\rangle) = \langle f(\alpha_1), f(\alpha_2), \ldots, f(\alpha_n)\rangle \in S_2$[1],

then the relational structures $\boldsymbol{Re}(S_1)$ *and* $\boldsymbol{Re}(S_2)$ *are homomorphic.*

Proof.

Let the relational structures $\boldsymbol{Re}(S_1) = \langle U(S_1), \boldsymbol{R}(S_1), \emptyset\rangle$ and $\boldsymbol{Re}(S_2) = \langle U(S_2), \boldsymbol{R}(S_2), \emptyset\rangle$ be given.

Let us determine the function of interpretation $int : \boldsymbol{R}(S_1) \longrightarrow \boldsymbol{R}(S_2)$ for the relational structures $\boldsymbol{Re}(S_1)$ and $\boldsymbol{Re}(S_1)$ in the following way:

(1) $\forall t \in S_1$ $(\{t\} \in \boldsymbol{R}(S_1) \Rightarrow int(\{t\}) = \{f(t)\})$.

It follows from assumption 1) that $\forall t \in S_1$ $(f(t) \in S_2)$. Hence, making use of the way in which we determine the family of the relation $\boldsymbol{R}(S_2)$, we obtain $\forall t \in S_1$ $(\{f(t)\} \in \boldsymbol{R}(S_2))$. Hence, on the basis of Formula (1) we have:

(2) $\forall \{t\} \in \boldsymbol{R}(S_1)$ $(f(t) \in int(\{t\}) \in \boldsymbol{R}(S_2))$.

Let us now define the function $h : U(S_1) \longrightarrow U(S_2)$ in the following way:

(3) $\forall \alpha \in U(S_1)$ $(h(\alpha) = f(\alpha))$.

The function h is well determined since for any $\alpha \in U(S_1)$, $f(\alpha) \in U(S_2)$, because from assumption 2) it follows that $f(\alpha)$ is a one-element tuple $\langle f(\alpha)\rangle$, composed from this element and, being identified with this tuple, belongs to $U(S_2)$. Using again assumption 2) and formulas (3) and (2), as well as the definition of homomorphism of structures, we can conclude that the function h establishes the homomorphism of the structure $\boldsymbol{Re}(S_1)$ into the structure $\boldsymbol{Re}(S_2)$. ■

Theorem 2.

Let two structures of tuples $S_1 \subseteq S(U)$ *and* $S_2 \subseteq S(U)$ *be given. If the relational structures* $\boldsymbol{Re}(S_1)$ *and* $\boldsymbol{Re}(S_2)$ *are homomorphic, then there exist a function* $f : S(U) \longrightarrow S(U)$ *satisfying the conditions:*

---

[1] Let us remind that the one-element tuples are identified with their elements, thus $f(\langle \alpha_i\rangle) = f(\alpha_i)$, for any $i \in N$.

1) $f(S_1) \subseteq S_2$,

and for any tuple $\langle \alpha_1, \alpha_2, \ldots, \alpha_n \rangle \in S_1$

2) $f(\langle \alpha_1, \alpha_2, \ldots, \alpha_n \rangle) = \langle f(\alpha_1), f(\alpha_2), \ldots, f(\alpha_n) \rangle \in S_2$.

Proof.

Let the function $h : U(S_1) \longrightarrow U(S_2)$ establish the homomorphism of the above-given relational structures $\boldsymbol{Re}(S_1)$ and $\boldsymbol{Re}(S_2)$.

Let the following function $H : S(U) \longrightarrow S(U)$, be an expansion of the function of homomorphism h:

a) $H(\alpha) = h(\alpha)$, for any $\alpha \in U(S_1)$,

b) $H(\langle \alpha_1, \alpha_2, \ldots, \alpha_n \rangle) = \langle h(\alpha_1), h(\alpha_2), \ldots, h(\alpha_n) \rangle$, for any $n \geq 1$ and $\langle \alpha_1, \alpha_2, \ldots, \alpha_n \rangle \in S_1 = BS^{\boldsymbol{B}(\boldsymbol{Re}(S_1))}$

c) If $t \notin (U(S_1) \cup S_1)$, than $H(t) = t$.

Let us note that for any $\langle \alpha_1, \alpha_2, \ldots, \alpha_n \rangle \in S_1$, $\langle h(\alpha_1), h(\alpha_2), \ldots, h(\alpha_n) \rangle$ belongs to a certain relation of the base $\boldsymbol{B}(\boldsymbol{Re}(S_2))$, and hence also to the base structure $BS^{\boldsymbol{B}(\boldsymbol{Re}(S_2))} = S_2$ (Fact 3b). From the assumptions a) and b) we have:

$$H(\langle \alpha_1, \alpha_2, \ldots, \alpha_n \rangle) = \langle H(\alpha_1), H(\alpha_2), \ldots, H(\alpha_n) \rangle \in S_2.$$

Thus: $H(S_1) \subseteq S_2$, and for the function H, the conditions 1) and 2) of the thesis of the theorem being proved are satisfied. ■

In the light of the above-presented theorems, the following definition is well justified:

Definition 9.

Let two structures of tuples $S_1 \subseteq S(U)$ *and* $S_2 \subseteq S(U)$ *be given and also let the function* $f : S(U) \longrightarrow S(U)$ *be given. The function* f *establishes a* ***homomorphism*** *of the structure of tuples* S_1 *into the structure of tuples* S_2 *(the structures of tuples* S_1 *and* S_2 *are* ***homomorphic****) iff*

1) $f(S_1) \subseteq S_2$

and for any tuple $\langle \alpha_1, \alpha_2, \ldots, \alpha_n \rangle \in S_1$

2) $f(\langle \alpha_1, \alpha_2, \ldots, \alpha_n \rangle) = \langle f(\alpha_1), f(\alpha_2), \ldots, f(\alpha_n) \rangle \in S_2$.

From Theorem 1, Theorem 2 and Definition 9 there follows the following theorem:

Theorem 3.

Let two structures of tuples $S_1 \subseteq S(U)$ *and* $S_2 \subseteq S(U)$ *be given. The structures of tuples* S_1 *and* S_2 *are homomorphic iff the relational structures* $\boldsymbol{Re}(S_1)$ *and* $\boldsymbol{Re}(S_2)$ *are homomorphic.*

Theorem 4.

Let two relational structures $\boldsymbol{Re1} = \langle U_1, \boldsymbol{R}_1, I_1 \rangle$ *and* $\boldsymbol{Re2} = \langle U_2, \boldsymbol{R}_2, I_2 \rangle$ *be given, where* $U_1, U_2 \subseteq U$. *If the relational structures* $\boldsymbol{Re1}$ *and* $\boldsymbol{Re2}$ *are homomorphic, then also the base structures* $BS^{\boldsymbol{B}(\boldsymbol{Re1})}$ *and* $BS^{\boldsymbol{B}(\boldsymbol{Re2})}$ *are homomorphic.*

Proof.

Let the function $h : U_1 \longrightarrow U_2$ establish the homomorphism of the relational structures $\boldsymbol{Re1}$ and $\boldsymbol{Re2}$.

Let the following function $H : S(U) \longrightarrow S(U)$ be an expansion of the function of homomorphism h:

a) $H(\alpha) = h(\alpha)$, for any $\alpha \in U_1$,

b) $H(\langle \alpha_1, \alpha_2, \ldots, \alpha_n \rangle) = \langle h(\alpha_1), h(\alpha_2), \ldots, h(\alpha_n) \rangle$, for any $n \geq 1$ and $\langle \alpha_1, \alpha_2, \ldots, \alpha_n \rangle \in BS^{\boldsymbol{B}(\boldsymbol{Re1})}$

c) If $t \notin (U_1 \cup BS^{\boldsymbol{B}(\boldsymbol{Re1})})$, than $H(t) = t$.

Let us note that for any $\langle \alpha_1, \alpha_2, \ldots, \alpha_n \rangle \in S_1$, $\langle h(\alpha_1), h(\alpha_2), \ldots, h(\alpha_n) \rangle$ belongs to a certain relation of the base $\boldsymbol{B}(\boldsymbol{Re2})$, and hence also to the base structure $BS^{\boldsymbol{B}(\boldsymbol{Re2})}$. From the assumptions a) and b) we have:

$$H(\langle \alpha_1, \alpha_2, \ldots, \alpha_n \rangle) = \langle H(\alpha_1), H(\alpha_2), \ldots, H(\alpha_n) \rangle \in BS^{\boldsymbol{B}(\boldsymbol{Re2})}.$$

Thus: $H(BS^{\boldsymbol{B}(\boldsymbol{Re1})}) \subseteq BS^{\boldsymbol{B}(\boldsymbol{Re2})}$. Moreover, from the manner of determining the function H it follows that $H(\langle \alpha_1, \alpha_2, \ldots, \alpha_n \rangle) = \langle H(\alpha_1), H(\alpha_2), \ldots, H(\alpha_n) \rangle$, for any $\langle \alpha_1, \alpha_2, \ldots, \alpha_n \rangle \in BS^{\boldsymbol{B}(\boldsymbol{Re1})}$. Hence, on the basis of Definition 9, the function H establishes the homomorphism of the base structure $BS^{\boldsymbol{B}(\boldsymbol{Re1})}$ into the base structure $BS^{\boldsymbol{B}(\boldsymbol{Re2})}$. ■

Definition 10.

Two structures of tuples $S_1 \subseteq S(U)$ *and* $S_2 \subseteq S(U)$ *are called* ***isomorphic*** *(symbolically:* $S_1 \approx_{isom} S_2$*) iff*

1) *there exists a function establishing a homomorphism of the structure of tuples* S_1 *into the structure of tuples* S_2 *and*

2) *the function* f *is a one to one function and*

3) *the function* f^{-1} *– the inverse of* f, *establishes a homomorphism of the structure of tuples* S_2 *into the structure of tuples* S_1.

The condition of homomorphism is one of the conditions of the adequacy of structures. In order to put the problem of adequacy in a more general way, it is necessary to expand the theory of adequacy being formulated by a new axiom and a definition. Let us first introduce the axiom of the adequacy.

Axiom 1. (*of adequacy*)

For any one-argument formulas α, β, and the two-argument formula φ, for any objects X, Y

$$\alpha(X) \longmapsto_{\varphi} \beta(Y) \Leftrightarrow$$
$$\varphi(X,Y) \wedge \forall V\ \forall Z\ (\varphi(V,Z) \Rightarrow \alpha(V) \wedge \beta(Z)) \wedge \forall Z\ (\varphi(X,Z) \Rightarrow Y = Z).$$

The expression "$\alpha(X) \longmapsto_{\varphi} \beta(Y)$" is read: the formula φ establishes the adequacy of the object X to the object Y with respect to the fact the object X possesses the property α, and the object Y possesses the property β.

The axiom of the adequacy can be verbally formulated in the following way: for any objects X and Y the formula φ establishes the adequacy of the object X to the object Y with respect to the fact that the object X possesses property α, while the object Y possesses property β iff 1) when objects X and Y satisfy the formula φ, 2) if any objects V and Z satisfy the formula φ, then the object V possesses the property α, while the object Z possesses the property β, 3) there exists exactly one object which – together with the object X – satisfies the formula φ.

When the accepted notation of the object X points, in an unambiguous manner, to that it has the property α, and the accepted notation of the object Y indicates, implicitly, that it possesses the property β, and the formula φ is assumed to have been well defined prior to that, then instead of writing:

"$\alpha(X) \longmapsto_{\varphi} \beta(Y)$", we shall write as follows: "$X \longmapsto_{Adq} Y$"

and read the expression as: *the object X is adequate to the object Y.*

For example: "$\boldsymbol{Re}$" and "$S^{\boldsymbol{Re}}$" are the notations of sets and points, respectively, to their being a relational structure and being a structure of tuples defined by the relational structure $\boldsymbol{Re}$. We shall thus write as follows:

$$\boldsymbol{Re} \longmapsto_{Adq} S^{\boldsymbol{Re}} \text{ instead of: } \alpha(\boldsymbol{Re}) \longmapsto_{\varphi} \beta(S^{\boldsymbol{Re}}),$$

where α is a property of being a relational structure and β is a property of being a structure of tuples determined by a relational structure.

Fact 5.

Let α, β, γ be any one-argument formulas. Then

a) *there exists a two-argument formula φ, such that $\alpha(X) \longmapsto_{\varphi} \alpha(X)$,*

b) *for any two-argument formulas φ, φ_1 there exist such a two-argument formula φ_2, that*

$$\alpha(X) \longmapsto_{\varphi} \beta(Y) \wedge \beta(Y) \longmapsto_{\varphi_1} \gamma(Z) \Rightarrow \alpha(X) \longmapsto_{\varphi_2} \gamma(Z).$$

Proof.

In proof a) it is enough to choose the formula of identity as the formula φ, while in proof b) to select, as the formula φ_2, a formula defined by the following expression:

$$\varphi_2(X_1, X_3) \Leftrightarrow \varphi(X_1, X_2) \wedge \varphi_1(X_4, X_3) \wedge X_2 = X_4. \qquad \blacksquare$$

Let us note that the object $\boldsymbol{Re}$ has the property of being a relational structure, while the object $S^{\boldsymbol{Re}}$ has the property of being a structure of tuples. Assuming that φ is a formula which states that the structure of tuples $S^{\boldsymbol{Re}}$ is determined, in an unambiguous way, by the relational structure $\boldsymbol{Re}$ (in accordance with Definition 8), we have:

Fact 6.

$$\boldsymbol{Re} \longmapsto_{\varphi} S^{\boldsymbol{Re}}.$$

Establishing the adequacy of different structures, we shall make use of the following definition of the adequacy of objects:

Definition 11.

For any formulas $\varphi_1, \varphi_2, \varphi_3, \varphi_4$, the objects X and Y of the property α are adequate (symbolically: $\alpha(X) \approx_{Adq} \alpha(Y)$) iff there exist such objects V and Z of a certain property β that

(i) $\alpha(X) \longmapsto_{\varphi_1} \beta(V) \wedge \alpha(Y) \longmapsto_{\varphi_2} \beta(Z)$,

and

(ii) $\alpha(X) \longmapsto_{\varphi_3} \beta(Z) \wedge \alpha(Y) \longmapsto_{\varphi_4} \beta(V)$.

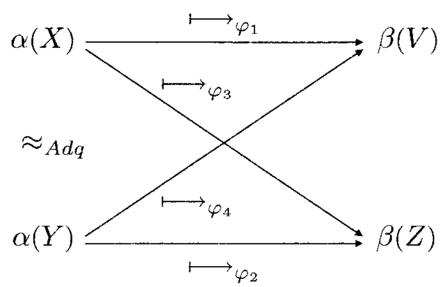
The expression "$\alpha(X) \approx_{Adq} \alpha(Y)$" is read: the objects X and Y are adequate with respect to the fact that the objects X and Y possess the property α.

When the accepted notations of the object X and the object Y explicitly point to the fact that they have the property α, and – moreover – the formulae $\varphi_1, \varphi_2, \varphi_3, \varphi_4$ are assumed to have been well defined prior to that, then instead of writing:

"$\alpha(X) \approx_{Adq} \alpha(Y)$", we shall write the following: $X \approx_{Adq} Y$

and read this expression as: *the objects X and Y are adequate.*

The definition of the adequacy is illustrated by the following diagram, in which the arrows symbolize the adequacy of one object in relation to another one:



The introduced axiom of the adequacy and the definition of adequacy of objects allow formulating certain theorems and conclusions that can be regarded as certain criteria of adequacy of structures.

Theorem 5.

If two structures of tuples $S^{\boldsymbol{Re1}}$ and $S^{\boldsymbol{Re2}}$ are isomorphic, then the relational structure $\boldsymbol{Re1}$ is adequate to the structure of tuples $S^{\boldsymbol{Re2}}$ and also the relational structure $\boldsymbol{Re2}$ is adequate to the structure of tuples $S^{\boldsymbol{Re1}}$. Symbolically:

$$S^{\boldsymbol{Re1}} \approx_{isom} S^{\boldsymbol{Re2}} \Rightarrow \boldsymbol{Re1} \longmapsto_{Adq} S^{\boldsymbol{Re2}} \wedge \boldsymbol{Re2} \longmapsto_{Adq} S^{\boldsymbol{Re1}}.$$

Proof.

Let the structures of tuples $S^{\boldsymbol{Re1}}$ and $S^{\boldsymbol{Re2}}$ be isomorphic. Since the structures of tuples $S^{\boldsymbol{Re1}}$ and $S^{\boldsymbol{Re2}}$ are isomorphic, there exists a function f establishing the homomorphism of the structure $S^{\boldsymbol{Re1}}$ into the structure $S^{\boldsymbol{Re2}}$, and being a one to one function, while the function f^{-1} – the inverse of f, establishes the homomorphism of the structure $S^{\boldsymbol{Re2}}$ into the structure $S^{\boldsymbol{Re1}}$.

Hence that the relational structure $\boldsymbol{Re1}$ is adequate to the structure of tuples $S^{\boldsymbol{Re1}}$ (see Fact 6) and that the function f establishes the homomorphism of the structure of tuples $S^{\boldsymbol{Re1}}$ into the structure of tuples $S^{\boldsymbol{Re2}}$ it follows that there exists such a formula φ that states that $\boldsymbol{Re1}$ determines $S^{\boldsymbol{Re2}}$ in an unambiguous manner. Since the set $\boldsymbol{Re1}$ has the property of being a relational structure, while the set $S^{\boldsymbol{Re2}}$ has the property of being a structure of tuples, it follows from the axiom of adequacy that $\boldsymbol{Re1} \longmapsto_{Adq} S^{\boldsymbol{Re2}}$.

We can show in an analogous way that $\boldsymbol{Re2} \longmapsto_{Adq} S^{\boldsymbol{Re1}}$, as it is sufficient to observe that the function f^{-1} establishes the homomorphism of the structure $S^{\boldsymbol{Re2}}$ into the structure $S^{\boldsymbol{Re1}}$. ■

From Theorem 5 and Definition 10 there follows:

Corollary 1.

If two structures of tuples $S^{\boldsymbol{Re1}}$ and $S^{\boldsymbol{Re2}}$ are identical, then the relational structure $\boldsymbol{Re1}$ is adequate to the structure of tuples $S^{\boldsymbol{Re2}}$ and also the relational structure $\boldsymbol{Re2}$ is adequate to the structure of tuples $S^{\boldsymbol{Re1}}$. Symbolically:

$$S^{\boldsymbol{Re1}} = S^{\boldsymbol{Re2}} \Rightarrow \boldsymbol{Re1} \longmapsto_{Adq} S^{\boldsymbol{Re2}} \wedge \boldsymbol{Re2} \longmapsto_{Adq} S^{\boldsymbol{Re1}}.$$

Theorem 6.

The relational structure $\boldsymbol{Re1}$ is adequate to the structure of tuples $S^{\boldsymbol{Re2}}$ and also the relational structure $\boldsymbol{Re2}$ is adequate to the structure of tuples $S^{\boldsymbol{Re1}}$ iff the relational structures $\boldsymbol{Re1}$ and $\boldsymbol{Re2}$ are adequate. Symbolically:

$$\boldsymbol{Re1} \longmapsto_{Adq} S^{\boldsymbol{Re2}} \wedge \boldsymbol{Re2} \longmapsto_{Adq} S^{\boldsymbol{Re1}} \Leftrightarrow \boldsymbol{Re1} \approx_{Adq} \boldsymbol{Re2}.$$

Proof.

($\Rightarrow$) Let the relational structure $\boldsymbol{Re1}$ be adequate to the structure of tuples $S^{\boldsymbol{Re2}}$ and let the relational structure $\boldsymbol{Re2}$ be adequate to the structure of tuples $S^{\boldsymbol{Re1}}$. Let us note that this assumption can be in accordance with condition (i) or condition (ii) of Definition 11. Let us assume that the assumption: $\boldsymbol{Re1} \longmapsto_{Adq} S^{\boldsymbol{Re2}} \wedge \boldsymbol{Re2} \longmapsto_{Adq} S^{\boldsymbol{Re1}}$ is in agreement with condition (i) of Definition 11. It follows from Fact 6 that the relational structure $\boldsymbol{Re1}$ is adequate to the structure of tuples $S^{\boldsymbol{Re1}}$ and that the relational structure $\boldsymbol{Re2}$ is adequate to the structure of tuples $S^{\boldsymbol{Re2}}$, thus condition (ii) of Definition 11 is also satisfied, i.e. $\boldsymbol{Re1} \longmapsto_{Adq} S^{\boldsymbol{Re1}} \wedge \boldsymbol{Re2} \longmapsto_{Adq} S^{\boldsymbol{Re2}}$. Hence, the relational structures $\boldsymbol{Re1}$ and $\boldsymbol{Re2}$ are adequate.

($\Leftarrow$) Let the relational structure $\boldsymbol{Re1}$ be adequate to the relational structure $\boldsymbol{Re2}$. When in Definition 11 instead of X, V, Y, Z we accept, in turn, the objects $\boldsymbol{Re1}, S^{\boldsymbol{Re1}}, \boldsymbol{Re2}, S^{\boldsymbol{Re2}}$, then from condition (ii) of this definition, it follows directly that: $\boldsymbol{Re1} \longmapsto_{Adq} S^{\boldsymbol{Re2}} \wedge \boldsymbol{Re2} \longmapsto_{Adq} S^{\boldsymbol{Re1}}$. ■

Theorem 7.

If the base structures $BS^{\boldsymbol{B(Re1)}}$ and $BS^{\boldsymbol{B(Re2)}}$ are isomorphic, then the relational structures $\boldsymbol{Re1}$ and $\boldsymbol{Re2}$ are adequate.

Proof.

Let the formula P_1 determine the function establishing the isomorphism of the base structure $BS^{\boldsymbol{B(Re1)}}$ onto $BS^{\boldsymbol{B(Re2)}}$, and let the formula P_2 be determined by the equivalence: $P_2(t,t') \Leftrightarrow P_1(t',t)$. The formula P_2 defines the function which establishes the isomorphism $BS^{\boldsymbol{B(Re2)}}$ onto $BS^{\boldsymbol{B(Re1)}}$.

For the formulas determined by the equations:

$$BS^{\boldsymbol{B(Re1)}} = \{t' : \exists t \in BS^{\boldsymbol{B(Re2)}} P_2(t,t')\},$$
$$BS^{\boldsymbol{B(Re2)}} = \{t' : \exists t \in BS^{\boldsymbol{B(Re1)}} P_1(t,t')\},$$

we have:

$$BS^{B(\boldsymbol{Re1})} \longmapsto_{Adq} BS^{B(\boldsymbol{Re2})} \text{ and } BS^{B(\boldsymbol{Re2})} \longmapsto_{Adq} BS^{B(\boldsymbol{Re1})}.$$

Since for the definition formula which determines the base structure for the relational structure we have:

$$\boldsymbol{Re1} \longmapsto_{Adq} BS^{B(\boldsymbol{Re1})} \text{ and } \boldsymbol{Re2} \longmapsto_{Adq} BS^{B(\boldsymbol{Re2})},$$

thus

1. $\boldsymbol{Re1} \longmapsto_{Adq} BS^{B(\boldsymbol{Re1})}$,
2. $BS^{B(\boldsymbol{Re1})} \longmapsto_{Adq} BS^{B(\boldsymbol{Re2})}$,
3. $BS^{B(\boldsymbol{Re2})} \longmapsto_{Adq} S^{\boldsymbol{Re2}}$,
4. $\boldsymbol{Re2} \longmapsto_{Adq} BS^{B(\boldsymbol{Re2})}$,
5. $BS^{B(\boldsymbol{Re2})} \longmapsto_{Adq} BS^{B(\boldsymbol{Re1})}$,
6. $BS^{B(\boldsymbol{Re1})} \longmapsto_{Adq} S^{\boldsymbol{Re1}}$.

Hence and from Fact 5b) we obtain, for certain formulas establishing adequacy, the following relations referring to adequacy:

7. $\boldsymbol{Re1} \longmapsto_{Adq} S^{\boldsymbol{Re2}}$,
8. $\boldsymbol{Re2} \longmapsto_{Adq} S^{\boldsymbol{Re1}}$.

Thus, on the basis of Theorem 6 as well as the formulas 7 and 8 it follows that relational structures ***Re1*** and ***Re2*** are adequate. ■

There follows directly from Theorems 5 and 6:

Theorem 8.

If the two structures of tuples $S^{\boldsymbol{Re1}}$ and $S^{\boldsymbol{Re2}}$ are isomorphic, then the relational structures ***Re1*** *and* ***Re2*** *are adequate. Symbolically:*

$$S^{\boldsymbol{Re1}} \approx_{isom} S^{\boldsymbol{Re2}} \Rightarrow \boldsymbol{Re1} \approx_{Adq} \boldsymbol{Re2}.$$

Theorem 9.

If the two relational structures ***Re1*** *and* ***Re2*** *are isomorphic, then the relational structure* ***Re1*** *is adequate to the structure of tuples $S^{\boldsymbol{Re2}}$, as well as the relational structure* ***Re2*** *is adequate to the structure of tuples $S^{\boldsymbol{Re1}}$. Symbolically:*

$$\boldsymbol{Re1} \sim_{isom} \boldsymbol{Re2} \Rightarrow \boldsymbol{Re1} \longmapsto_{Adq} S^{\boldsymbol{Re2}} \wedge \boldsymbol{Re2} \longmapsto_{Adq} S^{\boldsymbol{Re1}}.$$

Proof.

Let the relational structures ***Re1*** and ***Re2*** be isomorphic. Hence and from Theorem 4 and Definition 10 it follows that the base structures $BS^{B(\boldsymbol{Re1})}$ and $BS^{B(\boldsymbol{Re2})}$ are isomorphic. Thus, on the strength of Theorem 7

it follows that the relational structures ***Re1*** and ***Re2*** are adequate. The thesis of the theorem is obtained on the basis of Theorem 6. ■

From Theorem 9 and Theorem 6 there follows:

Corollary 2.

If the two relational structures ***Re1*** *and* ***Re2*** *are isomorphic, then the relational structures* ***Re1*** *and* ***Re2*** *are adequate. Symbolically:*

$$\boldsymbol{Re1} \sim_{isom} \boldsymbol{Re2} \Rightarrow \boldsymbol{Re1} \approx_{Adq} \boldsymbol{Re2}.$$

Theorem 10.

If the relational structure ***Re1*** *is isomorphically embedded in the relational structure* ***Re2****, then in the relational structure* ***Re2*** *there exists such relational substructure* ***Re'2*** *that the relational structures* ***Re'2*** *and* ***Re1*** *are adequate. Symbolically:*

$$\boldsymbol{Re1} \sim_{emb} \boldsymbol{Re2} \Rightarrow \exists \boldsymbol{Re'2} \subset_p \boldsymbol{Re2}\ (\boldsymbol{Re'2} \approx_{Adq} \boldsymbol{Re1})^2.$$

Proof.

Let the relational structure ***Re1*** be embedded isomorphically in the relational structure ***Re2***. Hence, it follows that in the relational structure ***Re2*** there exists a relational substructure which is isomorphic with the structure ***Re1***. We denote this relational substructure by ***Re'2***. Since the relational structures ***Re1*** and ***Re'2*** are isomorphic, it follows from Corollary 2 that the relational structures ***Re'2*** and ***Re1*** are adequate. ■

Theorem 11.

If the two structures of tuples $S^{\boldsymbol{Re1}}$ and $S^{\boldsymbol{Re2}}$ are isomorphic and also if there exists a well-formed formula which establishes, in an unambiguous manner, the isomorphism of the structures of tuples ***Re1*** *and* ***Re2****, then the structures of tuples $S^{\boldsymbol{Re1}}$ and $S^{\boldsymbol{Re2}}$ are adequate. Symbolically:*

$$S^{\boldsymbol{Re1}} \approx_{isom} S^{\boldsymbol{Re2}} \Rightarrow S^{\boldsymbol{Re1}} \approx_{Adq} S^{\boldsymbol{Re2}}.$$

Proof.

Let the structures of tuples $S^{\boldsymbol{Re1}}$ and $S^{\boldsymbol{Re2}}$ be isomorphic. Each structure of tuples is identical to itself and from Fact 5a) it follows that:

$$S^{\boldsymbol{Re1}} \longmapsto_{Adq} S^{\boldsymbol{Re1}} \text{ and } S^{\boldsymbol{Re2}} \longmapsto_{Adq} S^{\boldsymbol{Re2}}.$$

Since the structures of tuples $S^{\boldsymbol{Re1}}$ and $S^{\boldsymbol{Re2}}$ are isomorphic and there exists a well-formed formula which establishes the isomorphism of structures of tuples $S^{\boldsymbol{Re1}}$ and $S^{\boldsymbol{Re2}}$, hence:

---

[2] The notation: ***Re'2*** $\subset_p$ ***Re2*** was applied as an indication that ***Re'2*** is a relational substructure of the relational structure ***Re2***.

$$S^{Re1} \longmapsto_{Adq} S^{Re2} \text{ and } S^{Re2} \longmapsto_{Adq} S^{Re1}.$$

We showed that Conditions (i) and (ii) of Definition 11 are satisfied. Then the structures of tuples S^{Re1} and S^{Re2} are adequate. ■

Now we shall formulate certain theorems on the adequacy of relational structures which have the same universe.

Theorem 12.

If the base $B(Re1)$ *is a subbase of the base* $B(Re2)$ *and* $BS^{B(Re2)} \subseteq S^{Re1}$*, then relational structures* **Re1** *and* **Re2** *are adequate.*

Proof.

Let the base $B(Re1)$ be a subbase of the base $B(Re2)$ and $BS^{B(Re2)} \subseteq S^{Re1}$.

From the fact that the base $B(Re1)$ is a subbase of the base $B(Re2)$ and from Fact 1 it follows that $BS^{B(Re1)} \subseteq BS^{B(Re2)}$. Let us analyze the following two cases:

a) $BS^{B(Re1)} = BS^{B(Re2)}$,

b) $BS^{B(Re1)} \subset BS^{B(Re2)}$.

The proof in the case of a) is obvious. Let us analyze case b). From the assumption that the base $B(Re1)$ is a subbase of the base $B(Re2)$ and on the strength of Fact 4 we have: $S^{Re1} \subseteq S^{Re2}$.

From the assumption of the theorem: $BS^{B(Re2)} \subseteq S^{Re1}$, as well as from Definiction 8 it follows that the base structure $BS^{B(Re1)}$ generates tuples belonging to the base structure $BS^{B(Re2)}$ according to the Conditions C1-C4 of this definition. Thus, $BS^{B(Re1)}$ determines S^{Re2}. Thus, the structure of tuples S^{Re2}, which is obtained from the base structure $BS^{B(Re2)}$ according to the conditions C1-C4 of Definition 8, is included in the structure of tuples S^{Re1}.

Since $S^{Re1} \subseteq S^{Re2}$ and $S^{Re2} \subseteq S^{Re1}$, then $S^{Re1} = S^{Re2}$. Hence and from Corollary 1 and Theorem 6 it follows that the relational structures **Re1** and **Re2** are adequate. ■

Theorem 13.

If $BS^{B(Re2)} \subseteq S^{Re1}$ *and* $BS^{B(Re1)} \subseteq S^{Re2}$*, then relational structures* **Re1** *and* **Re2** *are adequate.*

Proof.

From the assumption that $BS^{B(Re2)} \subseteq S^{Re2}$ and from Definition 8 it follows that the base structure $BS^{B(Re1)}$ generates tuples belonging to the base structure $BS^{B(Re2)}$ according to the Conditions C1-C4 of this definition. Thus, $BS^{B(Re1)}$ determines S^{Re2}. Hence, the structure of tuples S^{Re2},

which is determined by the base structure $BS^{B(Re2)}$ according to the Conditions C1-C4 of Definition 8, is included in the structure of tuples S^{Re1}.

In an analogous way, we show that $S^{Re1} \subseteq S^{Re2}$. Because $S^{Re1} \subseteq S^{Re2}$ and $S^{Re2} \subseteq S^{Re1}$, then $S^{Re1} = S^{Re2}$. Hence, from Corollary 1 and Theorem 6 it follows that the relational structures **Re1** and **Re2** are adequate. ■

4. Ontological structures of an object and the ontological problem of the adequacy

Finally, we shall discuss the problem area connected with applications of the theory of the adequacy, taking as an example selected applications of this theory in ontology. First, let us observe that there have continuously been undertaken certain attempts at a precise determination of the notion of the substantial structure of object (structure of the matter, physical structure of bodies, chemical structure of substances, etc.), that is the notion of the ontological structure of the object. Making reference to certain theoretical propositions by J. Perzanowski (2004) and the concepts of adequacy introduced in the present paper, we can propose the following description of this notion:

Through the ***ontological structure of an object*** *we shall understand a structure described by the general theory of analysis and synthesis relativised to description of this object.*

The general theory of analysis and synthesis was formulated by Perzanowski (2004). In the sense of this theory, elements (components) of an object are the objects into which the given object can be decomposed or from which this object can be composed. The predicate that corresponds in this sense to being a component of the object is denoted by "$\prec$". The expression "$x \prec y$" is read: *the object x is a component of the object y or the object x is simpler, than the object y, its component.*

Let the object o be a distinguished object, whose structure we describe. Let us make the description relative only to components of this object. We accept, thus, that all the considered objects are components of the object o, and hence $\forall x\ (x \prec o)$.

Let us consider the following notions:

The object x is a superelement of the object o (symbolically: $SE(x)$) iff

$(SE) \quad \forall y\ (x \prec y)$.

The object x is a simple component of the object o (symbolically: $S(x)$) iff

(S) $\forall y\ (\neg y \prec x)$.

The object x is an atom of the object o (symbolically: $A(x)$) iff

(A) $\forall y\ (y \prec x \Rightarrow x = y)$.

The object x is an element of the object o (symbolically: $E(x)$) iff

(E) $\forall y\ (y \prec x \Rightarrow x = y \vee SE(x))$.

The ontological structure of the object o is well defined, when for any condition $\alpha \in \{(SE), (S), (A), (E)\}$ there exists such a component x of the object o that $\alpha(x)$ is satisfied.

In order to precisely verify the knowledge about whether the ontological structure of the object o is well described, in other words, to answer the question whether this knowledge is adequate, the best thing to do is to interpret this knowledge on the basis of formal ontology, e.g. the set theory.

Thus, let U^o be the set of all components of the object o possible to be distinguished in the process of conceptualization or through experience by means of analysis or synthesis. A descriptive model of the ontological structure of the object o will then be described set-theoretically by the relational structure $\boldsymbol{Re}^o = \langle U^o, <, \subseteq\rangle$, where "$<$" denotes a *relation of being a component in the process of analysis*, and "$\subseteq$" denotes a *relation of being a component in the process of synthesis*. Predicate "$\prec$" will be interpreted as the first or the second relation ($x \prec y \Leftrightarrow x \subseteq y \vee x < y$). Because the appearance of objects in the process of synthesis and their decomposition into components in the process of analysis denotes that the components are connected to form tuples (chains), and relationships among the elements (links) of tuples (chains) are determined through analysis and synthesis. The set of all such tuples S^o is a certain structure of tuples. This structure, set by experience, can be or may not be determined by means of a theoretical model $\boldsymbol{Re}^o$. In other words: theoretical knowledge can be adequate to experimental knowledge or not. However, we can verify it precisely on the ground of formal ontology. Therefore there appears the following question: **is structure $\boldsymbol{Re}^o$ adequate to structure S^o?** We identify this question with the **ontological problem of the adequacy**. Offering, on the grounds of the theory of adequacy, some criteria of adequacy that are necessary to solve this problem, goes considerably beyond the intended framework of this paper.

References

Marciszewski W. (Ed.), 1987, *Formal Logic. An Encyclopedic Outline* (*Logika formalna. Zarys encyklopedyczny*), PWN, Warszawa;

Marciszewski W. (Ed.), *Dictionary of Logic as Applied to the Study of Language: Concepts, Methods, Theories*, Marrtinus Nijhoff, Haga 1981.

Perzanowski J., 2004, 'Towards Combination Metaphysics', *Reports on Mathematical Logic*, No 38, Cracow: Jagiellonian University Press, pp. 93–116.

Putnam H., 1989, *Representation and Reality*, Bradford Books, The MIT Press.

Wójcicki R., 1991, *Theories in Science. Introduction to Logics, Methodology and Philosophy of Science Teorie w nauce* (*Wstęp do logiki, metodologii i filozofii nauk*), volume 1, Warsaw.

Wójcicki R., 1996, 'From Tarski to Davidson' ('Od Tarskiego do Davidsona'), in: Żegleń U. (Ed.) *Discussions with Donald Davidson about truth, language and mind* (Dyskusje z Donaldem Davidsonem o prawdzie, języku i umyśle), Lublin, pp. 65–68.

Jacek Waldmajer
Section of Communication and Logic
University of Opole
e-mail: jwaldmajer@uni.opole.pl